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## Accelerated Phase-lock-loop Frequency Control Methods of User's Equipment in Perspective Radio Navigation Systems

**Evgeny V. Kuzmin\***

*Siberian Federal University,  
79 Svobodny, Krasnoyarsk, 660041 Russia <sup>1</sup>*

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*This paper investigates noise-immunity of accelerated phase-lock-loop frequency control algorithms of user equipment in perspective ground-based radio navigation systems. Three algorithms of accelerated phase-lock-loop frequency control are suggested and described. Statistic simulations of signal processing in involved system are given.*

*Key words: Radio navigation, spread-spectrum signal, minimum shift keying, phase-shift discriminator, phase synchronization system, accelerated phase-locked-loop frequency control, phase-tracing error, statistical modeling, quasi-optimal algorithm.*

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### Introduction

Spread spectrum signals with minimum shift keying (MSK) are widely used in modern radio navigation systems (RNS), e.g.: GEOLoc (France). High accuracy of coordinate measuring in the whole RNS working area requires providing phase shift measurements with root-mean-square (RMS) error  $\sigma_{\varphi} \leq 3^\circ$ , when signal-to-noise ratio threshold equals to  $-40 \text{ dB}$  (in the band of MSK-signal). That is why, the meaning of phase-lock-loop frequency control pass band equals to  $0,1 \text{ Hz}$ . Thus, locking time is  $600 \text{ s}$ , and can grow by a factor of 10 under noise and jamming influence [1].

Recently, researchers have shown an increased interest in Kalman filtering, because it can provide high accuracy of phase tracing measurements. But Kalman filter has a significant disadvantage – computational complexity, therefore, in the foreseeable future it can't be used for preprocessing algorithms.

Due to limits in computational technology, it's necessary to investigate phase tracking algorithms with performance objectives: small values of locking time and RMS error. So, the hypothesis that will be tested is that multistage (several meanings of pass band) phase-lock-loop frequency control algorithms can provide adequate accuracy of phase-tracing measurements and greatly smaller locking time. Consequently, investigation of accelerated phase-lock-loop frequency control algorithms with invariable phase shift accuracy is a topical scientific problem.

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\* Corresponding author E-mail address: kuzminev@mail.ru

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## 1. Navigation signal model of perspective RNS

Total realization of received MSK-signal and additive white Gaussian noise (AWGN) can be described as:

$$y(t) = \text{Re}\left\{\dot{S}(t)\exp\left[j\left(2\pi(f_0 \pm F_d)t - \varphi_s\right)\right]\right\} + \xi(t), \quad (1)$$

here  $j$  – imaginary unit;  $f_0$  – carrier frequency;  $F_d$  – Doppler frequency shift;  $\varphi_s$  – starting phase of signal;  $\xi(t)$  – AWGN;  $\dot{S}(t)$  – complex envelope of MSK-signal:

$$\dot{S}(t) = D(t)\sqrt{2P_s}\exp[j\theta(t)], \quad (2)$$

where  $P_s$  – signal's power;  $D(t) = \pm 1$  – information signal;  $\theta(t) = \frac{\pi}{2T} \int_0^t d(t')dt'$  – function which determines angle modulation,  $d(t) = \sum_{i=0}^{N-1} d_i \text{rect}(t - iT)$ ,  $\{d_i\}$  – pseudorandom sequence (PRS) of  $N$ -length,  $T$  – one's bit PRS duration,  $\text{rect}(t)$  – square pulse with  $T$  duration [2].

## 2. Phase synchronization system of MSK-signal receiver

Structural chart of MSK-signal receiver's digital phase-lock-loop frequency control system (PLFS) is presented in Fig. 1. Values  $y_i = y(t_i)$  ( $t_i = i\Delta t$ ,  $i = 0, 1, \dots$ ,  $\Delta t$  – sampling interval) are incoming observations to digital phase-shift discriminator (DPD), formed by analog-digital converter (ADC).

Reference signals of carrier frequency  $\cos \hat{\Phi}_i(k) = \cos(2\pi(f_0 \pm \hat{F}_d(k))t_i)$  and  $\sin \hat{\Phi}_i(k) = \sin(2\pi(f_0 \pm \hat{F}_d(k))t_i)$  come into supporting inputs of DPD. These signals are formed by digital synthesizer (DS) and based on Doppler frequency shift estimation  $\hat{F}_d(k)$  in each  $k$ -period of filtering. Reference signals  $Q_i = \sin \theta_i$  and  $I_i = \cos \theta_i$ , which are synchronous with quadrature components of MSK-signal, are formed by delay lock system. Quadrature components of bandwidth compressing signal (after MSK-detection) are formed by summarizing of multiplications of quadrature components of realization (1) and reference signals  $I_i$ ,  $Q_i$  and integration on intervals  $t \in [kT_p, (k+1)T_p]$ ,  $k = 0, 1, \dots$ , ( $T_p = 40 \text{ ms}$  – MSK-signal's period). Time of one cycle radio-range beacon transmission equals  $T_c = 25T_p$ . Error signal which is proportional to phase mismatch forms in compliance with quasi-optimal algorithm [3]:

$$Z_d(k) = \text{sign}(z_1(k))z_2(k) = \hat{D}(k)z_2(k), \quad (3)$$

where  $\text{sign}(x)$  – sign function,  $\hat{D}(k)$  – estimation of information signal  $D(t)$  on  $k$ -period of filtering,  $z_1(k)$  and  $z_2(k)$  – quadrature components of correlation, computed on interval  $t \in [kT_p, (k+1)T_p]$ . Error signal  $Z_d(k)$  comes into digital filter (DF). Output signal of DF used to control signals  $\cos \hat{\Phi}_i(k)$  and  $\sin \hat{\Phi}_i(k)$  frequencies. When there is no noise, discrimination characteristic can be described as

$$Z_d(\varphi) = \frac{1}{2}M \text{sign}(\cos \varphi) \sin \varphi$$

Structural chart of the DPD is presented in Fig. 2, where  $\times$  – multiplier;  $+$  – adder;  $\Sigma$  – adder accumulator (digital integrator), which interrogated in  $kT_p$  moments,  $k = 0, 1, \dots$ ;  $M = T_p / \Delta t$  – integer.

Normalized discrimination (curves 1, 2) and fluctuation (curve 3) characteristics of DPD are presented in Fig. 3. At that, curve 1 corresponds with no-noise case, and curves 2, 3 present discrimination and fluctuation characteristics respectively. Curves 2, 3 are the statistical simulation

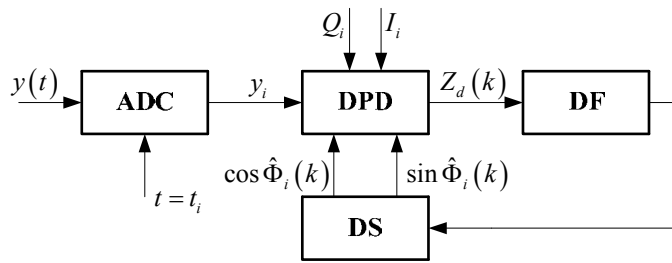


Fig. 1

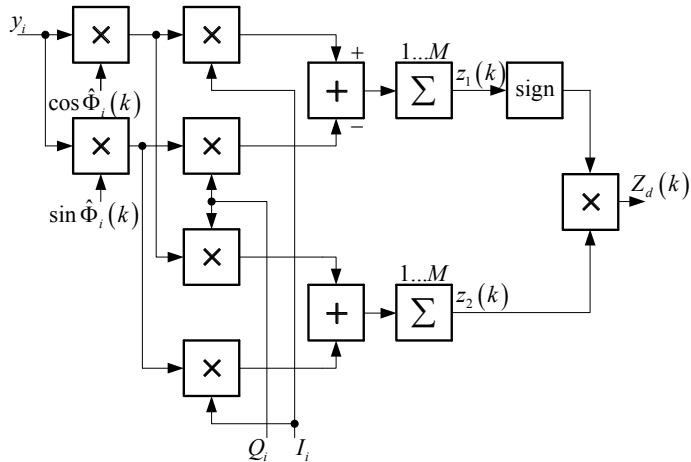


Fig. 2

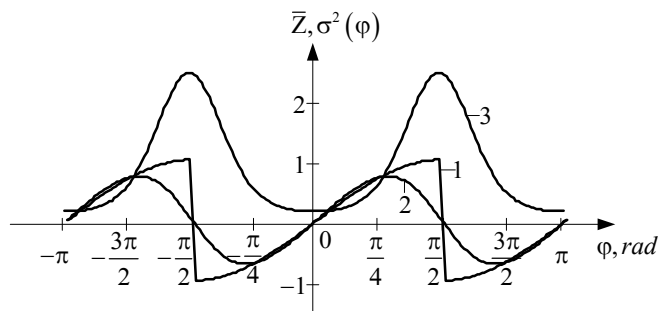


Fig. 3

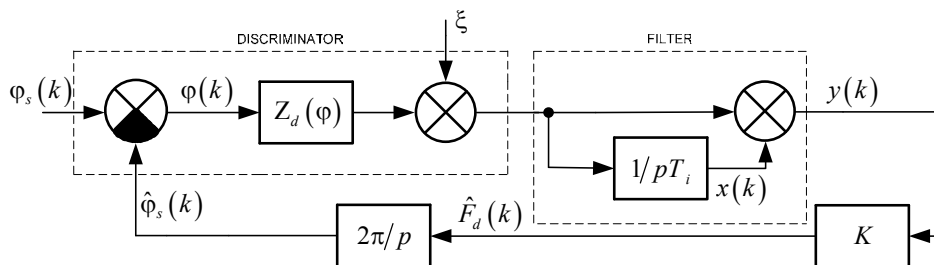


Fig. 4

data then signal-to-noise ratio equals to  $-40\text{ dB}$ . Length of using PRS  $N = 2^{14} - 1 = 16383$ . Number of statistical examinations equals to  $10^4$ .

The model of PLFS is presented in Fig. 4, where  $Z_d(\varphi)$  – discrimination characteristic of DPD;  $T_i$  – time constant of integrator;  $K = K_\phi K_c$  – instantaneous element, taking account of transfer constants of digital filter  $K_\phi$  and digital synthesizer  $K_c$ ; the meaning of another designation are clear without comments.

Doppler frequency shift on  $k$ -period of filtering is estimated in compliance with the following algorithm:

$$\hat{F}_d(k) = K \left( Z_d(k) + x(k-1) + \frac{T_p}{T_i} Z_d(k-1) \right). \quad (4)$$

Discriminator nonlinearity in case of using quasi-continuous analyzing method for digital synchronization systems is taking into account by it parameters, which depend on signal-to-noise ratio [4].

### 3. Accelerated phase synchronization target setting

In phase navigation systems RMS error of coordinate measuring (in meters) can be approximately determined as

$$\sigma_c \approx \frac{1}{2\pi} \lambda_0 \tilde{A} \sigma_\varphi, \quad (5)$$

where  $\lambda_0$  – wave-length,  $\tilde{A}$  – geometric quotient,  $\sigma_\varphi$  – RMS error of phase-shift measurements [5]. In steady-state regime phase-tracking error dispersion value can be determined by using quasi-continuous analyzing method for digital synchronization systems [6]:

$$\sigma_\varphi^2 = 2\sigma_e^2 T_p F_\varphi, \quad (6)$$

here  $\sigma_e^2$  – phase fluctuation dispersion, which can be calculated as

$$\sigma_e^2 = \frac{\sigma_d^2}{k_d^2}, \quad (7)$$

where  $\sigma_d^2 = \sigma_d^2(0)$  – fluctuation characteristic for algorithm (3) of phase mismatch failing;  $k_d = \partial Z_d(\varphi) / \partial \varphi|_{\varphi=0}$  – discrimination characteristic slope for algorithm (3), line from the top means statistical estimation. Noise pass band of PLFS can be written as

$$F_\varphi = \frac{1}{2\pi} \int_0^\infty |K(j\omega)|^2 d\omega, \quad (8)$$

where  $K(j\omega)$  – complex transfer coefficient of PLFS.

Using (5) it can be shown that in case of  $\Gamma = 1,5$  (rho-rho navigation),  $\lambda_0 = 150\text{ m}$  for attainment of coordinate measuring accuracy with RMS  $\sigma_c \leq 2\text{ m}$  needed RMS error of phase-shift measurements value is  $\sigma_\varphi \leq 3,3^\circ \approx 0,053\text{ rad}$ . Further, using results [3] for  $\sigma_d^2$  and  $k_d^2$ , when signal-to-noise ratio threshold equals to  $-40\text{ dB}$ , and using equation (6) let's compute required noise pass band of PLFS for MSK-signal receiver:

$$F_\varphi \leq \frac{\sigma_\varphi^2}{2\sigma_e^2 T_p} \leq \frac{0,053^2}{2 \cdot 0,364 \cdot 0,04} \approx 0,1\text{ Hz}. \quad (9)$$

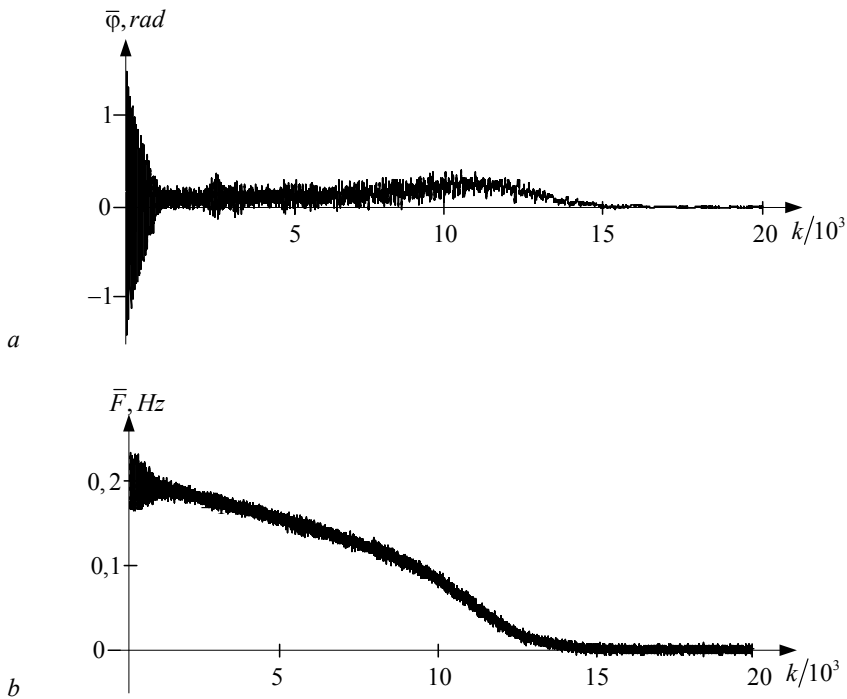


Fig. 5

Thus, PLFS must provide RMS error of phase-shift measurements value  $\sigma_{\varphi} \leq 0,05 \text{ rad}$  in case of noise pass band value  $F_{\varphi} \leq 0,1 \text{ Hz}$ .

Functional dependences of phase-tracking and frequency estimation error average values from discrete time  $k$  in digital PLFS are presented in Fig. 5, *a* and 5, *b* respectively. Computational approach conditions are equal to discriminator modeling, except number of statistical examinations –  $10^2$ .

Presented functional dependences are correspondent to noise pass band value  $F_{\varphi} = 0,1 \text{ Hz}$ , user's top speed equals  $V_{\max} = 100 \text{ km/h}$  (peak level of Doppler frequency shift  $|F_{d\max}| = 0,2 \text{ Hz}$ ) and capture probability  $P_c \rightarrow 1$ .

Analysis of statistic simulation data of digital PLFS (Fig. 5) shows that average locking time has intolerable level for perspective RNS for special users –  $\bar{t}_l \approx 15 \cdot 10^3 \cdot T_p = 600 \text{ s}$ .

#### 4. Digital PLFS statistical simulation

Progress in locking time decrease can be realized by varying of PLFS noise pass band. Thus, using “wide” noise pass band  $F_{\varphi_w} = 0,5 \text{ Hz}$  on the first time stage and “narrow”  $F_{\varphi_n} = 0,1 \text{ Hz}$  on the second time stage, it is possible to attain benefit in synchronization time.

Digital PLFS statistical simulation results, namely: phase  $\bar{\varphi}$  and frequency  $\bar{F}$  tracking errors average meanings (*a*, *c*), and RMS phase  $\sigma_{\varphi}$  and frequency  $\sigma_F$  tracking errors (*b*, *d*) are presented in Fig. 6 and in Fig. 7. All curves are functional dependences on discrete time  $k$ .

Curves 1, 2, and 3 are signifying Doppler frequency shifts: 0; 0,02; 0,2 Hz respectively. Noise pass bands are described by discrete time step functions (10). Function  $F'_{\varphi}(k)$  describes noise pass band for Fig. 6, and  $F''_{\varphi}(k)$  for Fig. 7.

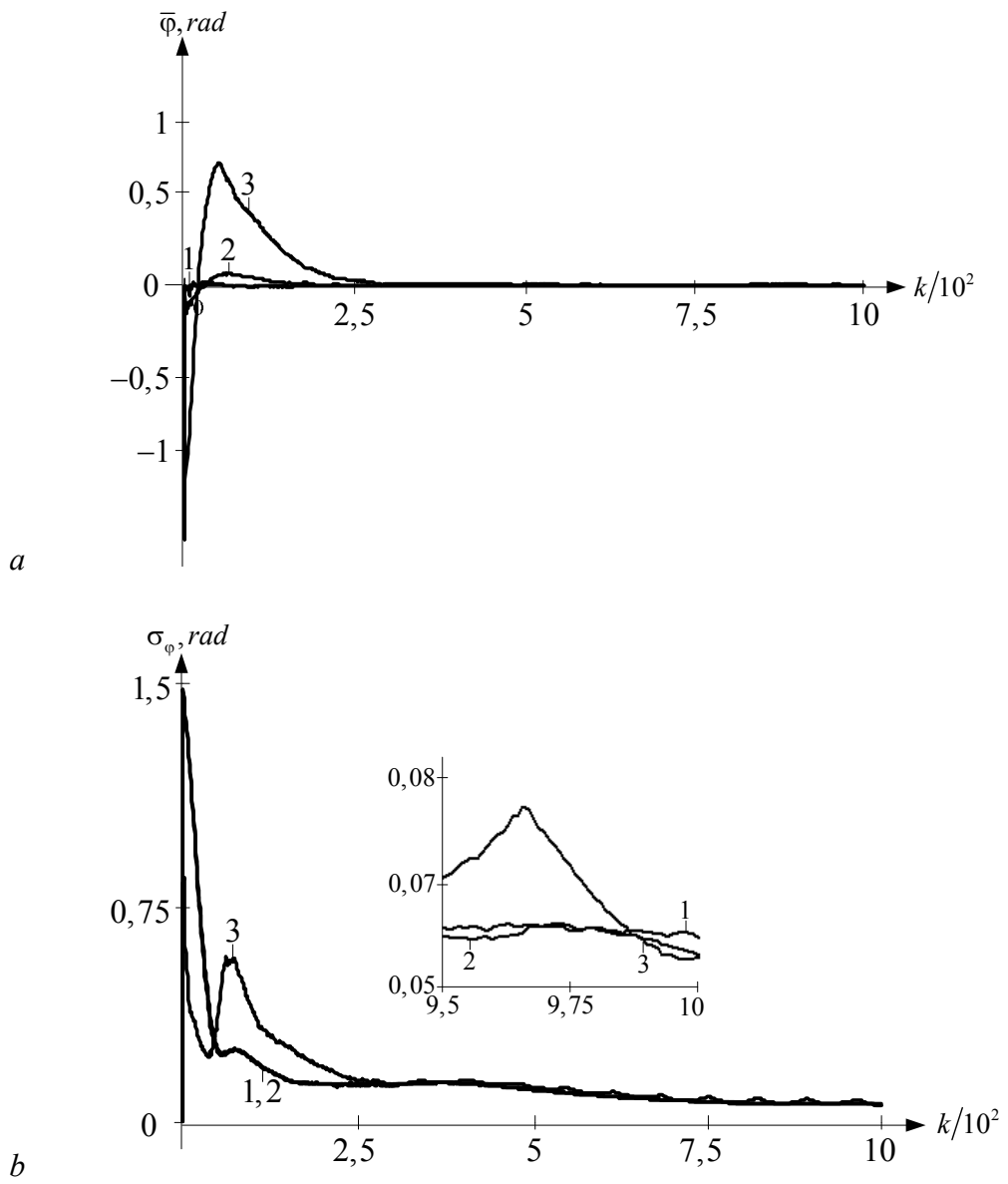


Fig. 6

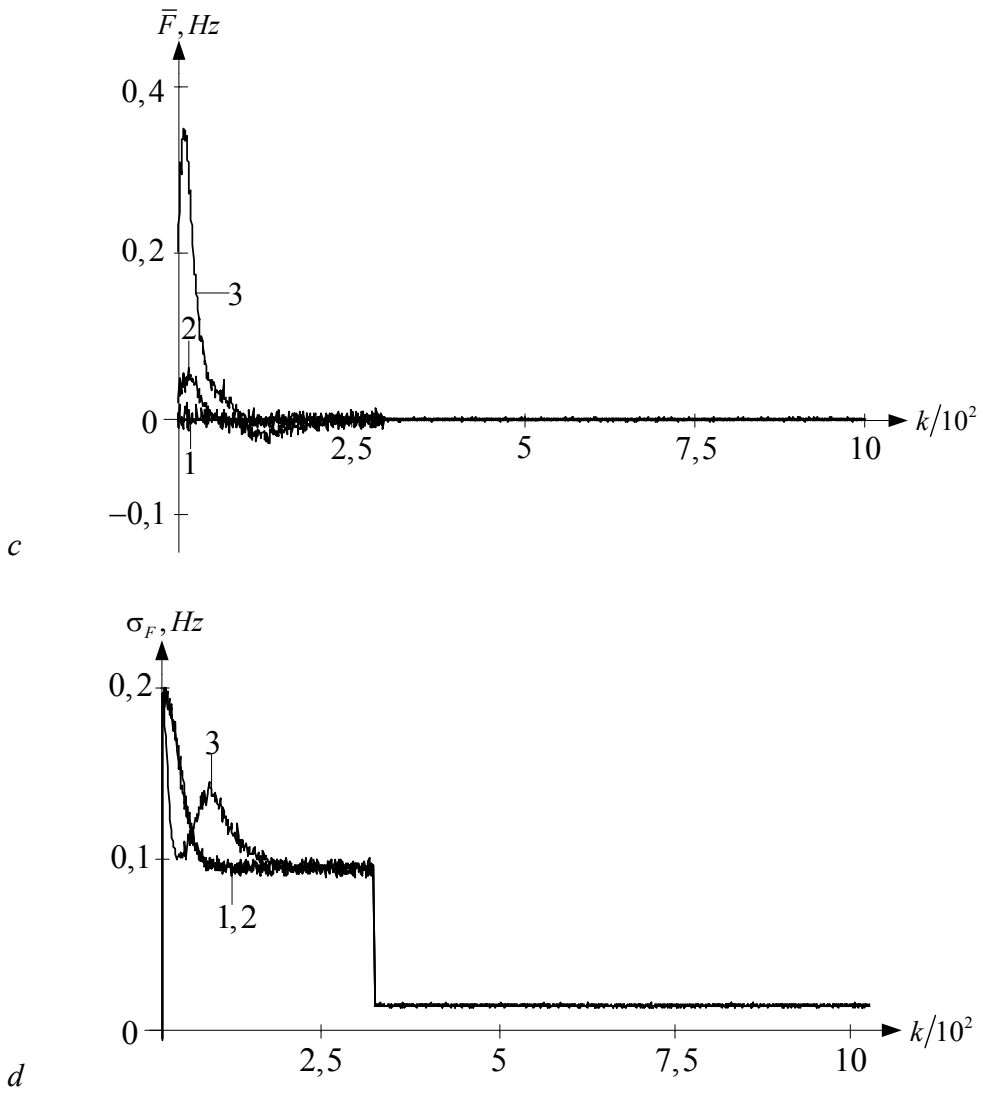


Fig. 6 (continue)

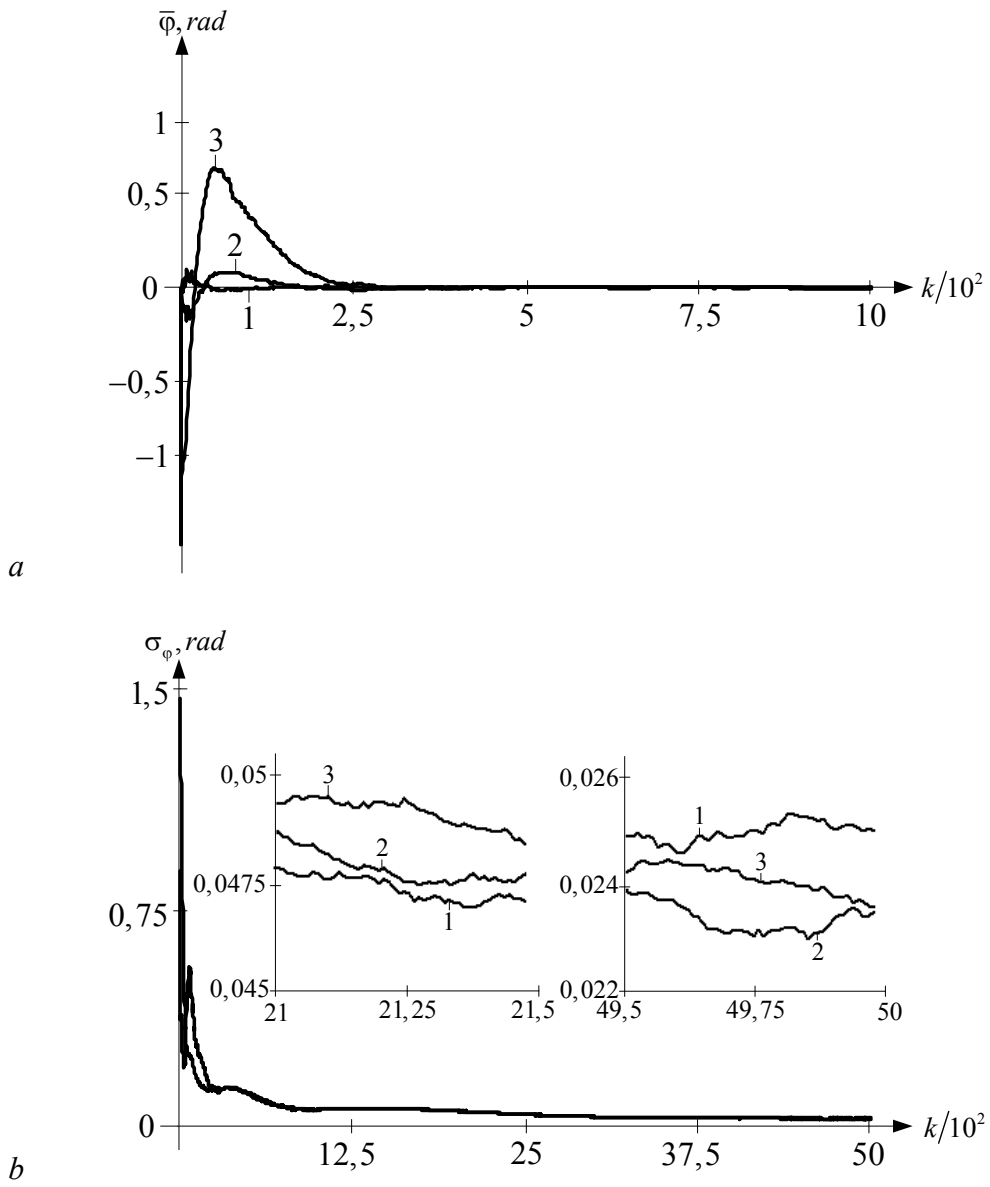


Fig. 7



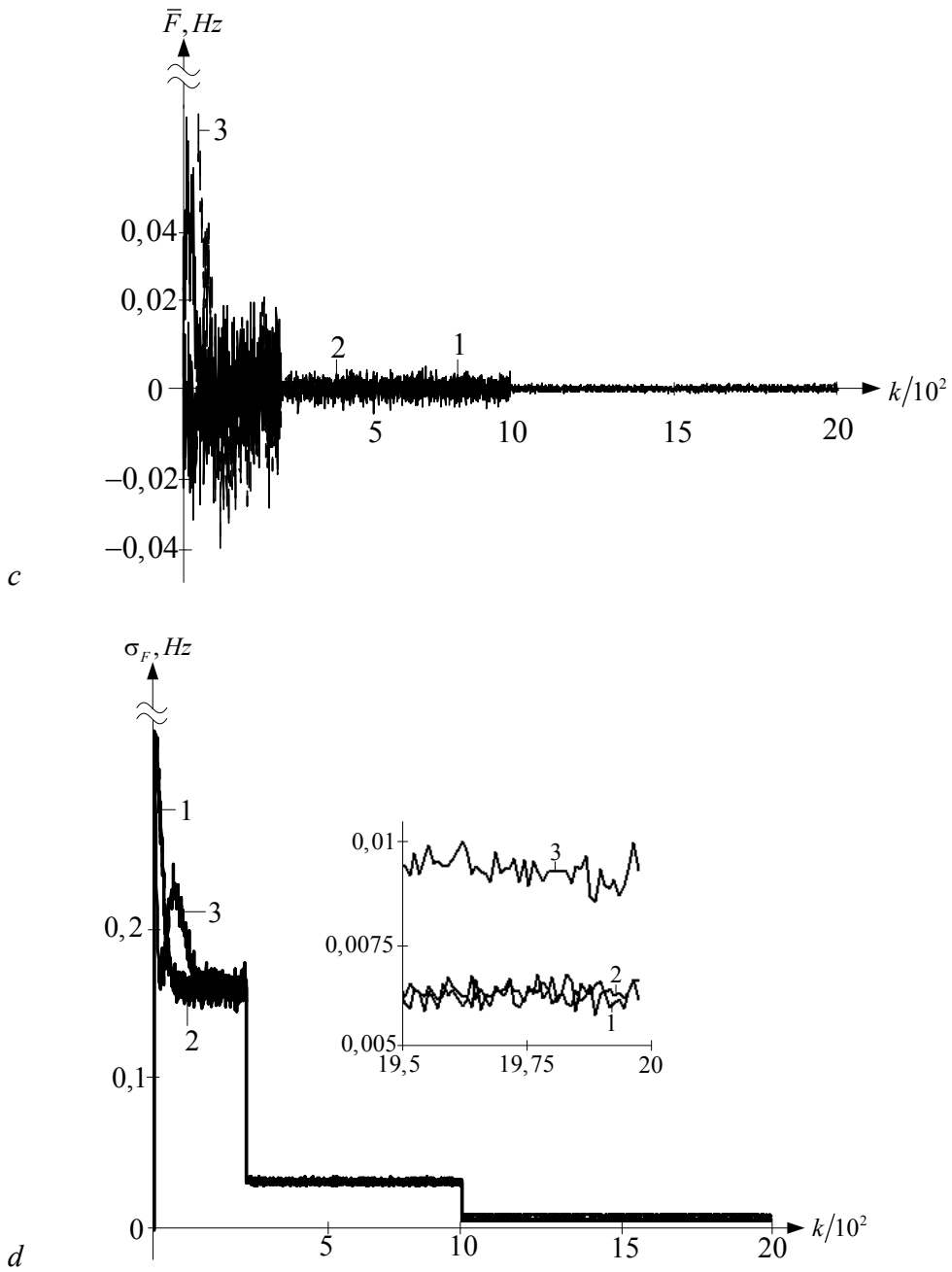


Fig. 7 (continue)

$$F'_\varphi(k) = \begin{cases} F_{\varphi w} = 0,5 \text{ Hz}, 0 \leq k \leq 300, \\ F_{\varphi n} = 0,1 \text{ Hz}, k > 300, \end{cases} \quad (10)$$

$$F''_\varphi(k) = \begin{cases} F_{\varphi w} = 0,5 \text{ Hz}, 0 \leq k \leq 300, \\ F_{\varphi n1} = 0,1 \text{ Hz}, 300 < k \leq 1000, \\ F_{\varphi n2} = 0,02 \text{ Hz}, k > 1000. \end{cases}$$

It becomes clear from Fig. 6, 7 that using multistage phase-lock-loop frequency control algorithms for MSK-signals receivers, average locking time can be significantly decreased (in comparison with autonomous algorithm  $F_\varphi = 0,1 \text{ Hz}$ ) to  $\bar{T}_l \approx 1000 \cdot T_p = 40 \text{ s}$ , with phase tracking RMS error desired value ( $\sigma_\varphi = 0,05 \text{ rad}$ ) in case of using function  $F'_\varphi(k)$ . Using function  $F''_\varphi(k)$ , it can be shown that phase tracking RMS error desired value is provided in time equal to  $40 \text{ s}$ . Also, using function  $F''_\varphi(k)$  it is possible to achieve  $\sigma_\varphi = 0,03 \text{ rad}$  in  $120 \text{ s}$  and in steady-state regime  $\sigma_\varphi = 0,02 \text{ rad}$  ( $k > 200 \text{ s}$ ).

Number of statistical examinations for Fig. 6 and Fig. 7 equals to  $10^3$ . In all examinations there are no tracking losses. Described two- and three-stage phase-lock-loop frequency control algorithms with discrete time step functions (10) can be used in MSK-signal receivers of perspective frequency-limited RNS.

### Conclusions

In present paper multistage phase-lock-loop frequency control algorithms of perspective RNS user's equipment are suggested. Statistical simulation was used to prove that a two-stage phase-lock-loop frequency control algorithm, using function  $F'_\varphi(k)$ , has gain in synchronization time equal to  $560 \text{ s}$  (in comparison with autonomous algorithm) and provides steady-state RMS error values  $\sigma_\varphi \leq 3^\circ$  and  $\sigma_f \leq 0,03 \text{ Hz}$ . It was also stated that a three-stage phase-lock-loop frequency control algorithm has two benefits: first, gain in synchronization time is not less than  $560 \text{ s}$ ; second, RMS error values in steady-state regime ( $k > 200 \text{ s}$ ) is  $\sigma_\varphi \leq 1,1^\circ$  and  $\sigma_f \leq 0,01 \text{ Hz}$  – better than required.

This article contains specific results which can be used in digital phase synchronization systems of user's equipment for perspective RNS with spread-spectrum MSK-signals. The investigated algorithms of accelerated phase synchronization can be easily realized on the basis of field programmable gate array technology (FPGA).

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