

Studying laminar flows of power-law fluids in the annular channel with eccentricity

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Abstract. The paper deals with numerical and experimental investigation of non-Newtonian flow of modeling drilling fluids in the annular channel. The Reynolds number was ranged from 100 to 1500. The parameters of the power-law model of drilling fluids were varied within the following ranges: $n=0.43-0.49$, $K=0.22-0.89$. The eccentricity was changed from 0 to 1. We have measured pressure drop in the annular channel and compared calculations with experimental data, achieving good agreement between calculations and experiment.

1. Introduction

Despite the huge number of theoretical, numerical, and experimental studies devoted to the drilling fluid flows in the boreholes, today it is impossible to fully provide the necessary information on all flow characteristics in the borehole within required wide range of drilling parameters. In practice, this flow is usually complicated by the presence of eccentricity e between axes of the outer and inner pipes of the annular channel as well as rotation of the inner pipe. In addition, drilling mud is usually non-Newtonian fluid. In the general case, no exact analytical solutions describing such a complicated flow are known, though there are many approximate and asymptotic solutions. Since no quite a satisfactory solution was received over the recent time, this problem is still being actively studied.

The first studies dealing with laminar flow of non-Newtonian fluids in annular channels were carried out by Volarovich and Gutkin [1], who suggested the approximation of analytical solution of the enforced flow of Bingham fluid in a concentric annular channel.

Fredrickson, together with Bird [2] and Laird [3], obtained the first exact solutions of this problem for power-law and Bingham fluids. Asymptotic solutions, taking into account the eccentricity were also proposed for these rheological models. This was carried out by Hanks [4] for power-law fluids, and Guckes [5] for Bingham fluid.

Many works devoted to the numerical modeling of such flows were appeared in recent years. First of all, it is worth noting the following studies: Hussain and Sharif [6] dealt with simulation of power-law and Herschel–Bulkley fluids flow in eccentric channel with a partial blockage and consideration of the rotation of the inner tube; another work was carried out by Escudier M. P., Gouldson I. W., Oliveira P. J., Pinho F. T. [7], where the authors studied the effect of the inner tube rotation.

At that, verification of these mathematical models requires experimental data, which at the moment are insufficient. This is especially true for non-Newtonian flows in channels with eccentricity. The laminar flow of power-law fluid in an annular channel without and with eccentricity was study in this work.

2. Experimental technique

An experimental test bench for verification the numerical algorithm of the motion of a solution of a non-Newtonian fluid in an annular channel (see figure 1) was created. The stand is a closed loop. The fluid is supplied from the tank (1) to the working section by a centrifugal pump (2). The flow rate is regulated by valves (3-4) and controlled by a flowmeter (5). Then the liquid hit the work area (6). The working area is an annular channel made of stainless steel, which is a pipe-in-pipe system. The outer diameter of the inner tube (D_{in}) is 12.5 mm, the inner diameter of the outer tube (D_{out}) is 21 mm, so the hydraulic diameter of the channel is 8.5 mm.

Measurements of pressure drop in the measuring section of the annular channel were carried out using a differential manometer (7). To measure the pressure drop in the outer tube, chokes were installed, to which a differential pressure gauge was connected. In the experiments, the length of the measuring section was 0.5 m. Before the measuring section, the inlet section of the flow stabilization was organized.

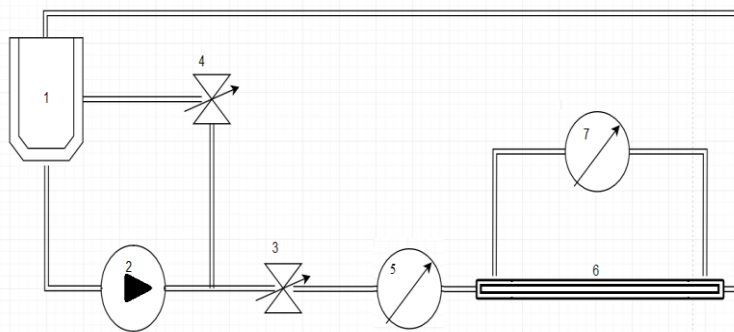


Figure 1. Schematic of the experimental test bench.

Experiments were carried out using preliminary prepared model drilling fluids. At that when preparing drilling fluids we used the recipe and the ingredients which are usually used for real drilling fluids. The following ingredients were used for the preparation of solutions: "GAMMAKSAN" – xanthan biopolymer (produced by Mirrico Company); low viscosity polyanionic cellulose (PAC-LV "OSNOPAK-N" TU 2231-001-70896713-2004, produced by Mirrico Company); liquid glass with density of 1.33-1.45 g/cm³; sodium chloride (GOST 4233-77), potassium chloride (GOST 4234-77), ammonium chloride (GOST 3773-72); POLY-SAL – a high quality starch (MI SWACO, Schlumberger Company); PLATINUM PAC – polyanionic cellulose (MI SWACO, Schlumberger Company); FLOTROL – starch (MI SWACO, Schlumberger Company); THRUTROL – specially modified derivatives of starch (MI SWACO, Schlumberger Company). Distilled water served the basis for drilling fluid prepared in laboratory conditions. The compositions of solutions are given in Table 1. The rheological properties of drilling fluids were measured by means of a rotational viscometer produced by OFITE MODEL 900 VISCOMETER company, fluid shear rate was varied from 50 to 1022 s⁻¹. Viscosity measurement showed that the rheology of the selected solutions was well described by a power-law model $\tau = K \cdot \gamma^n$. The rheological properties of the solutions are given in table 1. The density of the solutions was equal to 1100 kg/m³.

Table 1. The rheological properties of model solutions					
No.	Composition of the solution, % wt.			Rheological parameters of power-law fluid ($\tau = K \cdot \gamma^n$)	
	THRUTROL	FLOTPOL	NaCl	n	$K, Pa \cdot s^n$
1	0.25	–	9	0.4871	0.2240
2	0.5	–	–	0.4317	0.5289
3	–	0.5	–	0.4300	0.8900

3. Mathematical model

In the general case, a viscous fluid flow is described by a system of Navier–Stokes equations consisting of the mass conservation equation:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0$$

and the equations of motion or momentum conservation law:

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla(\rho \mathbf{v} \cdot \mathbf{v}) = -\nabla p + \nabla(\boldsymbol{\tau}) + \mathbf{F}$$

where \mathbf{v} – is the fluid velocity vector, $\boldsymbol{\tau}$ – is the tensor of viscous stresses, \mathbf{F} – is the body forces vector, p – is the static pressure, ρ – is the fluid density. Since in most cases the drilling mud is non-Newtonian fluid, for simulating non-Newtonian flows we used well-known approach [8-9], in which the medium is considered as a nonlinear viscous fluid with the introduction of effective fluid viscosity $\mu(\dot{\gamma})$ which in general is dependent on shear rate. At that, the tensor of viscous stresses $\boldsymbol{\tau}$ is determined as follows:

$$\boldsymbol{\tau} = \mu \mathbf{D}$$

The components of the strain velocity tensor \mathbf{D} are of the form:

$$\mathbf{D}_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i},$$

shear rate $\dot{\gamma}$ is the second invariant of the strain velocity tensor:

$$\dot{\gamma} = \sqrt{\frac{1}{2} \mathbf{D} \cdot \mathbf{D}}.$$

Depending on the drilling fluid rheology, the effective viscosity is: $\mu(\dot{\gamma}) = K$, for Newtonian medium (K – is the molecular viscosity of the fluid), $\mu(\dot{\gamma}) = K\dot{\gamma}^{n-1}$ for a power-law model, $\mu(\dot{\gamma}) = \frac{K\dot{\gamma} + \boldsymbol{\tau}_0}{\dot{\gamma}}$ for Bingham Plastic model, $\mu(\dot{\gamma}) = \frac{K\dot{\gamma}^n + \boldsymbol{\tau}_0}{\dot{\gamma}}$ for Herschel– Bulkley model, where n and K – are the coefficients of the rheological models, $\boldsymbol{\tau}_0$ – is the yield limit of viscoplastic fluid.

In our calculation we used structured grid consisting of 30 nodes along the radius, 120 nodes along the circumference, and 30 nodes along the channel length. The flow rate values obtained in the experiment were set at the inlet to the channel.

Figure 2 shows the calculated axial velocity profiles in the annular channel for these solutions. It is seen that with decrease in power exponent n velocity profile becomes flatter. In the general case this should contribute to the better removal of cuttings during drilling.

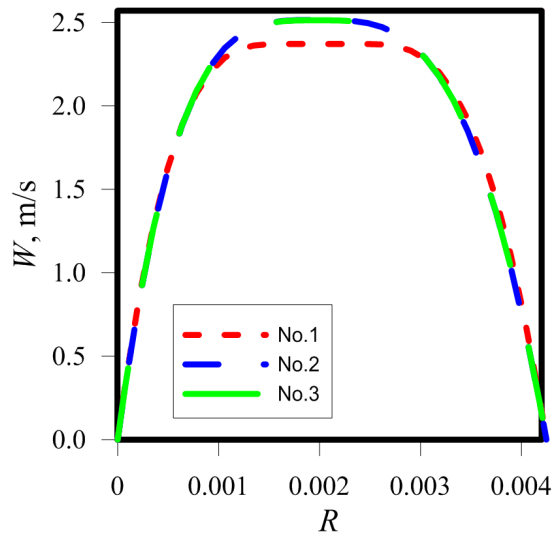


Figure 2. The velocity profiles in the annular channel ($D_{out}=21\text{mm}$, $D_{in}=12.5\text{mm}$) for various drilling fluids for $G=0.473\text{ kg/s}$ and $e=0$.

Comparison of calculated and experimental values of the pressure drop in concentric annular channel for the considered solutions is given in figure 3. As can be seen, all cases show very good agreement between calculations and experiments. The maximum discrepancy does not exceed 10%.

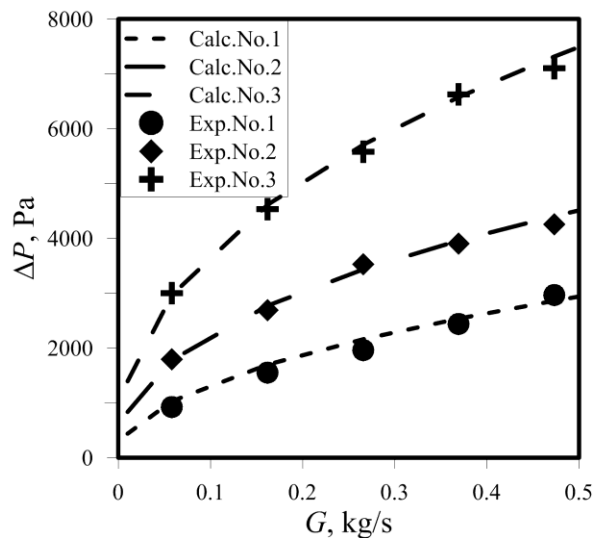


Figure 3. Pressure drop in the concentric annular channel ($D_{out}=21\text{mm}$, $D_{in}=12.5\text{mm}$) versus fluid flow rate. Solid lines correspond to calculation, symbols are experiment.

Further we investigated the effect of eccentricity on flow parameters and the pressure drop. Figure 4 shows the behavior of axial velocity of drilling fluid No.3 at the flow rate $G=0.473\text{ kg/s}$ depending on increasing eccentricity. As seen when the inner tube is displaced to the wall of the outer pipe, the flow of the drilling fluid is redistributed. With increasing eccentricity, the width of the annular gap in the wide part increases. In this zone, the maximum flow rate is observed. In the narrow part of the annular channel, zones with a reduced velocity are formed. The redistribution of the velocity in the channel

affects the pressure drop that is shown in figure5. As is obvious, the presence of the eccentricity leads to an almost twofold drop in the differential pressure in the channel. The experiment shows the same behavior. The calculated pressure drop for eccentricity e equal to 1 (where the inner pipe rests on the wall of the outer tube) is in a good agreement with the corresponding experimental value.

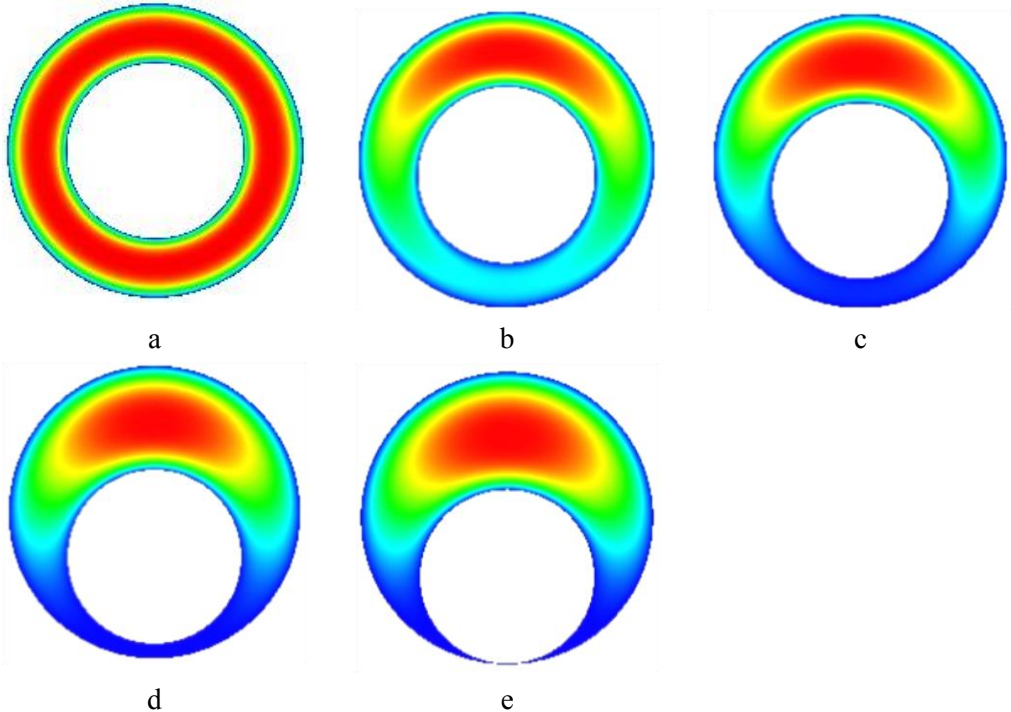


Figure 4. The isolines of the velocity modulus in the annular channel ($D_{out}=21\text{mm}$, $D_{in}=12.5\text{mm}$) for $G=0.473 \text{ kg/s}$ and different eccentricities, a) $e=0$ b) $e=0.25$ c) $e=0.5$ d) $e=0.75$ e) $e=1$.

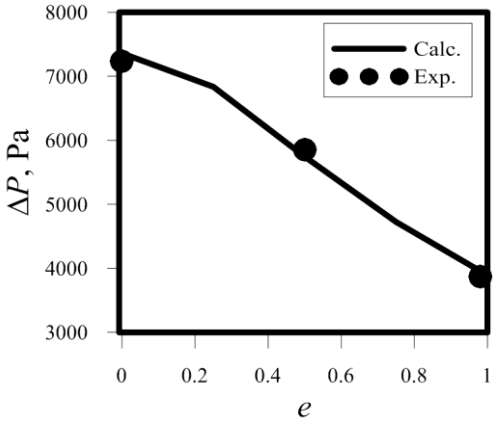


Figure 5. The pressure drop in the drilling fluid No.3 versus the eccentricity for flow rate $G=0.473 \text{ kg/s}$. Solid lines correspond to calculation, symbols are experiment.

4. Conclusion

The numerical and experimental investigation of laminar flow of power-law fluid has been conducted in an annular channel with the eccentricity. Three model solutions with properties close to real drilling fluids were examined. Experimental values of pressure drop are obtained depending on the rheological properties of drilling fluids, flow rates, and eccentricity. The obtained experimental data were used for

verification of the proposed numerical methodology. The calculated data coincided with good accuracy with the experimental data. In the future we plan to extend these studies to the case of turbulent flows.

Acknowledgments

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