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## Influence of an Initial States on the Critical Relaxation of Ising-like Spin Systems

Vyacheslav V. Krizhanovskiy\*

Pavel V. Prudnikov†

Vladimir V. Prudnikov‡

Omsk State University  
Mira, 55a, Omsk, 644077  
Russia

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*The non-equilibrium critical behaviour of the three-dimensional pure and site-diluted spin systems described by Ising model is studied for different spin concentrations with evolution from various initial magnetic states. The universal scaling functions are determined for the magnetization.*

*Keywords: non-equilibrium critical behaviour, Ising model, initial state, structure defects.*

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The problem of describing phase transitions and critical phenomena is one of the most difficult task of the statistical physics. Critical phenomena are characterized by abnormally large amplitudes and long-living fluctuations of some thermodynamic variables, which are called as the order parameters. The renormalization group  $\varepsilon$ -expansion method is often used for description of the critical phenomena. Along with these analytical approaches, the numerical methods are widely used for description of the phase transitions, especially Monte-Carlo methods. The development of powerful computer technology and the increase of their computing power contribute to it.

The effect of critical slowing-down is an one of the features appearing in the critical behaviour of various systems. This phenomenon is connected with an abnormal increase in the relaxation time of a system  $t_{rel}$  when the temperature is approached to the critical temperature  $T_c$  of the second type phase transition:  $t_{rel} \sim |T - T_c|^{-z\nu}$ . As a result of this, a system in the critical point turns out be unable to come to equilibrium state during the relaxation process. Therefore, an unusual non-equilibrium phenomena like aging, violation fluctuation-dissipation theorem and the influence of different initial non-equilibrium states arise in the behaviour of such systems at times  $t \ll t_{rel}$  [1].

### 1. Characteristics of non-equilibrium critical behaviour and results of simulations

In the studies of the effects of the initial states of the system on the non-equilibrium critical behaviour, we distinguish the high-temperature states created at  $T \gg T_c$  and characterized

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\*FehtunKrizh@gmail.com

†prudnikp@univer.omsk.su

‡prudnikov@univer.omsk.su

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by the initial magnetization  $m_0 = 0$  and the low-temperature initial state which is created at temperature  $T_0 = 0$  and  $m_0 = 1$ . A realization of the non-equilibrium process is performed in the following way. At the initial instant of time, the system is brought into contact with the thermal bath at the critical temperature  $T_c$  of the system. For systems that start from the low-temperature initial state, the time dependence of magnetization is characterized by the relation  $M(t) \sim t^{-\beta/(z\nu)}$ , and for systems which are started from the high-temperature initial state by the power law  $M(t) \sim t^{\theta'}$  with the initial slip exponent  $\theta' > 0$  [2]. On a non-equilibrium stage of the critical behaviour the system magnetization becomes be the generalized homogeneous function of the observation time and a new time scale  $t_m \sim m_0^{-1/k}$  associated with the initial magnetization  $m_0$  [3]. In this work we carried out numerical study by Monte Carlo methods of influence of different initial states on the critical relaxation of the magnetization of the three-dimensional pure and structurally disordered Ising model.

The Hamiltonian of the structurally disordered Ising model is given by:

$$H = -J \sum_{\langle i,j \rangle} p_i p_j S_i S_j, \quad (1)$$

where  $J > 0$  is the short-range exchange interaction integral between spins  $S_i = \pm 1$  fixed at the lattice sites. The random occupation numbers  $p_i$  take the values 0 or 1, and  $p_i$  equals 1 if the site of the lattice contains spin and 0 otherwise. We considered the cubic lattice with periodic boundary conditions. Let us denote  $N_s = pL^3$  as the number of spins in a lattice with linear size  $L = 64$  and 128, where  $p$  is the spin concentration.

A non-equilibrium process is characterized by the magnetization

$$M(t) = \frac{1}{V} \int d^d x [\langle S(x, t) \rangle] = \left[ \left\langle \frac{1}{N_s} \sum_{i=1}^{N_s} p_i S_i(t) \right\rangle \right], \quad (2)$$

where the angular brackets denote statistical averaging over the realizations of the initial state, the square brackets are for averaging over different configurations of distribution defects in the lattice.

Currently, it is found that the time dependence of the magnetization for systems starting from a low-temperature initial state with  $m_0 \neq 0$  satisfies the following scaling form [3]:

$$M(t, t_m) = A_M t^{-\beta/z\nu} F_M(t/t_m), \quad (3)$$

where  $t_m = B_m m_0^{-1/k}$  is a new time scale with exponent  $k = \theta' + \beta/(z\nu) > 0$ ,  $\theta'$  is the initial slip exponent. The scaling function  $F_M(t/t_m)$  isare finite at limit  $t/t_m \rightarrow 0$ ,  $A_M$  is non-universal amplitude.

At the beginning, we realized investigation of critical relaxation of magnetization  $M(t)$  for pure ( $p = 1$ ) and diluted systems ( $p = 0.8, 0.6$ , and  $0.5$ ) with various initial states  $m_0$  (Fig. 1 (a)), which demonstrate that the relaxation time is increased with growth of the impurity concentration  $c_{imp} = 1 - p$ . The curves in Fig. 1 (b) and (c) for pure and diluted systems with  $p = 0.6$  demonstrate an essential differences in relaxation from the high-temperature with  $m_0 \ll 1$  and low-temperature with  $m_0 = 1$  initial states, and from the intermediate initial states with  $0.2 \leq m_0 \leq 0.6$ . So, a remarkable property of the non-equilibrium critical relaxation from the high-temperature initial state is the increase of magnetization  $M(t) \sim t^{\theta'}$  with  $\theta' > 0$ . The initial rise of magnetization is changed to the well known decay  $M(t) \sim t^{-\beta/z\nu}$  for  $t > t_{cr} \sim m_0^{-1/k}$ . The critical relaxation from the completely ordered initial state with  $m_0 = 1$  is immediately characterized by power-law of magnetization decay  $M(t) \sim t^{-\beta/z\nu}$ . The intermediate cases with

$0.2 \leq m_0 \leq 0.6$  and  $0.4$  are characterized by a short stage of magnetization rise under the power-law  $M(t) \sim t^{\theta'}$  which is changed to much long-continued stage of relaxation with  $M(t) \sim t^{-\beta/z\nu}$ .

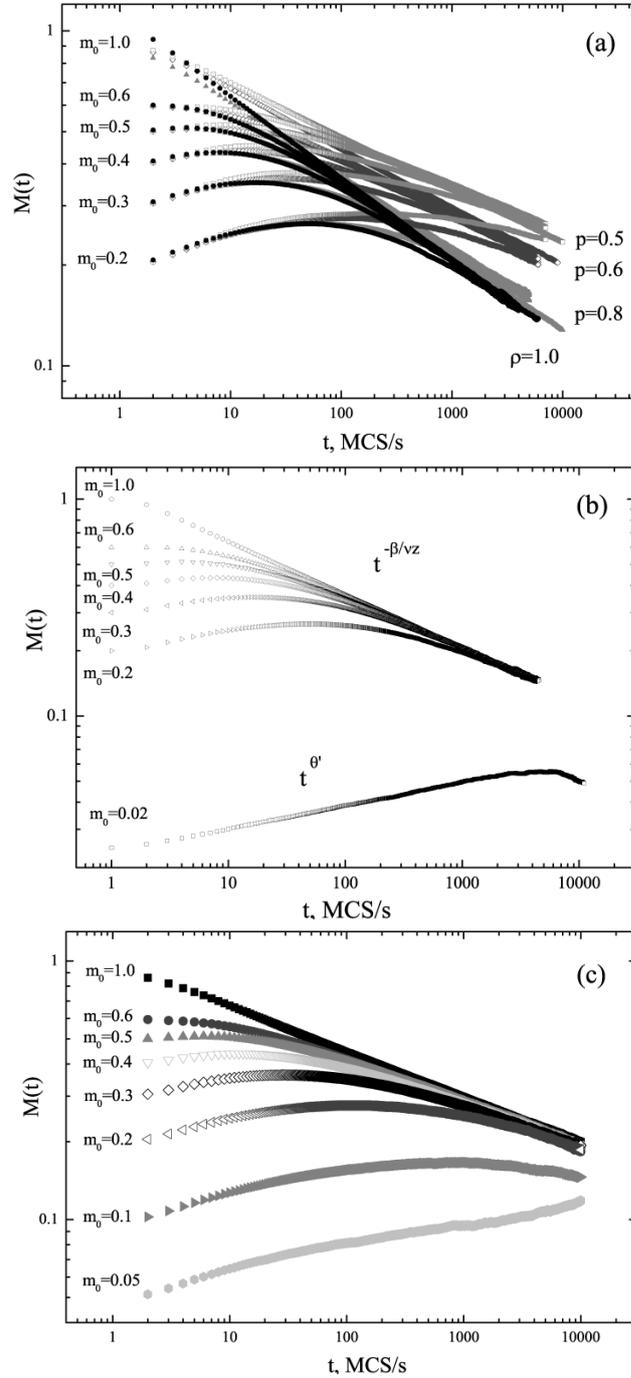


Fig. 1. Time dependence of the magnetization  $M(t)$  for various values of the initial magnetization  $m_0$  and different spin concentrations  $p$  (a), for pure model,  $p = 1.0$  (b), diluted system,  $p = 0.6$  (c)

For checking the scaling prediction for  $M(t)$  as function of the initial magnetization  $m_0$  give

by relation (3), we plot the dependence of  $t^{\beta/z\nu} M(t)$  versus  $x = tm_0^{1/k}$  in Fig. 2 with the use of exponent values for the pure 3D Ising model:  $\beta/\nu = 0.516(2)$ ,  $z = 2.024(6)$ , and diluted systems:  $\beta/\nu = 0.508(16)$ ,  $z = 2.191(21)$  for  $p = 0.8$  and  $\beta/\nu = 0.462(40)$ ,  $z = 2.663(30)$  for  $p = 0.6$  [4].

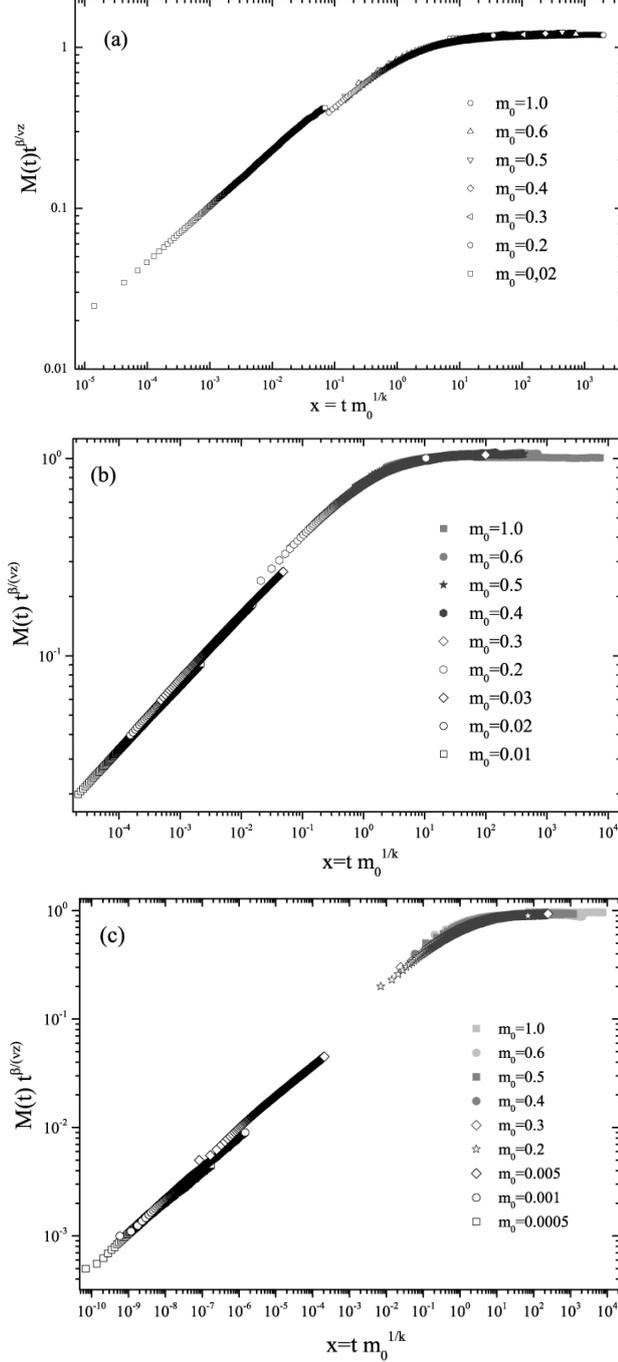


Fig. 2. Dependence of the magnetization scaling functions  $F_M(t/t_m) = M(t)t^{\beta/z\nu}$  on  $x = tm_0^{1/k}$  for different initial magnetizations  $m_0$  and spin concentrations  $p = 1.0$  (a),  $p = 0.8$  (b) and  $p = 0.6$  (c)

We can see in Fig. 2 (a), (b) and (c) the collapse of  $F_M(x)$  data for different  $m_0$  with fixed spin concentration  $p$  into a single universal line with linear initial line piece (in log - log scale) with  $F_M(x) \sim x^k$ . We obtained the following values of exponent  $k$ : for pure case  $k = 0.361(3)$ , for systems with  $p = 0.8$   $k = 0.353(4)$  and with  $p = 0.6$   $k = 0.325(7)$ .

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## References

- [1] P.Calabrese, A.Gambassi, *J. Phys. A*, **38**(2005), R133.
- [2] V.V.Prudnikov, P.V.Prudnikov, E.A.Pospelov, *J. Stat. Mech.*, (2016), 043303.
- [3] P.Calabrese, A.Gambassi, F.Krzakala, Critical ageing of Ising ferromagnets relaxing from an ordered state, *J. Stat. Mech.*, 2006, P06016.
- [4] P.V.Prudnikov, V.V.Prudnikov, E.A.Pospelov, P.N.Malyarenko, A.N.Vakilov, *Prog. Theor. Exp. Phys*, (2015), 053A01.

## Влияние начальных состояний на критическую релаксацию в изингоподобных спиновых системах

Вячеслав В. Крижановский  
Павел В. Прудников  
Владимир В. Прудников  
Омский государственный университет  
Мира, 55а, Омск, 644077  
Россия

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*Представлены результаты численного исследования особенностей неравновесного критического поведения в трехмерной чистой и структурно неупорядоченной модели Изинга с широким спектром изменения спиновой концентрации при ее эволюции из различных начальных магнитных состояний.*

*Ключевые слова: неравновесное критическое поведение, модель Изинга, начальное состояние, дефекты структуры.*