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Abstract. This article discusses coordinate measurement errors in short-range pseudolite navigation systems. An analysis of the components of the measurement error of radio navigating parameters has been performed; the geometric factor values have been calculated for various variants of constructing the system. The peculiarities of measuring the spatial orientation of objects using interferometric methods in on-ground short-range navigation systems have been ascertained.

Keywords: short-range radio navigation system, pseudo-satellite (pseudolite), geometric factor, coordinate measurement errors, spatial orientation.

1. Introduction

Modern traffic control systems − especially air and water traffic control systems − are challenged by the significant increase in the density of this traffic. In these circumstances modern high-precision positioning systems based on radio navigation are introduced at an ever-growing rate. Despite their increasing popularity, global navigation satellite systems (GNSS) are rarely used for autonomous navigating maintenance activities such as providing flight support services in the vicinity of aerodromes, supporting flight for unmanned aerial vehicles, assisting vessels in navigating in coastal waters and rivers, providing geodetic, cartographic and special services. The reason for this is the low noise immunity of GNSS which occasionally makes it impossible to comply with requirements of continuity and accuracy for navigation purposes.

One way to resolve this contradiction is by supporting the GNSS with a ground-based radio navigation system located in the vicinity of the navigation activities of a specified user. This enables the construction of a navigation field in a given area using ground-based pseudo-satellites (pseudolites) [1]. In such a radio navigation system, the high precision measurements for navigation parameters is proved by highly accurate geodetic coordinate

definition and mutual synchronization of the pseudolites, which are stationary navigation signal transmitters similar to the signals of GLONASS navigation satellites.

This arrangement for a navigation system significantly differs from local, regional or wideband differential GNSS subsystems which can be used only if there is a stable GLONASS signal, whereas the proposed navigation system based pseudolites can provide navigation services even in the absence of signals from a designated satellite.

2. Possible errors occurring during coordinate measurements of radio navigation systems based on ground-based pseudolites

The disposition of pseudolites within a specific area as well as their number in a short-range navigation system largely depends on the local terrain and the size of the coverage area. Thus, the area of an aerodrome would include the glide path of the aircraft; if we are speaking about river navigation then the main channel of the river would be the area covered by the navigation system. Despite this area being highly susceptible to various interference, it is necessary ensure high-precision continuous navigation services irrespective of the prevailing conditions. The limits of this are correspond to the boundaries in which the requirements for coordinate accuracy measurements and special orientation of angles are performed by the consumers of navigation services.

Various factors may influence the precision for measuring the coordinates of a moving object in a given area. These are some of the most widespread:

- The geometry of the pseudolite's location in relation to the navigation receiver (it is known as *the geometric factor*);

- Pseudo range measurement errors may be caused by the structure of the navigation signals and their selected processing method;

- Errors caused by the geodetic positioning quality of the pseudolite and the quality of mutual synchronization between transceivers and the Russian State Standard of the Coordinated Universal Time (UTC) scale of the state standard (similar to the ephemeris error in GNSS);

- Errors introduced by the signal propagation path;

- The power ratio of the desired received signal and interference.

The formation of a pseudolite short range navigation system arises a complex of tasks, including the selecting the optimal location for the pseudolites, the power output of the navigation signals, the structure of the navigation receivers surveillance schemes, which will provide a desired accuracy.

Measurements of coordinates of objects (*X*, *Y*, *Z*) in a short-range navigation system are performed using the standard range-difference (pseudorange) method:

$$R_i = \sqrt{\left(X_{mi} - X\right)^2 + \left(Y_{mi} - Y\right)^2 + \left(Z_{mi} - Z\right)^2} + C\Delta t, \qquad 1)$$

where X_{mi}, Y_{mi}, Z_{mi} are the coordinates of the *i*-th pseudo-satellite; *X*, *Y*, *Z* are the coordinates of the navigation receiver; *C* is the speed of light; Δt is the systematic error caused by discrepancy in time scales.

The value of Δt includes additional uncontrolled signal delays emerging on the signal propagation route.

The error in the measurement of the object's coordinates will depend on the hardware error

of pseudorange measurement, the measurement error of the pseudolites' coordinates and errors in the path of signal propagation. For ground-based short-range navigation systems based on pseudolites using GLONASS signals the magnitude of the hardware pseudorange measurement error will be approximately equal to a similar error in the GNSS: it will be approximately 1.5 m [2].

The error of inaccurate coordinate determination for pseudolites and their mutual timing (analog ephemeris errors in GNSS) for a discussed navigation system will be significantly less than in the GNSS. This is the result of a high-precision definition of geodetic coordinates of the pseudolites and the possibility of their mutual synchronization using highly stable time and frequency standards. The error in the signal propagation path cannot be ignored due to rather small distances in the system.

Errors which depend on the properties and methods of navigation signal processing – so-called internal errors – are planned during the design stage for such a short-range navigation system. One advantage provided by the ground-based system over GNSS is the ability to plan external caused by geometric factors. It is possible to determine the location of a pseudolite in such a way, which would minimize its impact on an important navigation sector.

The practice of solving navigation problems demonstrates that the values of the geometric factor in the measurement of planimetric coordinates and altitude may significantly vary. Therefore it is necessary to individually assess its allowable values to ascertain any errors in the various components of coordinate positioning.

The essence of the geometric factor is as follows. To calculate the coordinates of an object, a system of nonlinear equations is used (1).The number of these equations is determined by the number of pseudolites. Nonlinear equations are characterized by a radius of curvature of the wave front, which is equal to the distance from the object to pseudolite. Since the radius of the wave front curvature is significantly larger than the range measurement error, the system of equations can be linearized at the point of signal reception. In this situation the conversion error will be close to linear. When performing linear transformation, the coordinate positioning error is described by a covariance matrix.

The covariance matrix may be obtained from a gradient matrix:

$$cov(r) = Gr^T Gr, \qquad (2)$$

where *Gr* is the gradient matrix:

$$
Gr = \left\|
\begin{matrix}
\dfrac{\partial R_1}{\partial X} & \dfrac{\partial R_1}{\partial Y} & \dfrac{\partial R_1}{\partial Z} & \dfrac{\partial R_1}{\partial (C\Delta t)} \\
\dfrac{\partial R_2}{\partial X} & \dfrac{\partial R_2}{\partial Y} & \dfrac{\partial R_2}{\partial Z} & \dfrac{\partial R_2}{\partial (C\Delta t)} \\
\dots & \dots & \dots & \dots \\
\dfrac{\partial R_N}{\partial X} & \dfrac{\partial R_N}{\partial Y} & \dfrac{\partial R_N}{\partial Z} & \dfrac{\partial R_N}{\partial (C\Delta t)}
\end{matrix}
\right\|
=
\left\|
\begin{matrix}
\dfrac{X_{m1}-X}{R_1} & \dfrac{Y_{m1}-Y}{R_1} & \dfrac{Z_{m1}-Z}{R_1} & 1 \\
\dfrac{X_{m2}-X}{R_2} & \dfrac{Y_{m2}-Y}{R_2} & \dfrac{Z_{m2}-Z}{R_2} & 1 \\
\dots & \dots & \dots & \dots \\
\dfrac{X_{mN}-X}{R_N} & \dfrac{Y_{mN}-Y}{R_N} & \dfrac{Z_{mN}-Z}{R_N} & 1
\end{matrix}
\right\|
\qquad (3)
$$

,

where $\dfrac{X_{mN} - X}{R_{iN}} = \cos\alpha_i;$ $\dfrac{Y_{mN} - Y}{R_{iN}} = \cos\beta_i;$ $\dfrac{Z_{mN} - Z}{R_{iN}} = \cos\gamma_i$ is the direction cosines of the radius vector joining the N-th user i-th and the pseudolite; R_{iN} is the distance between the i-th pseudolites and N-this the user.

If it is assumed that the pseudorange measurements are of equal accuracy, it is possible to divide the measurement errors and transformations. In this case, the coordinate determination error will be equal to the product of the covariance matrix and the dispersion ranging; the resulting error can be estimated through the trace of the covariance matrix.

Generally, the covariance matrix contains non-zero off-diagonal elements which describe individual components of the coordinate correlation error. However, we can choose a system of coordinates in which the coordinates are uncorrelated errors, whereas the covariance matrix is diagonal. The elements of the main diagonal are the variance component coordinates; the resulting variance will be equal to their sum. At the same time, the sum of the diagonal elements of the covariance matrix for linear transformation does not change as it is a linear invariant tensor of rank 2.Consequently, the trace of a matrix − a ratio that shows what fold increases the variance of coordinates relative to the dispersion of the measured distance. The shape of the covariance matrix depends only on the relative position of pseudolites and navigation receiver, hence, giving it its name − the geometric factor.

The coefficients of the gradient matrix are the direction cosines in the pseudolite. Practically, the square root of the covariance matrix path, which shows the degree of increase in the standard deviation of the positioning error relative to the standard deviation range of the measurement error, is used:

$$
\begin{aligned}
\sigma_{x,y} &= \sqrt{cov\left(r_{11}\right) + cov\left(r_{22}\right)}, \\
\sigma_z &= \sqrt{cov\left(r_{33}\right)},
\end{aligned}
\tag{4}
$$

where $\sigma_{x,\,y}$, σ_z is, respectively, the standard deviation of the error in determining coordinates in the horizontal and altitude.

3. Calculating geometric factors for various pseudolite location configurations
Let's perform the calculation of the values of the geometric factor for a variety of pseudolite location configurations on the ground using the Math lab mathematical software environment [3].
3.1 Configuration No. 1
In the first configuration three pseudolites are placed on the corners of an equilateral triangle of side 50 km; one more pseudolite is positioned in the center of the triangle. Nominally, the pseudolites are situated on the earth and at the same altitude ($Y_m = 0$). Figs. 1a, 1b demonstrate the calculated values of the geometric factor for measurements of planimetric coordinates and altitude.

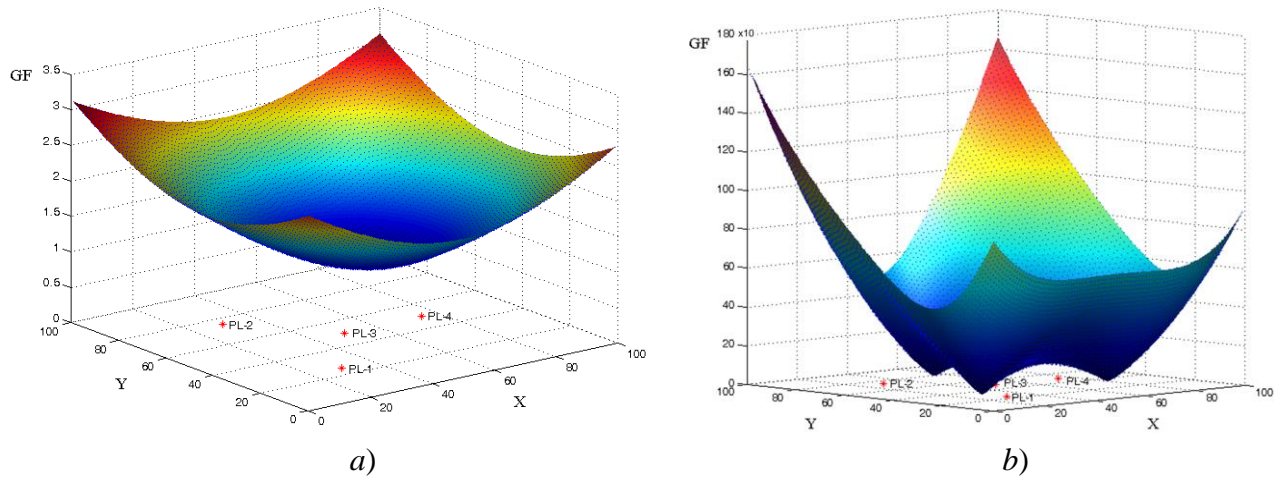*a)*   *b)*

Figure 1. Configuration No. 1: *a* is the geometric factor for measurement of planimetric coordinates; *b* is the geometric factor for altitude measurements

The geometric factor for measuring the coordinate's altitude exceeds the geometric factor for measuring planimetric coordinates. This can be explained by the fact that the pseudolites are located in one plane; as the object is brought closer to this plane, the altitude measurement error increases (Fig. 1*b*), whereas the planimetric coordinate error is almost unaffected by the object's altitude and is 1 ... 2.

For on-ground short-range navigation systems with the pseudolite positioned at the same altitude it is almost impossible to minimize the measurement error for the altitude with a large geometric factor. One option to solve this problem is by positioning the pseudolites on high masts, installing them on the tops of buildings, or by using the local terrain.

3.2 Configuration No. 2

Let's consider a configuration in which six pseudolites are situated at an altitude of 0 m on the sides of a square of side 60 km; two pseudolites are located in the center of the square at an altitude of 500 m (Fig. 2a).
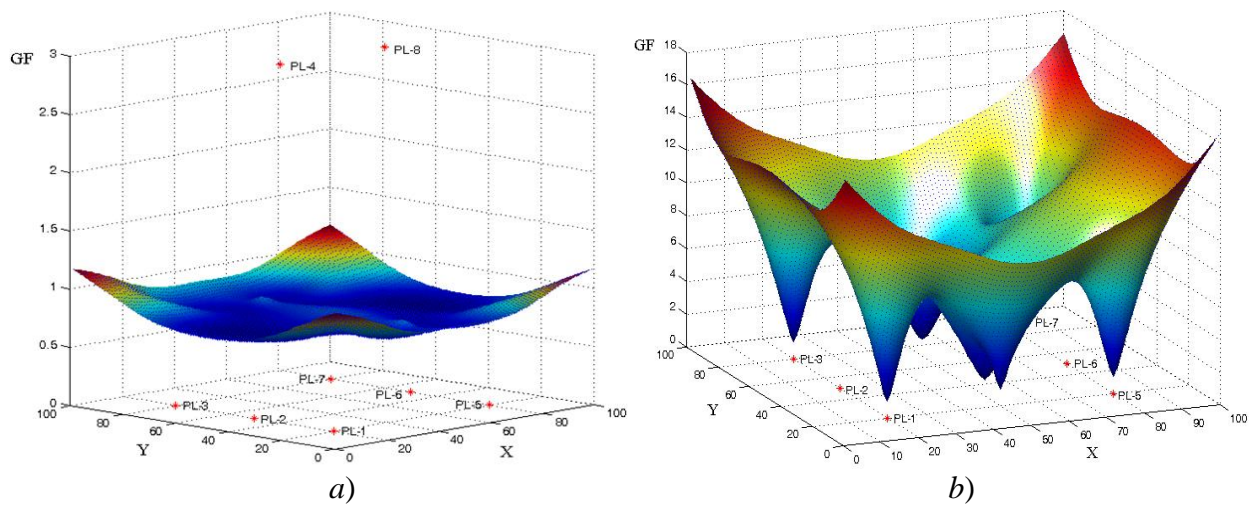

a)   *b)*

Figure 2. Configuration No. 2: *a* is the geometric factor for measurement of planimetric coordinates; *b* is the geometric factor for altitude measurements

The geometric factor for measuring planimetric coordinates by shifting the location of the pseudolite configuration has not changed and does not exceed the value of 1 ... 1.3. Positioning the two pseudolites at an altitude of 500 meters significantly reduces the geometric factor for altitude measurements (10 in the short-range and 50 in the middle range (Fig. 2*b*).

3.3 Configuration No. 3

In the following configuration four pseudolites are situated at four corners of a square (the distance between them is 60 km); four more pseudolites are elevated to an altitude of 1,000 m (Fig. 3*a*).

The altitude of the suspended pseudolites can increase the area in which the value of the geometric adjustment factor is less than 10 (Fig 3*b*) while the geometric factor for coordinates in the short-range zone has not changed.
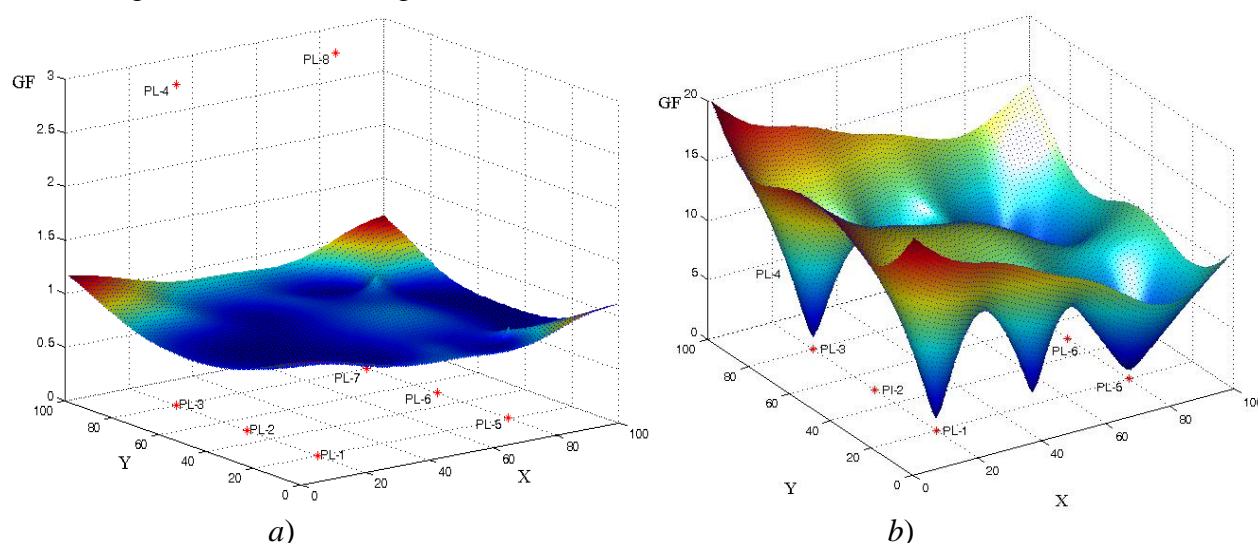

a)   *b*)

Figure 3. Configuration 3: *a* is the geometric factor for measuring planimetric coordinates; *b* is the geometric factor for altitude measurements

Thus, the ground-based radio navigation system enables pseudolites to provide sufficient (for land and river transport) improvement for the precise measurement of planimetric coordinates. Air transport, naturally, requires higher accuracy for altitude measurements; these are defined by ICAO standards. In order to minimize the altitude measurement error, it is necessary to elevate one or more pseudolites to a certain height. In this case, the maximum value of the geometric factor and, respectively, the maximum altitude measurement error will occur at half the altitude of the elevated pseudolite.

Options for raising some pseudolites above the height of the rest of the configuration may include high masts or, if such are available, mountain peaks.

4. Measuring the spatial orientation of objects

Currently, it is possible to observe increasing interest to the expansion of the functionality in the consumer's navigation equipment. Among the various existing options for extra functions, determining an object's spatial orientation is a promising direction for research. In existing radio navigation systems the highly precise measurements for spatial orientation angles is provided only by interferometric methods, which involve the application of an antenna array [4–6]. In this case, we assess the difference in the path of the navigation signals

$\Delta R_{1,\,2}$ received by means of antenna diversity, through phase taper $\Delta R_{1,2} = \dfrac{\lambda \Delta \varphi_{1,2}}{2\pi}$. During this, the signal reception is performed by diverse antennas A_1, A_2 and A_3 which constitute the two vectors-bases of length B_1 and B_2 (Fig. 4). Not necessarily, these vectors-bases are perpendicular. More commonly, however, one vector-base is oriented parallel to the longitudinal axis of the object, the second is parallel to the transverse.

The phase shift of the pseudolite signal, received by two diverse antennas and the cosine of the angle between the vector-base and the vector-direction of the pseudolite can be expressed as

$$\cos \alpha_{1,2} = \frac{\Delta R_{1,2}}{B_{1,2}} = \frac{\lambda \Delta \varphi_{1,2}}{2\pi B_{1,2}}, \tag{5}$$

where λ is the wavelength of the pseudolite signal; α is the angle between the vector-base and the vector-direction of the pseudolite.
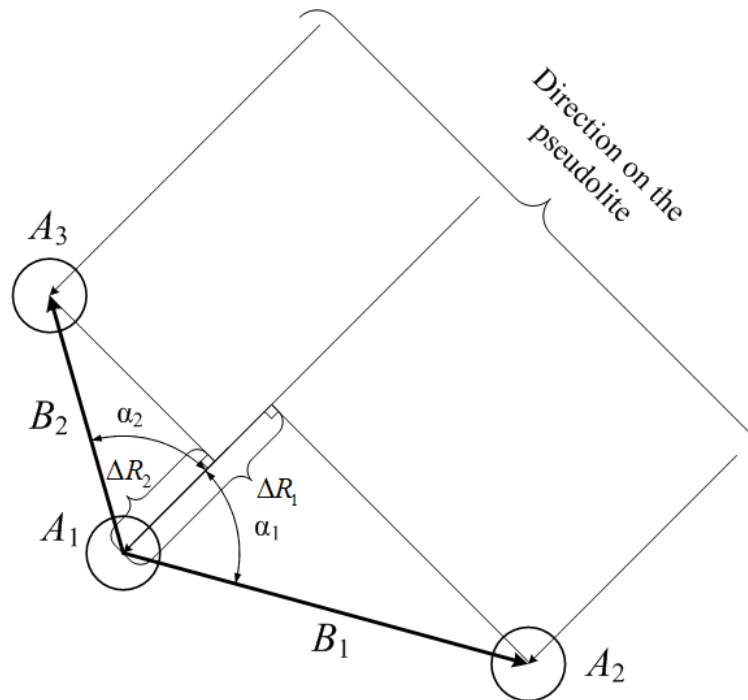

Figure 4. Position of navigation receiver antennas during spatial orientation of angular measurements

The direction cosines of the vector-base can be determined by an equation, based on the scalar product of vectors:

$$B \cdot \cos \alpha = k_x \cdot X + k_y \cdot Y + k_z \cdot Z, \tag{6}$$

There is a constraint equation between the components of the vector-base coordinates of the vector-base:

$$B_{1,2} = \sqrt{X_{1,2}^2 + Y_{1,2}^2 + Z_{1,2}^2}. \tag{7}$$

If there are two bases and considering (5) and (7), we shall write the equation for determining the direction cosine of two vectors-bases as such:

$$\begin{cases} k_{xi}X_{m1} + k_{yi}Y_{m1} + k_{zi}Z_{m1} = \Phi_{1i}, \\ k_{xi}X_{m2} + k_{yi}Y_{m2} + k_{zi}Z_{m2} = \Phi_{2i}, \\ B_{1,2} = \sqrt{X_{1,2}^2 + Y_{1,2}^2 + Z_{1,2}^2}. \end{cases} \tag{8}$$

where k_{xi}, k_{yi}, k_{zi} are the direction cosines in the pseudolite; $\Phi = \dfrac{\lambda\Delta\varphi}{2\pi}$ is the phase shift of the signals between the receiving antennas, expressed in units of length.

To find the position of a base-vector in space it is necessary to measure the phase shifts of signals from two pseudolites. Thus, the system of equations (8) comprises two linear equations based on the results phase shift Φ measurements and one equation of relation between the coordinates X, Y, Z.

The system of equations (8) is non-linear because of the nonlinear coupling equations. Consequently, due to the non-linearity, the accuracy of calculating orientation depends from the angular position of the vector-base.

Thus, as it can be observed from the features of this short-range navigation system based on pseudolites, positioned near the horizontal plane, in the system of equations (8) with the position of the vector-bases in the horizontal plane, the vertical directional cosine in the direction of the pseudolite k_y is equal (or is close) to zero.

From the linear part of the system of equations (8) we can only determine the planimetric coordinates of vector-base X and Z; the vertical component is obtained from a non-linear system of equations (7).

$$Y = \sqrt{B^2 - X^2 - Z^2} = \sqrt{B^2 - B_{plane}^2} \tag{9}$$

where B_{plane} is the projection of the vector-base on a horizontal plane.

Therefore, by using a vector-base along the longitudinal or transverse axes of the object it is impossible to determine the angles of pitch and roll.

The best solution to this contradiction arises by analyzing expressions (7), (8) and (9): it is the measurement of the angles of pitch and roll using a vertical base. The linear part of the system of equations (8) gives the planimetric coordinates for components of the vector-base; they increases with the pitch and roll angles. The vertical component is determined from the non-linear equation (7), while the sign for the vertical component is known in advance. Thus, we can determine the position of the vertical axis of an object on a vertically positioned base. At a certain course angle it is possible to determine the angles of pitch and roll. The measurement error, in this case, is comparable to the accuracy of determining the angle of

course. An intermediate option is to position the arrangement of the bases at an inclined angle which will enable us to determine the angles of course, pitch and roll.

5. Conclusion

Thus we have demonstrated that ground-based navigation systems, containing pseudolites, are capable of significantly increasing the accuracy of measuring planemetric coordinates if compared to conventional GNSS: the error is reduced from 12 to 0.4 m. In order to measure an object's altitude using a pseudolite system, it is necessary to suspend part of the pseudolites in one configuration above the surface of the Earth; this variant, however, is not always possible.

In order to measure the spatial orientation of an object using a pseudolite-based system, it is necessary to arrange the receiver antenna system is such a manner that the vector-base is positioned vertically or inclined.

Thus, the possibility for selecting different locations for pseudolites enables us, considering the terrain, to adapt the navigation area for a designated area for specific navigation objects taking into account various features of the terrain, such as mountain peaks and hilltops, as well as to form the desired geometric factor in a given area. It is desirable to use 5 ... 10 pseudolites, arranged uniformly along the perimeter of the coverage area, while several pseudolites should be situated at various altitudes to improve the navigation signal above the ground. For a more accurate measurement of an aircraft's altitude it is recommended combine autonomous on-board barometers, altimeters and GNSS receivers with the receivers of short-distance navigation systems based on pseudolites.

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