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On Traces in Hardy Type Analytic Spaces in Bounded Strictly Pseudoconvex Domains and in Tubular Domains over Symmetric Cones

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We provide some new estimates on Traces in new mixed norm Hardy type spaces and related new results on Bergman type intergal operators in Hardy type spaces in tubular domains over symmetric cones and bounded striclty pseudoconvex domains with smooth boundary. We generalize a well-known one dimensional result concerning Traces of Hardy spaces obtained previously in the unit disk by various authors.

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In this note we give new estimates for traces of new mixed norm Hardy type spaces in tubular domains over symmetric cones and in bounded strictly pseudoconvex domains. We generalize a well-known one dimensional result concerning traces of Hardy spaces obtained previously in the unit disk by various authors. This trace problem in Hardy spaces(diagonal map problem) in the unit disk and in the unit polydisk was considered by many authors during last several decades. In polydisk some estimates related with this problem can be seen, for example, in [1, 2]. In polydisk, but in particular values of parameters in [3] and in some papers from list of references of [1, 2]. A rather long history related with these type estimates of traces of Hardy spaces can be read in [4] and in [2] also. In this paper we extend a known crucial estimate related with this problem to more general and complicated cases of tubular domains over symmetric cones and pseudoconvex domains with smooth boundary putting a natural condition on Bergman kernel of these domains. This paper can be considered also as continuation of a long series of papers of first author on traces of function spaces (see, for example, [5–8] and various references there).

In recent decades many papers appeared where various Hardy and other analytic spaces were studied from various points of views in higher dimension in various domains in $\mathbb{C}^n$. We refer for example to a series of papers of Krantz and coauthors (see [9–11] in particular) and also [12–15] in this direction. For some new interesting results on analytic spaces in tubular domains over symmetric cones we refer the reader to [16–19] and various references there also. We will heavily use nice techniques which was developed in these papers related with so-called lattices.

We start hoverer with a result in the unit ball. Then we define new mixed norm Hardy type classes in tubular domains and pseudoconvex domains (see [1, 2] for much simpler case of the unit polydisk). Then we provide a complete proof of our assertion in polyball and then provide assertions in bounded pseudoconvex domains with smooth boundary and in tubular domains over symmetric cones. In all cases proofs actually are the same.

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Note that first such type sharp trace results in more complicated bounded pseudoconvex domains were provided recently in [5, 6, 20] and in unbounded tubular domains over symmetric cones in [8]. Hoverer these papers recently covered only various analytic Bergman type and Herz type spaces cases.

The situation with Hardy spaces is more complicated even in simplest case of product domains that is the case of the unit polydisk (see [1, 2] and references there). We note trace results have many applications (see [20] for example). Note in more general than unit disk and unit polydisk case the case of polyball such type results were proved earlier in [5–7] in BMOA type and other analytic function spaces.

To provide our assertion we will need some basic notations and lemmas in the case of the unit ball and the unit polyball, in tubular domains over symmetric cones and in bounded strongly pseudoconvex domains with smooth boundary (see, for example, [5–7] and references there).

Let B be the unit ball in $\mathbb{C}^n$, let $H(B)$ be the space of all analytic functions in B. Moreover, let $d\nu$ denote the Lebesgue measure on B, normalizes such that $\nu(B) = 1$, and, for any $\alpha \in \mathbb{R}$, let $d\nu_\alpha(z) = c_\alpha \left(1 - |z|^2\right)^\alpha d\nu(z)$ for $z \in B_n$. Here, if $\alpha \leqslant -1$, $c_\alpha = 1$ and if $\alpha > -1$, $c_\alpha = \dfrac{\Gamma(n+\alpha+1)}{\Gamma(n+1)\Gamma(\alpha+1)}$ is the normalizing constant so that ν_α has unit total mass. The Bergman metric on B is $\beta(z,w) = \dfrac{1}{2}\log\dfrac{1+|\varphi_z(w)|}{1-|\varphi_z(w)|}$, where φ_z is the Mobius transformation of B that interchanges 0 and z. Let $D(a,r) = \{z \in B : \beta(z,a) < r\}$ denote the Bergman metric ball centered at $a \in B$ with radius $r > 0$.

The proofs of the following properties of the Bergman balls can be found in [21] (see Lemmas 1.24, 2.20, 2.24 and 2.27 in [21]).

Lemma A.

a) *There exists a positive number $N \geqslant 1$ such that, for any $0 < r \leqslant 1$, we can find a sequence $\{v_k\}_{k=1}^{\infty}$ in B to be r-lattice in the Bergman metric of B. This means that $B = \bigcup\limits_{k=1}^{\infty} D(v_k, r)$, $D(v_l, r/4) \cap D(v_k, r/4) = 0$ if $k \neq l$ and each $z \in B$ belongs to at most N of the sets $D(v_k, 2r)$.*

b) *For any $r > 0$, there is a constant $C > 0$ so that $\dfrac{1}{C} \leqslant \left|\dfrac{1-\langle z,w\rangle}{1-\langle z,v\rangle}\right| \leqslant C$ for all $z \in B$ and all w, v with $\beta(w,v) < r$.*

c) *For any $\alpha > -1$ and $r > 0$, $\displaystyle\int_{D(z,r)} (1-|w|^2)^\alpha dv(w)$ is comparable with $(1-|z|^2)^{n+1+\alpha}$ for all $z \in B$.*

d) *Suppose $r > 0$, $p > 0$ and $\alpha > -1$. Then there is a constant $C > 0$ such that*

$$|f(z)|^p \leqslant \frac{c}{(1-|z|^2)^{n+1+\alpha}} \int_{D(z,r)} |f(z)|^p d\nu_\alpha(w),$$

for all $f \in H(B)$ and all $z \in B$.

Let

$$(T_\alpha f)(z_1, \ldots, z_m) = c_\alpha \int_B \frac{f(w)(1-|w|^2)^\alpha dv(w)}{\prod\limits_{j=1}^{m} (1-\langle z_j, w\rangle)^{\frac{n+1+\alpha}{m}}},$$

$z_j \in B$, $j = 1, \ldots, m$, $\alpha > -1$, $z_j \in B$, $j = 1, \ldots, m$, $c_\alpha = \dfrac{\Gamma(m+\alpha+1)}{\Gamma(\alpha+1)n!}$. For $m = 1$, $(Tf)(z) = f(z)$, $z \in B$, $\alpha > -1$. The question is to study the action of this operator in Hardy, Bergman spaces. For $m = 1$, $(Tf)(z) = f(z)$ there is nothing to show, because of Bergman representation formula. For general case this operator or it is analogues were used before by author in various estimates related with trace problem in Hardy, Bergman and BMOA classes in tubular and pseudoconevx domains (see [5–8, 22]).

Let $B^m = B \times \cdots \times B$, $m > 1$. Let $H(B^m)$ be the space of all analytic functions in B^m.

Let

$$H^{p_1,\dots,p_m}(B^m) = \left\{ g \in H(B^m) : \sup_{r_1,\dots,r_m<1} \int_S \left(\dots \int_S \left(\int_S |g(r_1\zeta_1, \dots, r_m\zeta_m)|^{p_1} d\sigma(\zeta_1) \right)^{\frac{p_2}{p_1}} \times \right. \right.$$
$$\left. \times\ d\sigma(\zeta_2) \dots \right)^{\frac{p_m}{p_{m-1}}} d\sigma(\zeta_m) < \infty \Big\},$$

$0 < p_i < \infty$, $i = 1, \dots, m$, where σ is a Lebegues measure on $S = \{z : |z| = 1\}$, $S = \partial B$. This is a new mixed norm analytic Hardy class on products of unit balls (polyball).

Similarly based on definition of one domain case we can define mixed norm Hardy classes on products of tubular and bounded strongly pseudoconvex domains as subspaces of $H(\Omega^m)$ and $H(T_\Omega^m)$, $m \in \mathbb{N}$ of spaces of all analytic functions on product domains $\Omega^m = \Omega \times \cdots \times \Omega$ or $T_\Omega^m = T_\Omega \times \cdots \times T_\Omega$. The well-known definitions of classical Hardy spaces on tubular and pseudoconevx domains can be seen in [15, 16, 18, 23, 24]. We denote them by $H^{p_1,\dots,p_m}(\Omega^m)$ and $H^{p_1,\dots,p_m}(T_\Omega^m)$, for all positive values of parameters. In case of bounded stongly pseudocnvex domains with smooth boundary the sphere S in our defintion must be simply replaced by special $\partial\Omega_\varepsilon$ domains closely related with so-called defining r function of pseudoconvex D or Ω domains to be more precise it is a set of those points of domain for which our defining function r is constant (see for standard analytic Hardy classes in bounded pseudoconvex domains with smooth boundary, for example, [9–11, 15]).

We provide first a full and rather short proof of our main result in polyball case based on Lemmas we just formulated. Then we add some vital comments concerning this proof and formulate complete analogues of this result in tubular and pseudoconvex domains under an additional natural condition on Bergman kernel of these domains.

Theorem 1. *Let* $f \in H(B)$; $0 < p_j < 1$; $p_j > p_{j+1}$, $j = 1, \dots, m$. *If*

$$\int_B |f(z)|^{p_m} (1 - |z|)^{np_m(\sum_{i=1}^{m-1} \frac{1}{p_i} - 1) - 1} dv(z) < \infty$$

and $\alpha > \dfrac{2n}{p_m} - n - 1$ *then we have* $T_\alpha f \in H^{p_1,\dots,p_m}(B_n^m)$.

This result is well-known when $n = 1$, $p_i = p$, $i = 1, \dots, m$. (see [1, 2, 4] for example and references there for these values of parameters in the case of unit polydisk). For these values of parameters it provides a sharp embedding for traces of classical H^p Hardy spaces in the unit polydisk, (see [1, 2, 4]).

Proof of Theorem 1. The proof of Theorem 1 is fully based on properties of r-lattices of the unit ball (see Lemma A). We consider the case $m = 2$, i.e. if $\int_B |f(w)|^{p_2} (1 - |z|)^{\frac{np_2}{p_1} - 1} dv(z) < \infty$, $p_2, p_1 < 1$ we prove that

$$\sup_{r_1, r_2 < 1} \int_S \left(\int_S |T_\alpha f(z_1, z_2)|^{p_1} d\sigma(\zeta_1) \right)^{\frac{p_2}{p_1}} d\sigma(\zeta_2) < \infty;\ \zeta_j = r_j \zeta_j;\ j = 1, 2.$$

The general case is the same. We use properties of r-lattices listed above (see Lemma A).

We have the following estimates $\left(\gamma = \dfrac{n+1+\alpha}{2}\right)$:

$$J = \int_S \left(\int_S |T_\alpha f(z_1, z_2)|^{p_1} d\sigma(\zeta_1) \right)^{\frac{p_2}{p_1}} d\sigma(\zeta_2) \leqslant$$

$$\leqslant c \sum_{k=1}^{\infty} \sup_{z \in D(a_k, r)} |f(z)|^{p_2} \int_S \left(\int_S \left(\int_{D(a_k, r)} \frac{(1-|w|)^\alpha dv(w)}{\prod_{j=1}^{n} (|1 - \langle z_j, \zeta_j \rangle|^\gamma)} \right)^{p_1} d\sigma(\zeta_1) \right)^{\frac{p_2}{p_1}} d\sigma(\zeta_2) =$$

$$= c \sum_{k=1}^{\infty} (F_k G_k); \quad F_k = \sup_{z \in D(a_k, r)} |f(z)|^{p_2}.$$

Note

$$G_k \leqslant (1-|a_k|)^{p_2(n+1+\alpha)} \left(\sup_{r_2 < 1} \int_S \frac{d\sigma(\zeta_1)}{|1 - \langle r_1 \zeta_1, a_k \rangle|^{p_1 \gamma}} \right)^{\frac{p_2}{p_1}} \left(\sup_{r_2 < 1} \int_S \frac{d\sigma(\zeta_1)}{|1 - \langle r_2 \zeta_2, a_k \rangle|^{p_2 \gamma}} \right).$$

Hence we have that the following chain of estimates is valid based on Lemma A

$$J \leqslant c \sum_{k=1}^{\infty} \sup_{z \in D(a_k, r)} |f(z)|^{p_2} (1 - |a_k|)^{n + \frac{n p_2}{p_1}} \leqslant$$

$$\leqslant c \sum_{k=1}^{\infty} \int_{D(a_k, 2r)} |f(z)|^{p_2} (1-|z|)^{n \frac{p_2}{p_1} - 1} dv(z) \leqslant c_N \int_B |f(z)|^{p_2} (1-|z|)^{n \frac{p_2}{p_1} - 1} dv(z) < \infty.$$

Note this chain of estimates is valid also in tubular and pseudoconevx domains based on properties of r-lattices of these domains, we refer to [25] and [26] for this.

This completes the proof of theorem in case of the unit ball. □

We provide some discussion on this proof. A carefull analysis of this proof shows similar results should be valid also in other domains. Indeed our arguments are valid, for example, in tubular domains and strongly pseudoconvex domains based on r-lattices of those domains (see [16, 26]). First for the proof in tubular, pseudoconvex domains we need a version of Lemma A in these domains. Complete analogues of first, third and forth assertions of Lemma A in tubular domains and pseudoconvex domains can be seen in pseudoconvex domains in [24, 25], in tubular domains in [16–18, 23]. The second assertion of Lemma A is also valid in these domains, but only in milder form [25, 26]. This hoverer, is vital for our proof and will stand in our formulations below as an additional condition on Bergman kernel both in tube and pseudoconvex domains.

Note further the last chain of estimates in our proof is valid also in polydisk, bounded symmetric domains, in tubular domains over symmetric cones and in bounded pseudoconvex domains with smooth boundary based on same properties of r-lattices of these domains (we refer to [25] and [26] for these standard arguments in tubular domains and pseudoconvex domains, to [4] for polydisk case).

To repeat estimates of G_k (see again our proof in the ball) in context of tube and pseudoconevx domains we refer to theorem 2 of [27] for pseudoconvex domains and to basic properties of r defining function of the pseudoconvex domain (see [27]) and for tubular domains to a Lemma from [17], and [19] concerning integrability of Δ function. To be more precise, for example, in tubular domains over symmetric cones it is the following well-known basic integrability properity for the determinant Δ function(see for this assertion [16] or [18]). It is complete analogues in pseudoconvex domains can be seen in [27]

Lemma B. *The integral*

$$J_\alpha(y) = \int\limits_{\mathbb{R}^n} \left| \Delta^{-\alpha} \left(\frac{x+iy}{i} \right) \right| dx$$

converges if and only if $\alpha > 2\frac{n}{r} - 1$. *In that case*

$$J_\alpha(y) = \widetilde{C}_\alpha \Delta^{-\alpha+n/r}(y),$$

$\alpha \in R, y \in \Omega$.

In addition we must use the fact that Δ function is "monotone" on Ω cone (see [16] or [18] for these basic properties). We omit easy details.

We denote by dv and dV as usual Lebegues measures on tube T_Ω and pseudoconvex Ω domains.

The Bergman kernels in tubular domains and pseudoconvex domains will be denoted as usual by $K_t(z,w)$ and $B_\nu(z,w)$ (see, for example, [16, 18, 24, 25]). The Bergman B_ν kernel for tubular domains can be written via so-called Δ function in the following manner

$$B_\nu(z,w) = C_\nu \Delta^{-(\nu+\frac{n}{r})}((z-\overline{w})/i),$$

$z \in T_\Omega,\ w \in T_\Omega$ (see [16, 28] for more details).

We formulate analogues of our first theorem in tubular domains over symmetric cones and in bounded strictly pseudoconvex domains with smooth boundary. We will omit calculations leaving them to interested readers.

We first define the complete analogue of Bergman type T_α integral operator in pseudoconvex and tubular domains using Bergman kernels in the following way. In the tubular domain over symmetric cones we define them as the following Bergman-type integral operators.

$$T_\alpha(f)(\vec{z}) = \int_{T_\Omega} \frac{f(w)\Delta^\alpha(\operatorname{Im} w)dv(w)}{\prod\limits_{j=1}^{m} \Delta^{\frac{\alpha+2\frac{n}{r}}{m}}(\frac{z_j-\bar{w}}{i})};\ f \in L^1(T_\Omega),$$

$\alpha > -1$, $z_j \in T_\Omega$, $j = 1, \dots, m$.

In bounded pseudoconvex domains with smooth boundary Ω we define them as

$$T_\alpha f(\vec{z}) = \int_\Omega f(w) \prod_{j=1}^{m} K_\tau(z_j,w)\delta^\alpha(w)dv(w);\quad \tau = \frac{\alpha+n+1}{m},$$

$f \in L^1(\Omega)$, $\alpha > -1$, $z_j \in \Omega$, $j = 1, \dots, m$. Such type operators were studied before in [5–7] in our papers. They play a crucial role in various Trace theorems related with Herz and Bergman spaces in such type domains (see [5–7],and also in [8]) . We define new mixed norm analytic Hardy spaces in products of tubular domains over symmetric cones as follows. Let further for all positive values of parameters

$$H^{p_1,\dots,p_m}(T^m_\Omega) = \left\{ f \in H(T^m_\Omega) : \sup_{y_j \in \Omega} \left[\int_{R^n} \int_{R^n} \left(\int_{R^n} |f(\vec{x}+i\vec{y})|^{p_1} dx_1 \right)^{\frac{p_2}{p_1}} \dots dx_m \right]^{\frac{1}{p_m}} < \infty \right\},$$

$0 < p_i < \infty$, $i = 1, \dots, m$. We assume that for $B_t(z,w)$ function (the Bergman kernel of T_Ω domain) the following condition is valid. $|B_t(z,w)| \asymp |B_t(z,w_k)|$, $w \in B_{T_\Omega}(w_k,R)$; $z \in T_\Omega$, where t is positive and $B_{T_\Omega}(w,R)$ is a Bergman ball in tubular domain (see, for example, [16, 18, 23] for definitions of these objects and more discussions). We refer to [26] for milder version of this condition on Bergman kernel.

Theorem 2. *Let $f \in H(T_\Omega)$ and let $0 < p_i < 1,\ p_i > p_{i+1},\ i = 1, \dots, m$, if*

$$\int_{T_\Omega} |f(z)|^{p_m} \Delta(Im\ z)^{\frac{n}{r} p_m \sum_{i-1}^{m-1} \frac{1}{p_i} - 1} d\tilde{V}(z) < \infty$$

and if $\alpha > \alpha_0$ for large enough α_0, then $T_\alpha f \in H^{p_1, \dots, p_m}(T_\Omega^m)$.

Let further,

$$H^{p_1,\dots,p_m}(\Omega^m) = \left\{ f \in H(\Omega^m) : \sup_{\varepsilon > 0} \left[\int_{\Omega_\varepsilon} \int_{\Omega_\varepsilon} \left(\int_{\Omega_\varepsilon} |f(\vec{w})|^{p_1} dw_1 \right)^{\frac{p_2}{p_1}} \dots dw_m \right]^{\frac{1}{p_m}} < \infty \right\}.$$

We assume that for the Bergman K_t kernel of Ω bounded pseudoconvex domains with smooth boundary the following assertion is valid. $|K_t(z, w)| \asymp |K_t(z, w_k)|$, $w \in B_\Omega(w_k, R)$, $z \in \Omega$,where $B_\Omega(w, R)$ is a Kobayashi ball (we refer for more details on these objects, for example, to [24,25]). We note in these papers the same condition on Bergman kernel, but in a bit milder form can be also found.

Theorem 3. *Let $f \in H(\Omega)$, $0 < p_i < 1,\ p_i > p_{i+1},\ i = 1, \dots, m$ if*

$$\int_\Omega |f(z)|^{p_m} \delta^{n p_m \sum_{i=1}^{m-1} \frac{1}{p_i} - 1}(z) d\tilde{v}(z) < \infty$$

and if $\alpha > \alpha_0$ for large enough α_0 then $T_\alpha f \in H^{p_1, \dots, p_m}(\Omega^m)$.

Finnaly we remark some analogues of these assertions with similar proofs may be true in some other types of complicated domains, for example, in minimal homogenious domains, in bounded symmetric domains, but again our proof may work under some natural additional condition on Bergman kernel which in some cases probably may be removed. We note for these domains we also have similar properties of r-lattices and the complete analogue of Lemma A (see, for example [29, 30] and various references there).

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О следах в аналитических пространствах типа Харди в ограниченных строго псевдовыпуклых областях и в трубчатых областях над симметрическими конусами

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В работе представлены некоторые новые оценки для следов в новых пространствах типа Харди со смешанной нормой и связанные с ними результаты, касающиеся итегральных операторов типа Бергмана в пространствах типа Харди в трубчатых областях над симметрическими конусами и ограниченных строго псевдовыпуклых областях с гладкой границей. Мы расширяем хорошо известный одномерный результат, полученный ранее для случая следов в пространствах Харди в единичном круге различными авторами.

Ключевые слова: ограниченные строго псевдовпуклые области, трубчатые области над симметрическими конусами, пространства типа Харди.