Journal of Siberian Federal University. Mathematics & Physics 2013, 6(2), 150–156

УДК 517.55

Mellin Transform for Monomial Functions of the Solution to the General Polynomial System

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Received 10.12.2012, received in revised form 10.01.2013, accepted 20.01.2013

In the present paper we give the calculation of Mellin transform for the monomial function of the vector-solution to the general polynomial system. We essentially use linearization of the system. In scalar case it defines bijective change of variables. In case of the system of equations we weaken requirements on the mapping given by the linearization: it is proper and its degree is equal to one.

Keywords: Mellin transform, algebraic equation.

Introduction

Consider a polynomial map

$$P = (P_1, \dots, P_n) : \mathbb{C}^n \to \mathbb{C}^n.$$

We assume that the sets $A^{(i)} \subset \mathbb{Z}_+^n$ of exponents of the monomials in polynomials P_i are fixed while all the coefficients vary. Then we say that P is a general polynomial map from $\mathbb{C}^n$ to $\mathbb{C}^n$. It defines a *general system of polynomial equations* of the form

$$\sum_{\lambda \in A^{(i)}} a_\lambda^{(i)} y^\lambda = 0, \ i = 1, \dots, n \tag{1}$$

with unknown $y = (y_1, \dots, y_n) \in \mathbb{C}^n$ and variable coefficients $a_\lambda^{(i)}$, where $A^{(i)} \subset \mathbb{Z}_+^n$ are fixed finite subsets, $\lambda = (\lambda_1, \dots, \lambda_n)$, $y^\lambda = y_1^{\lambda_1} \dots y_1^{\lambda_n}$.

In 1921 Mellin [1] presented an integral formula and series expansion for the positive power $y^\mu(x)$ of the function $y(x)$ defined by the general (reduced) algebraic equation

$$y^m + x_1 y^{\lambda^1} + \dots + x_p y^{\lambda^p} - 1 = 0, \tag{2}$$

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$m > \lambda^1 > \cdots > \lambda^p \geqslant 1$. This result was extensively applied by the Krasnoyarsk School (see, for example [2],[3]) in the investigation of the monodromy of the general algebraic function.

The aim of our study is to calculate the Mellin transform for monomial function

$$\frac{1}{y^{\mu}(-x)} := \frac{1}{y_1^{\mu_1}(-x)\cdots y_n^{\mu_n}(-x)},\ \mu = (\mu_1, \ldots, \mu_n),\ \mu_i > 0, \tag{3}$$

composed of coordinates of the solution $y(x)$ to the *reduced polynomial system* of the form

$$y^{d^{(i)}} + \sum_{\lambda\in\Lambda^{(i)}} x_{\lambda}^{(i)} y^{\lambda} - 1 = 0,\ \ i = 1, \ldots, n, \tag{4}$$

where the matrix of the columns of distinguished exponents $\left(d^{(1)}, \ldots, d^{(n)}\right) =: D$ is nondegenerate and $\Lambda^{(i)} := A^{(i)} \setminus \left\{d^{(i)}, 0\right\}$. As a rule, the system (1) can be reduced to the form (4) (see [4]). Now we consider the case of diagonal matrix D with positive integer diagonal elements $m_1, \ldots, m_n$. Let Λ be the disjunctive union of $\Lambda^{(i)}$, and let N be the cardinality of Λ, i.e., the number of coefficients in system (4). The set of these coefficients is a vector space $\mathbb{C}^{\Lambda} \cong \mathbb{C}_x^{\Lambda}$, where the coordinates of points $x = (x_\lambda)$ are indexed by the elements $\lambda \in \Lambda$. We usually distinguish the group of coordinates corresponding to the indices $\lambda \in \Lambda^{(i)}$ by writing $x^{(i)}$. Let $\sharp\Lambda^{(i)}$ be the cardinality of the set $\Lambda^{(i)}$. We will assume that all sets $\Lambda^{(i)}$ lie in the interior of the simplex with vertices $0, m_1e_1, \ldots, m_ne_n$, where $e_1, \ldots, e_n$ is a basis in $\mathbb{Z}^n$. In other words all points $\lambda \in \Lambda$ have nonzero coordinates and satisfy the following condition

$$\sum_{i=1}^{n} \left\langle \widetilde{d}^{(i)}, \lambda \right\rangle < 1, \tag{5}$$

where $\widetilde{d}^{(i)}$ are columns of the matrix $\widetilde{D} := D^{-1}$.

In Section 1 we discuss the definition and the Jacobian of linearization $(\xi, W) \to (x, y)$ of system (4) which is the main tool in calculations of the Mellin transform for the monomial function (3). Section 2 contains the main results of the present paper. The first one is Theorem 1 which states that the mapping Φ defined by the linearization $(\xi, W) \to (x, y)$ is proper and its degree $\deg \Phi$ is equal to one. The second one is Theorem 2 which gives the Mellin transform for the monomial function (3).

1. Linearization of the system (4)

Let $\mathbb{T}^n$ be the complex algebraic torus. We regard (4) as a system of equations in the space $\mathbb{C}^{\Lambda} \times \mathbb{T}^n$ with coordinates $x = \left(x_{\lambda}^{(i)}\right)$, $y = (y_1, \ldots, y_n)$ and introduce a change of variables $(\xi, W) \to (x, y)$ in $\mathbb{C}^{\Lambda} \times \mathbb{T}^n$ by putting

$$\begin{aligned} &x_{\lambda}^{(i)} = -\xi_{\lambda}^{(i)} W^{-\widetilde{D}\lambda},\ \ \lambda \in \Lambda^{(i)},\ \ i = 1, \ldots, n,\\ &y = W^{\widetilde{D}}, \end{aligned} \tag{6}$$

where $W = (W_1, \ldots, W_n)$, $W^{-\widetilde{D}\lambda} = \prod\limits_{i=1}^{n} W_i^{-\left\langle \widetilde{d}^{(i)}, \lambda\right\rangle}$, $W^{\widetilde{D}} = \left(W^{\widetilde{d}^{(1)}}, \ldots, W^{\widetilde{d}^{(n)}}\right)$, $\xi = \left(\xi_{\lambda}^{(i)}\right)$.

Therefore we can write (4) as a system of linear equations

$$W_i = 1 + \sum_{\lambda\in\Lambda^{(i)}} \xi_{\lambda}^{(i)},\ \ i = 1, \ldots, n. \tag{7}$$

If we define a change of variables $\xi \to x$ in the space of coefficients $\mathbb{C}^{\Lambda}$ by the formula

$$x_{\lambda}^{(i)} = \xi_{\lambda}^{(i)} \prod_{k=1}^{n} \left(1 + \sum_{\tau \in \Lambda^{(k)}} \xi_{\tau}^{(k)}\right)^{-\left\langle \widetilde{d}^{(k)}, \lambda \right\rangle}, \quad \lambda \in \Lambda^{(i)}, \ i = 1, \dots, n, \tag{8}$$

then vector $y\left(-x(\xi)\right)$ takes the following form

$$y\left(-x(\xi)\right) = \left(W(\xi)\right)^{\widetilde{D}}, \tag{9}$$

where $W(\xi) = \left(W_1\left(\xi^{(1)}\right), \dots, W_n\left(\xi^{(n)}\right)\right)$ is formed from the linear functions (7).

The idea of linearizing an algebraic equation by using a change of variables of this type is due to Mellin. It was realized by Mellin in [1] (see also [5]) in order to obtain the integral representation and series expansion for the solution of algebraic equation (2). In [6] this trick was applied to the "triangular" system when the first equation depends only on y_1, the second one depends only on y_1, y_2 and so on. Analogous linearization of the system (4) was given in [7] with a view to calculate Mellin transform for the monomial function of the following type

$$y^{\mu}(x) := y_1^{\mu_1}(x) \cdots y_n^{\mu_n}(x), \ \mu = \left(\mu_1, \dots, \mu_n\right), \ \mu_i > 0.$$

The main result of [7] is the power series expansion of the function $y^{\mu}(x)$. This expansion was given using the formal calculation of the Mellin transform for $M\left[y^{\mu}(x)\right]$.

Further we need to consider the restriction Φ of the mapping $\mathbb{C}_{\xi}^{N} \to \mathbb{C}_{x}^{N}$ given by (8) on the positive orthant $\mathbb{R}_{+}^{N}$.

Lemma 1. *The Jacobian of the mapping Φ is equal to*

$$\frac{\partial(x)}{\partial(\xi)} = \prod_{i=1}^{n} W_i^{-\sum\limits_{\lambda \in \Lambda^{(i)}} \left\langle \widetilde{d}^{(i)}, \lambda \right\rangle - 1} \left(1 + \sum_{q=1}^{n} \sum_{|I|=q} \sum_{\tau \in \Lambda^{I}} \begin{vmatrix} 1 - \frac{\tau_{i_1}^{i_1}}{m_{i_1}} & \dots & -\frac{\tau_{i_q}^{i_1}}{m_{i_1}} \\ \dots & \dots & \dots \\ -\frac{\tau_{i_1}^{i_q}}{m_{i_q}} & \dots & 1 - \frac{\tau_{i_q}^{i_q}}{m_{i_q}} \end{vmatrix} \xi_{\tau}\right), \tag{10}$$

where I is an odered set $1 \leqslant i_1 < \dots < i_q \leqslant n$, $\tau = \left(\tau^{i_1}, \dots, \tau^{i_q}\right)$ is an element of Cartesian product $\Lambda^{I} = \Lambda^{(i_1)} \times \dots \times \Lambda^{(i_q)}$, and $\xi_{\tau} = \prod\limits_{i \in I} \xi_{\tau^{i}}$.

Proof. The Jacobi matrix of the change of variables (8) has a block structure with n^2 blocks. There are square blocks on the diagonal of the matrix. The diagonal elements of ith diagonal block are of the following type

$$\frac{\partial x_{\lambda}^{(i)}}{\partial \xi_{\lambda}^{(i)}} = \left(1 - \left\langle \widetilde{d}^{(i)}, \lambda \right\rangle \xi_{\lambda}^{(i)} W_i^{-1}\right) W^{-\widetilde{D}\lambda};$$

the elements outside the diagonal in this block are

$$\frac{\partial x_{\lambda}^{(i)}}{\partial \xi_{\tau}^{(i)}} = -\left\langle \widetilde{d}^{(i)}, \lambda \right\rangle \xi_{\lambda}^{(i)} W_i^{-1} W^{-\widetilde{D}\lambda}.$$

The nondiagonal (i,k)-block contains the following elements

$$\frac{\partial x_{\lambda}^{(i)}}{\partial \xi_{\tau}^{(k)}} = -\left\langle \widetilde{d}^{(i)}, \lambda \right\rangle \xi_{\lambda}^{(i)} W_i^{-1} W^{-\widetilde{D}\lambda}, \ k \neq i.$$

The computation gives us

$$\frac{\partial(x)}{\partial(\xi)} = \prod_{i=1}^{n} W_i^{-\sum\limits_{\lambda\in\Lambda^{(i)}} \left\langle \widetilde{d}^{(i)},\lambda\right\rangle -1} \det\left(\delta_k^j W_j - \sum_{\lambda\in\Lambda^{(k)}} \left\langle \widetilde{d}^{(j)},\lambda\right\rangle \xi_\lambda^{(k)}\right)_{j,k=\overline{1,n}},$$

where δ_k^j is the Kronecker symbol, $j,k = 1,\dots,n$.

The last determinant may be reduced by the formula:

$$\det(E+A) = 1 + \sum_{q=1}^{n}\sum_{|I|=q} \begin{vmatrix} A_{i_1}^{i_1} & \dots & A_{i_q}^{i_1} \\ \dots & \dots & \dots \\ A_{i_1}^{i_q} & \dots & A_{i_q}^{i_q} \end{vmatrix},$$

where E is an unit matrix, $A = (A_i^j)$ is an arbitrary matrix of order n, and I is an ordered set $1 \leqslant i_1 < \ldots < i_q \leqslant n$.

All that now remains to be written is that

$$\frac{\partial(x)}{\partial(\xi)} = \prod_{i=1}^{n} W_i^{-\sum\limits_{\lambda\in\Lambda^{(i)}} \left\langle \widetilde{d}^{(i)},\lambda\right\rangle -1} \left(1 + \sum_{q=1}^{n}\sum_{|I|=q} \begin{vmatrix} A_{i_1}^{i_1} & \dots & A_{i_q}^{i_1} \\ \dots & \dots & \dots \\ A_{i_1}^{i_q} & \dots & A_{i_q}^{i_q} \end{vmatrix}\right),$$

where $A_{i_l}^{i_r} = \sum\limits_{\lambda\in\Lambda^{(i_r)}} \left(\delta_l^r - \left\langle \widetilde{d}^{(i_l)},\lambda\right\rangle\right)\xi_\lambda^{(i_r)}$, $r,l = 1,\dots,q$, and δ_l^r is the Kronecker symbol. Hence we get the desired formula (10). □

2. Mellin transform for the function $\frac{1}{\mathbf{y}^\mu(-\mathbf{x})}$

We consider the monomial function (3) where $y_j(-x)$ are branches with conditions $y_i(0) = 1$, $i = 1,\dots,n$. Let us recall that Mellin transform of the function $\frac{1}{y^\mu(-x)}$ is defined by the following integral

$$M\left[\frac{1}{y^\mu(-x)}\right](z) = \int_{\mathbb{R}_+^N} \frac{1}{y^\mu(-x)} x^{z-I}\,dx, \tag{11}$$

where $x^{z-I} = x_1^{z_1-1}\cdots x_N^{z_N-1}$, $dx = dx_1\cdots dx_N$ (see, for example [5]). To calculate the integral (11) we consider the transformation $\xi \to x$ (or a mapping $\Phi : \mathbb{R}_+^N \to \mathbb{R}_+^N$) given by (8).

Theorem 1. *The mapping* Φ *is proper. Its degree* $\deg\Phi$ *is correctly defined and* $\deg\Phi = 1$.

Proof. We prove that $\Phi(\partial\mathbb{R}_+^N) = \partial\mathbb{R}_+^N$. Further we miss the upper index in the notations $x_\lambda^{(i)}, \xi_\lambda^{(i)}$, and write x_λ, ξ_λ for simplicity. Note that any coordinate plane $\xi_\lambda = 0$ is mapped to coordinate plane $x_\lambda = 0$. Moreover, the boundary points of the orthant are mapped only to the boundary points. If a sequence $\xi^{(k)} \in \mathbb{R}_+^N$, $k \in \mathbb{N}$, converges to the boundary point $\xi \in \partial\mathbb{R}_+^N$ then the sequence of images $\Phi(\xi^{(k)})$, $k \in \mathbb{N}$, also converges to a boundary point of the orthant. We are now going to prove that condition $\xi^{(k)} \to +\infty$ implies $x^{(k)} = \Phi(\xi^{(k)}) \to +\infty$. Note that coordinate $x_\lambda^{(k)}$ may be finite in case when the corresponding coordinate $\xi_\lambda^{(k)}$ tends to $+\infty$, but

$$\left|x^{(k)}\right| := \sum_{\lambda\in\Lambda} x_\lambda^{(k)} \to +\infty$$

when

$$\left|\xi^{(k)}\right| = \sum_{\lambda\in\Lambda} \xi_\lambda^{(k)} \to +\infty.$$

Using assumption (5) for any $\lambda \in \Lambda$ one can choose such a real positive n-dimensional vectors $r^\lambda = (r_j^\lambda)$, $p^\lambda = (p_j^\lambda)$ that

$$\left\langle \widetilde{d}^{(j)}, \lambda \right\rangle + r_j^\lambda = p_j^\lambda,\ |p^\lambda| = 1. \tag{12}$$

Using the well-known Jensen inequality

$$a_1^{p_1} \dots a_n^{p_n} \leqslant p_1 a_1 + \dots + p_n a_n,$$

which is valid for any positive numbers $a_1, \dots, a_n$, $p_1, \dots, p_n$, $\sum p_i = 1$, and conditions (12) we get the following estimates:

$$\sum_{\lambda\in\Lambda} x_\lambda^{(k)} = \sum_{\lambda\in\Lambda} \xi_\lambda^{(k)} \left(W(\xi^{(k)})\right)^{-\widetilde{D}\lambda} = \sum_{\lambda\in\Lambda} \xi_\lambda^{(k)} \left(W(\xi^{(k)})\right)^{r^\lambda} \left(W(\xi^{(k)})\right)^{-p^\lambda} \geqslant$$

$$\geqslant \sum_{\lambda\in\Lambda} \xi_\lambda^{(k)} \frac{\left(W(\xi^{(k)})\right)^{r^\lambda}}{\langle p^\lambda, W(\xi^{(k)})\rangle} \geqslant \left(W(\xi^{(k)})\right)^r \sum_{\lambda\in\Lambda} \frac{\xi_\lambda^{(k)}}{\langle p^\lambda, W(\xi^{(k)})\rangle} \geqslant \left(W(\xi^{(k)})\right)^r \frac{\sum\limits_{\lambda\in\Lambda} \xi_\lambda^{(k)}}{1 + \sum\limits_{\lambda\in\Lambda} \xi_\lambda^{(k)}}, \tag{13}$$

where $r = (r_1, \dots, r_n)$, $r_j = \min\limits_{\lambda\in\Lambda} r_j^\lambda$. If $\sum\limits_{\lambda\in\Lambda} \xi_\lambda^{(k)} \to +\infty$, then the last term in (13) tends to infinity also. So, we proved that mapping Φ is proper. In spherical compactification of the space $\mathbb{R}^N$ the closure of the orthant $\mathbb{R}_+^N$ is a manifold with a piecewise smooth boundary $\partial\mathbb{R}_+^N$. Consequently, the mapping Φ transforms a manifold with boundary to a manifold with boundary.

As the mapping Φ is proper, it follows that for any inner noncritical value $x \in \left(\mathbb{R}_+^N\right)^o$ full preimage $\Phi^{-1}(x)$ being discrete and compact is a finite set. Hence one can define the degree of the mapping $\Phi : \mathbb{R}_+^N \to \mathbb{R}_+^N$ as follows

$$\deg_x \Phi = \sum_{\xi:\Phi(\xi)=x} \operatorname{sgn} \frac{\partial(x)}{\partial(\xi)}. \tag{14}$$

The degree does not depend on the choice of noncritical value $x \in \mathbb{R}_+^N$. To prove that we fix an arbitrary bounded neighborhood $V \subset \mathbb{R}_+^N$ of the point $x \in \mathbb{R}_+^N$ and a set U such that $V \subset \Phi(U)$. One can represent the degree $\deg_x \Phi$ by the following integral

$$\deg_x \Phi = \int_{\partial U} \omega(\Phi(\xi) - x),$$

where ω is the Poincaré form (see [8, II, Ch. 3]). This integral takes integer values and continuously depends on $x \in V$ therefore $\deg \Phi$ is a constant. So we proved that degree of the mapping Φ does not depend on the choice of noncritical point x.

All that now remains to be shown is that $\deg\Phi = 1$. In order to do that we use the following fact: the degree of the restriction $\Phi|_{\partial\mathbb{R}_+^N}$ coincides with the degree $\deg\Phi$ (see [8, II, Ch. 3]). We apply induction on dimension N. In the case $N = 1$ the mapping Φ is

$$x = \xi(1+\xi)^{-r},\ 0 < r < 1. \tag{15}$$

The function (15) is increasing so it implies that $\deg \Phi = 1$. In the case $N = 2$ the restriction $\Phi|_{\xi_\lambda = 0}$ also has the form (15) therefore $\deg \Phi|_{\xi_\lambda=0} = 1$. So the degree of the mapping $\Phi : \mathbb{R}_+^2 \to$

$\mathbb{R}^2_+$ is equal to one also. For an arbitrary dimension N the restriction of the mapping Φ on every face $\{\xi_\lambda = 0\}$ of the orthant $\mathbb{R}^N_+$ remains in the same class of the mappings. By inductive assumption we conclude that $\deg\Phi = 1$. $\square$

We shall interpret the set Λ as a matrix $\Lambda = \left(\Lambda^{(1)}, \ldots, \Lambda^{(n)}\right) = \left(\lambda^1, \ldots, \lambda^N\right)$, whose columns are the vectors $\lambda^k = \left(\lambda^k_1, \ldots, \lambda^k_n\right)$ of exponents of the monomials in (4). Here the ordering of the columns λ^k inside each block $\Lambda^{(i)}$ is arbitrary but fixed.

We introduce two $n \times N$ matrices $\Psi := \widetilde{D}\Lambda$, $\widetilde{\Psi} := \Psi - \chi$, where χ is the matrix whose ith row χ_i is the characteristic function of the subset $\Lambda^{(i)} \subset \Lambda$, that is, it has 1 on places $\lambda \in \Lambda^{(i)}$ and 0 elsewhere. We denote the rows of the matrices Ψ and $\widetilde{\Psi}$ by $\psi_1, \ldots, \psi_n$ and $\widetilde{\psi}_1, \ldots, \widetilde{\psi}_n$ respectively. For any ordered set $J = \{j_1, \ldots, j_q\}$ $1 \leqslant j_1 < \ldots < j_q \leqslant n$, we fix the set of columns $\tau = \left(\tau^{j_1}, \ldots, \tau^{j_q}\right) \in \Lambda^J := \Lambda^{(j_1)} \times \ldots \times \Lambda^{(j_q)}$ and introduce $q \times q$-matrix $\Psi_J(\tau)$. Denote by $\Delta_J(\tau)$ the determinant of the matrix $E - \Psi_J(\tau)$, where E is q-order unit matrix. The same determinants $\Delta_I(\tau)$ arise in formula (10).

Theorem 2. *The Mellin transform defined by the integral (11) is equal to*

$$M\left[\frac{1}{y^\mu(-x)}\right](z) = \prod_{j=1}^{n} \frac{\Gamma\left(\left\langle \widetilde{d}^{(j)}, \mu \right\rangle + \left\langle \widetilde{\psi}_j, z \right\rangle\right) \Gamma\left(\langle \chi_j, z\rangle\right)}{\Gamma\left(\left\langle \widetilde{d}^{(j)}, \mu \right\rangle + \langle \psi_j, z\rangle + 1\right)} Q(z), \tag{16}$$

where $Q(z)$ is a polynomial of the form

$$Q(z) = \sum_{q=0}^{n} \sum_{|J|=q} \prod_{j\notin J} \left(\left\langle \widetilde{d}^{(j)}, \mu \right\rangle + \left\langle \widetilde{\psi}_j, z \right\rangle\right) \sum_{\tau \in \Lambda^J} \Delta_J(\tau) z_\tau.$$

The integral (11) converges under the following conditions

$$\operatorname{Re} z_\lambda > 0,\ \lambda \in \Lambda,\ \operatorname{Re}\left(\left\langle \widetilde{d}^{(i)}, \mu \right\rangle + \left\langle \widetilde{\psi}_i, z \right\rangle\right) > 0,\ i = 1, \ldots, n.$$

Proof. Theorem 1 argues validity of the change of variables (8) in the integral (11). Thus it follows from Lemma 1 that the integral (11) in coordinates ξ looks like:

$$\int_{\mathbb{R}^N_+} \left(1 + \sum_{q=1}^{n} \sum_{|J|=q} \sum_{\tau\in\Lambda^J} \Delta_J(\tau)\xi_\tau\right) \frac{\xi^{z-I}\, d\xi}{W^{\widetilde{D}\mu + \Psi z + I}} =$$

$$= \prod_{i=1}^{n} \int_{\mathbb{R}^{\sharp\Lambda^{(i)}}_+} \frac{{\xi^{(i)}}^{z^{(i)}-I}\, d\xi^{(i)}}{W_i^{\left\langle \widetilde{d}^{(i)},\mu\right\rangle + \langle\psi_i, z\rangle + 1}} + \sum_{q=1}^{n} \sum_{|J|=q} \sum_{\tau\in\Lambda^J} \Delta_J(\tau) \times$$

$$\times \prod_{i=1}^{n} \int_{\mathbb{R}^{\sharp\Lambda^{(i)}}_+} \frac{{\xi^{(i)}}^{z^{(i)}-I+\delta^i_J}\, d\xi^{(i)}}{W_i^{\left\langle \widetilde{d}^{(i)},\mu\right\rangle + \langle\psi_i, z\rangle + 1}},\ \delta^i_J = \begin{cases} (1, \ldots, 1), & i \in J, \\ (0, \ldots, 0), & i \notin J. \end{cases} \tag{17}$$

All integrals in (17) may be calculated by the formula (see [9, formula 4.638(2)]):

$$\int_{\mathbb{R}^q_+} \frac{x^{z-I}\, dx}{(1 + x_1 + \cdots + x_q)^s} = \frac{\Gamma(z_1)\cdot \ldots \cdot \Gamma(z_q)\Gamma(s - z_1 - \cdots - z_q)}{\Gamma(s)}. \tag{18}$$

In view of conditions $\operatorname{Re} s > 0$, $\operatorname{Re} z_i > 0$, $i = 1, \ldots, q$ on convergence of the integral (18) the integrals in (17) converge under the following conditions $\operatorname{Re} z_\lambda > 0$, $\lambda \in \Lambda$, $\operatorname{Re}\left(\left\langle \widetilde{d}^{(i)}, \mu \right\rangle + \langle \psi_i, z\rangle\right) > 0$, $i = 1, ..., n$.

All that remains is to apply the formula (18) to the integrals in (17). So we obtain the required expression (16) for the Mellin transform. □

The first author's work was carried out with the financial support of RFBR (grant 11-01-00852) and Ministry of Education and Science of Russian Federation (grant 1.34.11). The second author's work was carried out with the financial support of RFBR (grant 12-01-31021-mol_a).

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Преобразование Меллина мономиальных функций решения общей полиномиальной системы

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В настоящей статье вычисляется преобразование Меллина мономиальной функции решения общей полиномиальной системы. При этом существенно используется линеаризация системы, которая в скалярном случае определяет биективную замену переменной. В случае системы уравнений требования к линеаризации ослаблены: она определяет собственное отображение, степень которого равна единице.

Ключевые слова: преобразование Меллина, алгебраическое уравнение.