Journal of Siberian Federal University. Mathematics & Physics 2011, 4(4), 419–433

УДК 519.248: [33+301+159.9]

Many-event-based Models of Market Following from the Eventological H-theorem

Oleg Yu. Vorobyev*
Ellen E. Goldenok†
Institute of Mathematics,
Siberian Federal University,
Svobodny, 79, Krasnoyarsk, 660041
Russia

Received 09.02.2011, received in revised form 09.03.2011, accepted 09.05.2011

Many-events-based and many-agent eventological models of supply and demand are offered. In these models new concepts of equilibrium intervals of prices and subsets of goods are introduced for the first time. These models follow from the eventological H-theorem (eventological generalization of Boltzmann's H-theorem) which serves as a theoretical base for an application of eventological distributions of supply and demand in the mathematical description of many-agent and many-events-based market systems.

Keywords: event, probability, eventology, eventological H-theorem, demand, supply, market, many-agent system, many-events-based system, equilibrium interval of prices, equilibrium interval of sets of goods.

Nobody will stop a course of events.

Eventology [1], the theory of events, is a new area of philosophical, mathematical, and applied researches. This theory bases on some observations: "event is always co-being" [2]; "matter is a simply convenient way of linkage of events together" [3]; "mental behavior arises there and then, where and when an ability to make probabilistic choice is arisen" [4]; and "mind is a way of probabilistic linkage of events together within many-events-based co-being" [1]. Starting with the classical Kolmogorov's probability theory, we develop an axiomatic basis for the formulation of eventological laws of development of many-agent and many-events-based systems.

The eventology enters the agent directly into a subject of scientific and mathematical research in the form of a strict mathematical concept: an *eventological distribution (E-distribution)* of the set of his/her own events describing his/her individual physical, reasonable and spiritual behaviour. Since Kolmogorov's probability theory [5] we build an axiomatic basis of the new theory for the formulation of eventological laws of development of many-agent and many-events-based systems.

Studying of the supply and demand is the fundamental approach to economics, which researches and compares the supply of a good (service)‡ with demand for them (it is usual in the form of graphics of curves of supply and demand concerning prices of goods). Graphics of the "eventological supply and demand" are represented with a variable "price" on one axis and a new variable "probability" (of events-supplies or of events-demands) on another axis. Unlike classical "models of the supply and demand" eventological models of supply and demand use the new variable "probability" (of events-supplies or of events-demands), which plays a role of the variable "quantity of goods" (supplied or demanded), which is used in economics. Thus we offer the wider eventological market model which is intended for the mathematical description of E-distributions of sets of market events of supply or demand under a condition of many-agent

*vorob@akadem.ru
†ellen_gold@rambler.ru

‡Here it is possible to speak both about goods and about services.

and many-events-based uncertainty. This model allows to expand the classical concept of the equilibrium price of supply and demand by the new concept of an eventological equilibrium interval of the prices and of subsets of goods.

As theoretical base of the eventological model of many-events-based and many-agent market systems the eventological H-theorem, the eventological generalization of Boltzmann's H-theorems [6–8], serves. The eventological H-theorem approves, that joint distribution of sets of events of supply and demand, occurring in many-agent market systems, has extreme entropy properties and forms a class of E-distributions of the Boltzmann type or of the Gibbs type. These extreme E-distributions not only in a new fashion define the function of supply and the function of demand (the well-known cross of supply and demand from economics), but also introduce two newer functions of concurrence and distinction of supply and demand. These four functions open an opportunity to expand the classical concept of an equilibrium market price up to an equilibrium interval of prices and of subsets of goods.

1. Many-events-based and Many-agent Market as an Eventological Market Model

In the eventological theory, each market agent (the buyer, the seller) is characterized by his/her own distribution of a set of the market events connected with some set of goods, circulating within the given market. The set of all market agents, acting within the given commodity market, is divided on two parts: the set of all buyers which is characterized by total distribution of a set of events-demands, and the set of all sellers, which is characterized by total distribution of a set of events-supplies.

With each goods $x \in \mathfrak{X}$ from the finite set of market goods $\mathfrak{X}$ two basic *market events* connected (Fig. 1): the *event-demand* $x^{\downarrow}$ and the *event-supply* $x^{\uparrow}$. These two basic events generate derivative events: the *event-concurrence* $x^{\downarrow} \cap x^{\uparrow}$, the event-not-demand-supply $(x^{\downarrow})^c \cap x^{\uparrow}$, the event-demand-not-supply $x^{\downarrow} \cap (x^{\uparrow})^c$ and the event-absence of market $(x^{\downarrow})^c \cap (x^{\uparrow})^c$. Two of the derivative events jointed together make up the important market event: the *event-distinction* $x^{\downarrow} \Delta\, x^{\uparrow} = (x^{\downarrow})^c \cap x^{\uparrow} + x^{\downarrow} \cap (x^{\uparrow})^c$.

So any many-events-based market with finite set of goods $\mathfrak{X}$ is defined by the set of events-demands

$$\mathfrak{X}^{\downarrow} = \{x^{\downarrow}, x \in \mathfrak{X}\} \subseteq \mathcal{F},$$

and by the set of events-supplies

$$\mathfrak{X}^{\uparrow} = \{x^{\uparrow}, x \in \mathfrak{X}\} \subseteq \mathcal{F},$$

where $x^{\downarrow}$ is an event-demand for the goods $x \in \mathfrak{X}$, $x^{\uparrow}$ is an event-supply of the goods $x \in \mathfrak{X}$ at the given event market, and $\mathcal{F}$ is the algebra of events of the universal probability space $(\Omega, \mathcal{F}, \mathbf{P})$ [9].

Let's suppose, that

$$\{p^{\downarrow}(X^{\downarrow}), X^{\downarrow} \subseteq \mathfrak{X}^{\downarrow}\}, \quad \{p^{\uparrow}(Y^{\uparrow}), Y^{\uparrow} \subseteq \mathfrak{X}^{\uparrow}\} \tag{$\star$}$$

are *E-distributions*[§] of two sets of events $\mathfrak{X}^{\downarrow}$ and $\mathfrak{X}^{\uparrow}$, where

$$p^{\downarrow}(X^{\downarrow}) = \mathbf{P}\Big(\mathrm{ter}^{\downarrow}(X^{\downarrow})\Big) \tag{1}$$

is the probability of demand only for the subset of goods $X \subseteq \mathfrak{X}$,

$$p^{\uparrow}(X^{\uparrow}) = \mathbf{P}\Big(\mathrm{ter}^{\uparrow}(X^{\uparrow})\Big) \tag{1'}$$

[§]E-distributions $p^{\downarrow}$ and $p^{\uparrow}$ define probabilities of events-terraces $\mathrm{ter}^{\downarrow}(X) = \bigcap_{x \in X} x^{\downarrow} \bigcap_{x \in X^c} (x^{\downarrow})^c$ and $\mathrm{ter}^{\uparrow}(Y) = \bigcap_{x \in Y} x^{\uparrow} \bigcap_{x \in Y^c} (x^{\uparrow})^c$ accordingly for any subsets $X, Y \in 2^{\mathfrak{X}}$. Here $2^{\mathfrak{X}}$ is the set of all subsets of finite set of goods $\mathfrak{X}$.

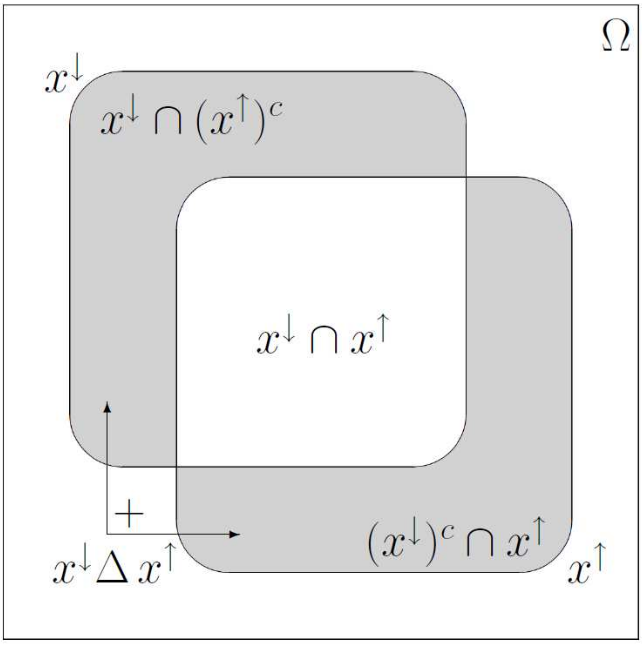

Fig. 1. The Venn diagram of the *event-demand* $x^{\downarrow}$ and the *event-supply* $x^{\uparrow}$ which are connected to any market goods $x \in \mathfrak{X}$. These two basic market events generate derivative events: the *event-concurrence* $x^{\downarrow} \cap x^{\uparrow}$, the event-not-demand-supply $(x^{\downarrow})^c \cap x^{\uparrow}$, the event-demand-not-supply $x^{\downarrow} \cap (x^{\uparrow})^c$ and the event-absence of market $(x^{\downarrow})^c \cap (x^{\uparrow})^c$. Two of the derivative events (jointed together) form the important event: the *event-distinction* $x^{\downarrow} \Delta x^{\uparrow} = (x^{\downarrow})^c \cap x^{\uparrow} + x^{\downarrow} \cap (x^{\uparrow})^c$

is the probability of supply only of the set of goods $X \subseteq \mathfrak{X}$, and

$$\{p^{\downarrow\uparrow}(X^{\downarrow}, Y^{\uparrow}), X^{\downarrow} \subseteq \mathfrak{X}^{\downarrow}, Y^{\uparrow} \subseteq \mathfrak{X}^{\uparrow}\} \tag{$\star\star$}$$

is the *joint E-distribution*[¶] of two sets of events $\mathfrak{X}^{\downarrow}$ and $\mathfrak{X}^{\uparrow}$, where

$$p^{\downarrow\uparrow}(X^{\downarrow}, Y^{\uparrow}) = \mathbf{P}\Big(\mathrm{ter}^{\downarrow}(X^{\downarrow}) \cap \mathrm{ter}^{\uparrow}(Y^{\uparrow})\Big) \tag{2}$$

is the probability of demand only for the subset of goods $X \subseteq \mathfrak{X}$ and of supplies only of the subset of goods $Y \subseteq \mathfrak{X}$.

Let's omit excessive arrows in formulas $(\star), (1), (1'), (\star\star)$ and (2) (and in similar formulas below) and rewrite these formulas in more compressed kind:

$$\{p^{\downarrow}(X), X \subseteq \mathfrak{X}\}, \quad \{p^{\uparrow}(Y), Y \subseteq \mathfrak{X}\}, \tag{$\star$}$$

$$p^{\downarrow}(X) = \mathbf{P}\Big(\mathrm{ter}^{\downarrow}(X)\Big), \tag{1}$$

$$p^{\uparrow}(X) = \mathbf{P}\Big(\mathrm{ter}^{\uparrow}(X)\Big), \tag{1'}$$

$$\{p^{\downarrow\uparrow}(X, Y), X \subseteq \mathfrak{X}, Y \subseteq \mathfrak{X}\}, \tag{$\star\star$}$$

[¶]Joint E-distribution $p^{\downarrow\uparrow}$ defines probabilities of events, which look like intersections of events-terraces: $\mathrm{ter}^{\downarrow}(X) \cap \mathrm{ter}^{\uparrow}(Y)$ for any pair of subsets $(X, Y) \in 2^{\mathfrak{X}} \times 2^{\mathfrak{X}}$. Here $2^{\mathfrak{X}} \times 2^{\mathfrak{X}} = \{(X, Y),\ X \subseteq \mathfrak{X}, Y \subseteq \mathfrak{X}\}$ is the direct (cartesian) product of the set $2^{\mathfrak{X}}$ on itself.

$$p^{\downarrow\uparrow}(X,Y) = \mathbf{P}\Big(\mathrm{ter}^{\downarrow}(X) \cap \mathrm{ter}^{\uparrow}(Y)\Big). \tag{2}$$

In eventological theory each set of events has two equivalent mathematical models. Therefore, it is possible to speak similarly as about *sets of events* $\mathfrak{X}^{\downarrow}$ and $\mathfrak{X}^{\uparrow}$, and about *random sets of events* $K^{\downarrow}$ and $K^{\uparrow}$ which have the same E-distributions of a kind (1).

Random sets of events $K^{\downarrow}$ and $K^{\uparrow}$ are random elements with outputs look like subsets of events-demands and of events-supplies accordingly which are defined as follows.

$$K^{\downarrow} : (\Omega, \mathcal{F}, \mathbf{P}) \to \left(2^{\mathfrak{X}^{\downarrow}}, 2^{2^{\mathfrak{X}^{\downarrow}}}\right)$$

is the *random set of events-demands* with the E-distribution[‖] for all $X \subseteq \mathfrak{X}$ of a kind:

$$p^{\downarrow}(X) = \mathbf{P}(K^{\downarrow} = X^{\downarrow}) \tag{1}$$

which coincides with the E-distribution of the set of events-demands $\mathfrak{X}^{\downarrow}$.

$$K^{\uparrow} : (\Omega, \mathcal{F}, \mathbf{P}) \to \left(2^{\mathfrak{X}^{\uparrow}}, 2^{2^{\mathfrak{X}^{\uparrow}}}\right)$$

is the *random set of events-supplies* with the E-distribution[**] for all $Y \subseteq \mathfrak{X}$ of a kind:

$$p^{\uparrow}(Y) = \mathbf{P}(K^{\uparrow} = Y^{\uparrow}) \tag{1$'$}$$

which coincides with the E-distribution of the set of events-supplies $\mathfrak{X}^{\uparrow}$.

The*pair of random sets* of events $K^{\downarrow}$ and $K^{\uparrow}$ has for all $X \subseteq \mathfrak{X}, Y \subseteq \mathfrak{X}$ the joint E-distribution[††] of a kind:

$$p^{\downarrow\uparrow}(X,Y) = \mathbf{P}(K^{\downarrow} = X^{\downarrow}, K^{\uparrow} = Y^{\uparrow}), \tag{2}$$

which coincides with the E-distribution of the pair of sets of events $\mathfrak{X}^{\downarrow}$ and $\mathfrak{X}^{\uparrow}$.

1.1. Eventological Characteristics of the Many-events-based and Many-agent Market

In the paper the *many-agent and many-events-based market of the set of goods* $\mathfrak{X}$ refers to the *market* for brevity in the further.

Each participant of the market (individual or total[‡‡]) has his/her own E-distribution of the set of market events-demands $\mathfrak{X}^{\downarrow}$ or the set of market events-supplies $\mathfrak{X}^{\uparrow}$ or that is equivalent has the E-distribution of random sets of events $K^{\downarrow}$ or $K^{\uparrow}$. The participant of a market demand and the participant of a market supply together have the joint E-distribution of pair of sets of events $\mathfrak{X}^{\downarrow}$ and $\mathfrak{X}^{\uparrow}$ or that is equivalent, have the joint E-distribution of pair of random sets of events $K^{\downarrow}$ or $K^{\uparrow}$.

So, E-distributions of corresponding market events act in a role of eventological mathematical models of behaviour of each participant of the market separately, and also their joint market behaviour. In the market with the set of goods $\mathfrak{X}$ two participants are considered: the *total buyer* with the own E-distribution of events-demands $p^{\downarrow}$, and the *total seller* with the own E-distribution of events-supplies $p^{\uparrow}$. An eventological model of many-agent and many-events-based

[‖]E-distribution $p^{\downarrow}$ defines probabilities of events-terrace $\mathrm{ter}^{\downarrow}(Y) = \{\omega : K^{\downarrow}(\omega) = X^{\downarrow}\}$ for any subsets $X \in 2^{\mathfrak{X}}$.

[**]E-distribution $p^{\uparrow}$ defines probabilities of events-terrace $\mathrm{ter}^{\uparrow}(X) = \{\omega : K^{\uparrow}(\omega) = Y^{\uparrow}\}$ for any subsets $Y \in 2^{\mathfrak{X}}$.

[††]Joint E-distribution $p^{\downarrow\uparrow}$ definesdetermines probabilities of events which look like crossings of events-terrace $\mathrm{ter}^{\downarrow}(X) \cap \mathrm{ter}^{\uparrow}(Y) = \{\omega : K^{\downarrow}(\omega) = X^{\downarrow}, K^{\uparrow}(\omega) = Y^{\uparrow}\}$ for any pair subsets $(X,Y) \in 2^{\mathfrak{X}} \times 2^{\mathfrak{X}}$.

[‡‡]E-distribution *the total participant of the market* is formed by a convex combination of E-distributions *individual participants* their participations weighed in view of shares in total market behaviour.

market interaction of the total buyer and the seller including all nuances of their mutual market behaviour is the joint E-distribution of events-demands and events-supplies $p^{\downarrow\uparrow}$.

It is necessary to notice, that the joint E-distribution of supply and demand $p^{\downarrow\uparrow}$ eventologically completely describes a market situation, including the E-distribution of only demand:

$$p^{\downarrow}(X) = \sum_{Y \subseteq \mathfrak{X}} p^{\downarrow\uparrow}(X, Y),$$

and the E-distribution of only supply:

$$p^{\uparrow}(Y) = \sum_{X \subseteq \mathfrak{X}} p^{\downarrow\uparrow}(X, Y).$$

The converse is incorrect. To define the joint E-distribution of supply and demand the knowledge of the marginal E-distribution of demand and of the marginal E-distribution of supply does not suffice. Because the joint E-distribution of supply and demand includes still incomparably more extensive information on many-agent and many-events-based interaction of supply and demand, which is absent in marginal E-distributions.

1.2. Local Characteristics

The eventological market model based on the joint E-distribution of supply and demand (of the total buyer and the total seller) allows to define following local event-based and probability characteristics of the market i.e. the characteristics connected with concrete sets of goods $X, Y \subseteq \mathfrak{X}$.

Event-demand for $X \subseteq \mathfrak{X}$:

$$\mathrm{ter}^{\downarrow}(X) = \bigcap_{x \in X} x^{\downarrow} \bigcap_{x \in X^c} (x^{\downarrow})^c = \{K^{\downarrow} = X^{\downarrow}\}.$$

Probability of demand for $X \subseteq \mathfrak{X}$:

$$p^{\downarrow}(X) = \mathbf{P}\left(\mathrm{ter}^{\downarrow}(X)\right) = \mathbf{P}\left(K^{\downarrow} = X^{\downarrow}\right).$$

Event-supply of $Y \subseteq \mathfrak{X}$:

$$\mathrm{ter}^{\uparrow}(Y) = \bigcap_{x \in Y} x^{\uparrow} \bigcap_{x \in Y^c} (x^{\uparrow})^c = \{K^{\uparrow} = Y^{\uparrow}\}.$$

Probability of supply of $Y \subseteq \mathfrak{X}$:

$$p^{\uparrow}(Y) = \mathbf{P}\left(\mathrm{ter}^{\uparrow}(Y)\right) = \mathbf{P}\left(K^{\uparrow} = Y^{\uparrow}\right).$$

1.3. Global Characteristics

The eventological market model based on the joint E-distribution of supply and demand (of the total buyer and the total seller), allows to define following global event-based and probability characteristics of the market.

Probability of the market:

$$\mathbf{P}(K^{\downarrow} \neq \varnothing, K^{\uparrow} \neq \varnothing)$$

Probability of absence of the market:

$$1 - \mathbf{P}(K^{\downarrow} \neq \varnothing, K^{\uparrow} \neq \varnothing) = p^{\downarrow}(\varnothing) + p^{\uparrow}(\varnothing) - p^{\downarrow\uparrow}(\varnothing, \varnothing).$$

Probability of demand in the market:

$$\mathbf{P}(K^{\downarrow} \neq \varnothing) = 1 - p^{\downarrow}(\varnothing).$$

Probability of supply in the market:

$$\mathbf{P}(K^{\uparrow} \neq \varnothing) = 1 - p^{\uparrow}(\varnothing)$$

2. Variants of Eventological Market Models

The eventological model of the many-events-based and many-agent market considers a supply and a demand exclusively as a *many-events-based demand* and a *many-events-based supply* in following sense. Any demand (or a supply) is considered as a special case of a *set demand* (or of a *set supply*). In particular, an absence of demand is considered as the demand for empty set of goods $\varnothing \subseteq \mathfrak{X}$, and the demand for one goods $x \in \mathfrak{X}$ is considered as the demand for monoplet of goods $\{x\} \subseteq \mathfrak{X}$, the set containing only one goods $x \in \mathfrak{X}$.

Assumption 1a (one-set demand). The eventological market model during each moment defines a demand only for any *one set* of goods by only any *one set* of agents*. In other words, the given model does not provide market situations of simultaneous demand for some sets of goods by several sets of agents. This assumption is completely not burdensome, as any variant of demand can be reduced to simultaneous demand only for one set of goods by one set of agents. For example, the demand for some separate goods $x, y, \dots, z$ in the given market model is treated as "slightly" divided in time a little separate consecutive demands for monoplets $\{x\}, \{y\}, \dots, \{z\}$, the sets containing only one goods.

Assumption 1b (one-set supply). The eventological market model during each moment defines a supply only any *one set* of goods by only any *one set* of agents. In other words, the given model does not provide market situations of simultaneous supply of several sets of goods by several sets of agents. This assumption also is not burdensome, as any variant of supply can be reduced to simultaneous supply only one set of goods by one set of agents. For example, the supply of several separate goods $x, y, \dots, z$ in the given market model is treated as " slightly" divided in time some separate consecutive supplies of monoplets $\{x\}, \{y\}, \dots, \{z\}$, the sets containing only one goods.

Assumption 2a (full-set demand). The eventological market model during each moment defines simultaneous demand for all subsets of any *one set* of goods. In other words, demand for only one set of goods $X \subseteq \mathfrak{X}$ is equivalent to simultaneous demand for all its subsets $Y \subseteq X$, i.e. for all set of its subsets 2^X.

Assumption 2b (full-set supply). The eventological market model during each moment defines simultaneous supply of all subsets of any *one set* of goods. In other words, the supply of only one set of goods $X \subseteq \mathfrak{X}$ is equivalent to simultaneous supply of all its subsets $Y \subseteq X$, i.e. of all set of its subsets 2^X.

Four variants of eventological market models (Fig. 2) include the simplest model based on assumptions 1a and 1b of one-set supply and demand. The most general model is based on assumptions 2a and 2b of full-set supply and demand. Assumptions 1a and 2b lead to the interesting model of one-set demand and full-set supply which looks quite naturally and can have wide applications. Assumptions 2a and 1b give the exotic market model which, in our opinion, meets seldom in practice.

All four variants of eventological market models differ among themselves with a kind of joint E-distribution of supply and demand and, in particular, with a definition of event-purchase-sale.

*In paper always only one fixed set of market agents, acting at the market, is given. Therefore, it is not used special designations for the given set of agents. The eventological market model provides the analysis of interaction of several markets in which various sets of agents, acting at corresponding markets, can be considered. We are going to analyze an eventological interaction between several markets in our further papers.

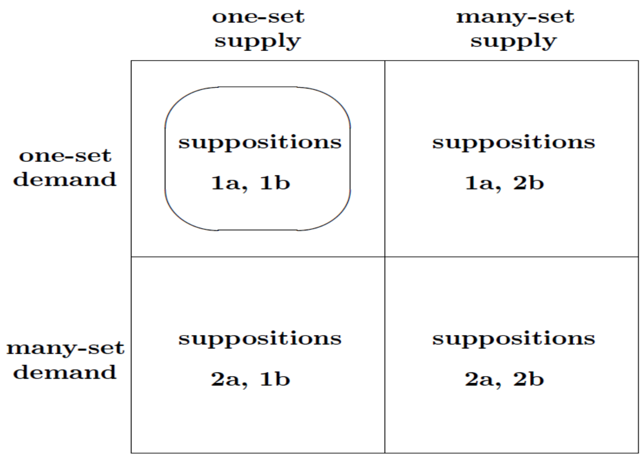

Fig. 2. Four variants of assumptions one- and full-set supply and demand, leading various eventological market models. In paper the simplest model of one-set demand and the one-set supply, based on assumptions 1a and 1b (the variant is allocated by the oval) is considered

In the paper the simplest eventological model of one-set demand and one-set supply (the variant 1a, 1b) is considered completely, and also the definition of event-purchase-sale is resulted under the one-set demand and full-set supply (the variant 1a, 2b).

3. Events-purchases-sales in Variants of an Eventological Market Model

3.1. One-set Supply and Demand (Variant 1a, 1b)

Assumptions of one-set supply and demand mean, that the event-purchase-sale comes only in the case when the demanded set of goods *coincides with* the supplied set of goods. Otherwise event-purchase-sale does not come.

Local characteristics. For the given set of goods $X \subseteq \mathfrak{X}$ the event-purchase-sale comes only when there comes both the event-demand $\text{ter}^{\downarrow}(X) = \{K^{\downarrow} = X\}$ and the event-supply $\text{ter}^{\uparrow}(X) = \{K^{\uparrow} = X\}$. In other words, the *event-purchase-sale* $X \subseteq \mathfrak{X}$ is an intersection of event-demand $X \subseteq \mathfrak{X}$ and event-supply $X \subseteq \mathfrak{X}$:

$$\text{ter}^{\downarrow\uparrow}(X, X) = \text{ter}^{\downarrow}(X) \cap \text{ter}^{\uparrow}(X) =$$
$$= \bigcap_{x \in X} x^{\downarrow} \bigcap_{x \in X^c} (x^{\downarrow})^c \bigcap_{x \in X} x^{\uparrow} \bigcap_{x \in X^c} (x^{\uparrow})^c =$$
$$= \{K^{\uparrow} = X^{\uparrow}, K^{\uparrow} = X^{\uparrow}\}.$$

The event-purchase-sale $\text{ter}^{\downarrow\uparrow}(X, X)$ will refer to also as the *event-concurrence-demand-supply* for the set of goods $X \subseteq \mathfrak{X}$.

Probability of purchase-sale of $X \subseteq \mathfrak{X}$ (probability of concurrence-demand-supply for $X \subseteq \mathfrak{X}$) is defined by "diagonal" probability from the joint E-distribution of supply and demand:

$$p^{\downarrow\uparrow}(X, X) =$$
$$= \mathbf{P}\left(\text{ter}^{\downarrow\uparrow}(X, X)\right) = \mathbf{P}\left(K^{\downarrow} = X^{\downarrow}, K^{\uparrow} = X^{\uparrow}\right). \quad (3)$$

Event-distinction-demand-supply of $X \subseteq \mathfrak{X}$ is defined as a result of operation of symmetric difference over events of supply and demand:

$$\text{ter}^{\downarrow}(X)\Delta\ \text{ter}^{\uparrow}(X).$$

This event comes when the event-purchase-sale of $X \subseteq \mathfrak{X}$ does not occur either under occurrence of event-demand for $X \subseteq \mathfrak{X}$, or under occurrence of event-supply of $X \subseteq \mathfrak{X}$.

Probability of distinction-demand-supply of $X \subseteq \mathfrak{X}$ is also defined by the joint E-distribution of supply and demand and calculated by the formula:

$$\begin{aligned} d^{\downarrow\uparrow}(X) &= \mathbf{P}(\text{ter}^{\downarrow}(X)\Delta\ \text{ter}^{\uparrow}(X)) = \\ &= p^{\downarrow}(X) + p^{\uparrow}(X) - 2p^{\downarrow\uparrow}(X,X). \end{aligned} \tag{4}$$

Global characteristics. The market as a whole is characterized by the event-concurrence-demand-supply

$$\sum_{X \neq \varnothing} \text{ter}^{\downarrow\uparrow}(X,X),$$

by the event-distinction-demand-supply[†]

$$\Omega - \big(\text{ter}^{\downarrow}(\varnothing) \cup \text{ter}^{\uparrow}(\varnothing)\big) - \sum_{X \neq \varnothing} \text{ter}^{\downarrow\uparrow}(X,X)$$

and by its probabilities.

Probability of concurrence-demand-supply (probability of purchase-sale) in the market:

$$\mathbf{P}(K^{\downarrow} = K^{\uparrow}, K^{\downarrow} \neq \varnothing) = \sum_{X \neq \varnothing} p^{\downarrow\uparrow}(X,X)$$

characterizes an activity of the market as a whole, allows to compare the various markets among themselves.

Probability of distinction-demand-supply in the market:

$$\begin{aligned} d^{\downarrow\uparrow} &= \mathbf{P}(K^{\downarrow} \neq K^{\uparrow}, K^{\downarrow} \neq \varnothing, K^{\uparrow} \neq \varnothing) = \\ &= 1 - \sum_{X \neq \varnothing} p^{\downarrow\uparrow}(X,X) - p^{\downarrow}(\varnothing) - p^{\uparrow}(\varnothing) + p^{\downarrow\uparrow}(\varnothing,\varnothing). \end{aligned}$$

3.2. One-set Demand and Full-set Supply (Variant 1a, 2b)

Assumptions of one-set demand and full-set supply mean, that the event-purchase-sale comes only in the case that the demanded set of goods *contains in* the supplied set of goods. Otherwise the event-purchase-sale does not come.

Probability of event-purchase-sale in the market as a whole[‡]

$$\mathbf{P}(K^{\downarrow} \subseteq K^{\uparrow}) = \sum_{X \subseteq \mathfrak{X}} \sum_{Y \subseteq \mathfrak{X}} p^{\downarrow\uparrow}(X,Y)\zeta(X,Y).$$

[†]The event-concurrence-demand-supply and the event-distinction-demand-supply together form the event $m^{\downarrow\uparrow} = \Omega - \big(\text{ter}^{\downarrow}(\varnothing) \cup \text{ter}^{\uparrow}(\varnothing)\big)$ which refers to the *event-market*. The event-market together with the *event-semiempty-market*: $\text{ter}^{\downarrow}(\varnothing) \cup \text{ter}^{\uparrow}(\varnothing) - \text{ter}^{\downarrow\uparrow}(\varnothing,\varnothing)$ and the *event-absence of market*: $\text{ter}^{\downarrow\uparrow}(\varnothing,\varnothing)$ form partition of Ω, and its probabilities give unit in the sum: $\mathbf{P}(m^{\downarrow\uparrow}) + p^{\downarrow}(\varnothing) + p^{\uparrow}(\varnothing) - p^{\downarrow\uparrow}(\varnothing,\varnothing) = 1$.

[‡]Here $\zeta(X,Y) = \begin{cases} 1, & X \subseteq Y, \\ 0, & X \not\subseteq Y \end{cases}$ is Riemann's zeta-function, which is used to allocate and to summarize only those probabilities from the sum, for which $X \subseteq Y$, as $\sum_{X\subseteq\mathfrak{X}}\sum_{Y\subseteq\mathfrak{X}} p^{\downarrow\uparrow}(X,Y)\zeta(X,Y) = \sum_{X\subseteq\mathfrak{X}}\sum_{X\subseteq Y} p^{\downarrow\uparrow}(X,Y)$.

Event-purchase[§] $X \subseteq \mathfrak{X}$

$$\mathrm{ter}^{\Downarrow}(X) = \mathrm{ter}^{\downarrow}(X) \cap \mathrm{ter}^{\uparrow}_{X} = \{K^{\downarrow} = X^{\downarrow}, X^{\uparrow} \subseteq K^{\uparrow}\}.$$

Probability of event-purchase $X \subseteq \mathfrak{X}$

$$p^{\Downarrow}(X) = \mathbf{P}(\mathrm{ter}^{\Downarrow}(X)) = \sum_{Y \subseteq \mathfrak{X}} p^{\downarrow\uparrow}(X, Y)\zeta(X, Y).$$

Event-sale[¶] $Y \subseteq \mathfrak{X}$

$$\mathrm{ter}^{\Uparrow}(Y) = \mathrm{ter}^{\downarrow Y} \cap \mathrm{ter}^{\uparrow}(Y) = \{K^{\downarrow} \subseteq Y^{\downarrow}, K^{\uparrow} = Y^{\uparrow}\}.$$

Probability of event-sale $Y \subseteq \mathfrak{X}$

$$p^{\Uparrow}(Y) = \mathbf{P}(\mathrm{ter}^{\Uparrow}(Y)) = \sum_{X \subseteq \mathfrak{X}} p^{\downarrow\uparrow}(X, Y)\zeta(X, Y).$$

Event-concurrence-purchase-sale $X \subseteq \mathfrak{X}$

$$\mathrm{ter}^{\Downarrow\Uparrow}(X) = \mathrm{ter}^{\Downarrow}(X) \cap \mathrm{ter}^{\Uparrow}(X) = \mathrm{ter}^{\downarrow}(X) \cap \mathrm{ter}^{\uparrow}(X).$$

Probability of event-concurrence-purchase-sale $X \subseteq \mathfrak{X}$

$$p^{\Downarrow\Uparrow}(X) = \mathbf{P}(\mathrm{ter}^{\Downarrow}(X) \cap \mathrm{ter}^{\Uparrow}(X)) = p^{\downarrow\uparrow}(X, X).$$

Event-distinction-purchase-sale $X \subseteq \mathfrak{X}$

$$\mathrm{ter}^{\Downarrow}(X)\Delta\ \mathrm{ter}^{\Uparrow}(X).$$

Probability of event-distinction-purchase-sale $X \subseteq \mathfrak{X}$

$$d^{\Downarrow\Uparrow}(X) = \mathbf{P}(\mathrm{ter}^{\Downarrow}(X)\Delta\ \mathrm{ter}^{\Uparrow}(X)) =$$

$$= p^{\Downarrow}(X) + p^{\Uparrow}(X) - 2p^{\downarrow\uparrow}(X, X).$$

4. Eventological Distributions of Supply and Demand of the Boltzmann and Gibbs Types

The eventological market model which is intended for the mathematical description of E-distributions of set of events-supplies or set of events-demands is based on the *eventological H-theorem*, the eventological generalization of Boltzmann's H-theorem [6–8]. The eventological H-theorem approves, that E-distributions of sets of events-supplies and events-demands have extreme entropy properties and form a class of E-distributions of the Boltzmann type or of the Gibbs type. These classes of extreme E-distributions define an exponential character of functional dependence of probabilities of supply and demand for a set of goods from the price of this set of goods and add two newer functions of concurrence and distinction of supply and demand to two exponential functions of supply of demand. The given four functions form the eventological

[§] This event is defined below by the event-terrace of a kind $\mathrm{ter}^{\uparrow}_{X} = \bigcap_{x \in X} x^{\uparrow}$ which means the supply of sets of goods Y such that $X \subseteq Y$.

[¶] This event is defined below by the event-terrace of a kind $\mathrm{ter}^{\downarrow Y} = \bigcap_{x \in Y^c} (x^{\downarrow})^c$ which means the demand for sets of goods X such, that $X \subseteq Y$.

"cross" of supply-demand-concurrence-distinction, the eventologically expanded analogue of the well-known "cross" of supply and demand from the economics.

Let $\mathcal{V} : 2^{\mathfrak{X}} \to \mathbb{R}$ is a function of market price of purchase-sale of subsets of goods from $\mathfrak{X}$. In other words, $\mathcal{V}(X)$ is a market price of purchase-sale of the set of goods $X \subseteq \mathfrak{X}$. E-distributions of supply and demand of the Boltzmann type for $X \subseteq \mathfrak{X}$ look like form:

$$p^{\downarrow}(X) = \frac{1}{Z^{\downarrow}} \exp\Big\{ -\beta\mathcal{V}(X)\Big\},$$

$$p^{\uparrow}(X) = \frac{1}{Z^{\uparrow}} \exp\Big\{ \gamma\mathcal{V}(X)\Big\},$$

where $Z^{\downarrow} = \sum_{X \subseteq \mathfrak{X}} \exp\Big\{ -\beta\mathcal{V}(X)\Big\}$, $Z^{\uparrow} = \sum_{X \subseteq \mathfrak{X}} \exp\Big\{ \gamma\mathcal{V}(X)\Big\}$ are multipliers providing probability normalizing. E-distributions of supply and demand of the Gibbs type look like form:

$$p^{\downarrow}(X) = \frac{1}{Z^{\downarrow}_{*}} \exp\Big\{ -\beta\mathcal{V}(X)\Big\} p^{\downarrow}_{*}(X),$$

$$p^{\uparrow}(X) = \frac{1}{Z^{\uparrow}_{*}} \exp\Big\{ \gamma\mathcal{V}(X)\Big\} p^{\uparrow}_{*}(X),$$

where $p^{\downarrow}_{*}$ and $p^{\uparrow}_{*}$ are some fixed E-distributions of supply and demand describing a market condition of the total buyer on $2^{\mathfrak{X}^{\downarrow}}$ and the total seller on $2^{\mathfrak{X}^{\uparrow}}$ accordingly; and $Z^{\downarrow}_{*} = \sum_{X \subseteq \mathfrak{X}} \exp\Big\{ -\beta\mathcal{V}(X)\Big\} p^{\downarrow}_{*}(X)$, $Z^{\uparrow}_{*} = \sum_{X \subseteq \mathfrak{X}} \exp\Big\{ \gamma\mathcal{V}(X)\Big\} p^{\uparrow}_{*}(X)$ are multipliers providing probability normalizing. Parameters $\beta > 0$ and $\gamma > 0$ characterize a mean of inverse activity in the given market of the total buyer and the total seller accordingly.

5. Equilibrium Intervals of Prices and of Sets of Goods

Let's suppose that the joint E-distribution of supply and demand $p^{\downarrow\uparrow}$ defines marginal E-distributions of supply and demand of the Boltzmann type or of the Gibbs type, functions of probabilities $p^{\downarrow}(X)$ and $p^{\uparrow}(X)$, exponentially depending on the price $\mathcal{V}(X)$. In assumptions 1a and 1b the function of probabilities of concurrence-demand-supply $p^{\downarrow\uparrow}(X, X)$ is defined by formulas (3), and the function of probabilities of distinction of demand-supply $p^{\downarrow\uparrow}(X)$ is defined by formulas (4). These four functions of probabilities form the eventological "cross" of supply and demand. Graphics of these functions for the triplet of goods $\mathfrak{X} = \{x, y, z\}$ are shown in Fig. 3.

In the classical economic theory, the equilibrium price of supply and demand is the price under which a demand is equal to a supply. The equilibrium price is defined by the point of crossing of curves of supply and demand. In the eventological theory of supply and demand (in which E-distributions of sets of events-demands and sets of events-supplies are studied) the point of crossing of "curves" of supply and demand defines the set of goods, event-demand and event-supply for which have the same probability. It is obvious, that an equality of probabilities of events does not imply (though does not exclude) an equality of events. The riches of the eventological collisions arising between events-demands and events-supplies under an equality of their probabilities or in other situations, are supervised by the joint E-distribution of a set of events-demands and a set of events-supplies. At concurrence of events, their probabilities coincide also. Besides the probability of their crossing achieves the maximum, and the probability of their distinction achieves the minimum. In other situations a concurrence of probabilities of events cannot mean a concurrence of events and therefore, cannot mean, that the probability of their crossing achieves the maximum, and probability of distinction achieves the minimum. This fact well-known in eventology [1]: the maximum of probability of concurrence-demand-supply and

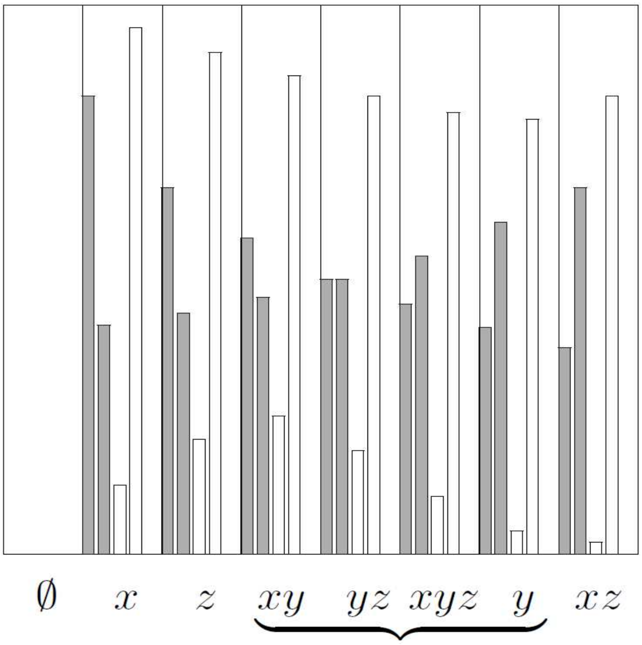

Fig. 3. Graphics of four functions of subsets of the triplet $X \subseteq \mathfrak{X} = \{x, y, z\}$, defining the eventological "cross" of supply and demand. Above each subset of the triplet values of four functions (from left to right) are shown: decreasing probability of demand $p^{\downarrow}(X)$ (dark rectangulars), increasing probability of supply $p^{\uparrow}(X)$ (dark rectangulars), probability of concurrence-demand-supply $p^{\downarrow\uparrow}(X, X)$, achieving the maximum on the subset $\{x, y\}$ (light rectangulars), probability of distinction-demand-supply $d^{\downarrow\uparrow}(X)$, achieving the minimum on the subset $\{y\}$ (light rectangulars). The equilibrium price is achieved on the equilibrium subset of goods $\{y, z\}$. The equilibrium interval of prices is defined by values of the function of price on the equilibrium interval of subsets of goods $\{\{x, y\}, \{y, z\}, \{x, y, z\}, \{y\}\}$ (it is shown by a horizontal brace). Subsets are designated in abbreviated form: for example, $xy = \{x, y\}$, etc.

the minimum of probability of distinction-demand-supply are not obliged to be achieved on the same set of goods.

These reasons underlie eventological opening which has added to the classical concept of equilibrium price defined by crossing of curves of supply and demand, two new key concepts of eventological modelling supply and demand, defined by curves of concurrence and distinction of a supply and demand. The first concept is the subset of goods X_1, on which the minimum of probability of distinction-demand-supply $d^{\downarrow\uparrow}(X_1)$ (4) is achieved. The second concept is the subset of goods X_2, on which the maximum of probability of concurrence-demand-supply $p^{\downarrow\uparrow}(X_2, X_2)$ (3) is achieved. These two extreme subsets of goods define the *equilibrium interval of prices*

$$[\mathcal{V}_1, \mathcal{V}_2] = \Big\{\mathcal{V}(X) : X \in [X_1, X_2]\Big\},$$

which is restricted by the prices of the given subsets of goods: $\mathcal{V}_1 = \mathcal{V}(X_1), \mathcal{V}_2 = \mathcal{V}(X_2)$:

$$[\mathcal{V}_1, \mathcal{V}_2] = \begin{cases} \{\mathcal{V}(X) : \mathcal{V}_1 \leq \mathcal{V}(X) \leq \mathcal{V}_2, \text{and } \mathcal{V}_1 \leq \mathcal{V}_2\}, \\ \{\mathcal{V}(X) : \mathcal{V}_2 \leq \mathcal{V}(X) \leq \mathcal{V}_1, \text{and } \mathcal{V}_2 \leq \mathcal{V}_1\}. \end{cases}$$

Moreover, the eventological theory allows to "decipher" the *equilibrium interval of prices* in terms of subsets of goods as the *equilibrium interval of subsets of goods* (see a Fig. 3):

$$[X_1, X_2] = \begin{cases} \{X : \mathcal{V}_1 \le \mathcal{V}(X) \le \mathcal{V}_2, \text{and } \mathcal{V}_1 \le \mathcal{V}_2\}, \\ \{X : \mathcal{V}_2 \le \mathcal{V}(X) \le \mathcal{V}_1, \text{and } \mathcal{V}_2 \le \mathcal{V}_1\}. \end{cases}$$

The eventological "cross" of supply and demand, consisting of four curves (demand, supply, concurrence and distinction), is entirely defined by the joint E-distribution of supply and demand $p^{\downarrow\uparrow}$. This E-distribution supervises marginal E-distributions of supply and demand and all structure of interdependence of sets of events-demands and events-supplies among themselves. Marginal E-distributions of supply and demand do not contain the information on interaction of supply and demand. Therefore, the point of crossing of curves of supply and demand, defining the classical equilibrium price, also does not reflect the information on interdependence of supply and demand. The eventological theory, suggesting to add to two curves of supply and demand also curves of concurrence and distinction, opens an opportunity of the account of many-events-based and many-agent structures of interdependence of supply and demand in two new concepts: the *equilibrium interval of prices* and the *equilibrium interval of subsets of goods*.

6. Eventological H-Theorem of Distributions of Supply and Demand

The theoretical foundation of eventological model of many-events-based and many-agent market follows from the *eventological H-theorem*, the eventological generalization of Boltzmann's H-theorem [6–8]. The eventological H-theorem approves, that E-distributions of sets of events of supply and demand extreme entropy properties and form a class of E-distributions of the Boltzmann type (theorem 1) or of the Gibbs type (theorem 2).

Theorem 1 (eventological H-theorem on E-distributions of the Boltzmann types). *Let $(\Omega, \mathcal{F}, \mathbf{P})$ is an E-space, $\mathfrak{X} \subseteq \mathcal{F}$ is a finite set of events, $\mathcal{V}(X)$ is a non-negative bounded function*[‖] *on $2^{\mathfrak{X}}$, and E-distributions $p(X)$ on $2^{\mathfrak{X}}$ provide the mean value of the function $\mathcal{V}(X)$ at the given level*

$$\langle \mathcal{V} \rangle = \sum_{X \subseteq \mathfrak{X}} p(X)\mathcal{V}(X.) \tag{$\mathcal{V}$}$$

Then the maximum of entropy

$$\mathcal{H}_p = -\sum_{X \subseteq \mathfrak{X}} p(X) \ln p(X) \to \max_p$$

among E-distributions p, satisfying to restriction ($\mathcal{V}$), is achieved for $X \subseteq \mathfrak{X}$ on E-distributions of two Boltzamnn types ($\alpha < 0, \alpha > 0$):

$$p(X) = \frac{1}{\mathcal{Z}} \exp\Big\{\alpha \mathcal{V}(X)\Big\}, \tag{5}$$

which without normalizing multiplier $\mathcal{Z} = \sum_{X \subseteq \mathfrak{X}} \exp\Big\{\alpha \mathcal{V}(X)\Big\}$ is written in the equivalent kind:

$$\frac{p(X)}{p(\varnothing)} = \exp\Big\{\alpha(\mathcal{V}(X) - \mathcal{V}(\varnothing))\Big\}, \tag{5$'$}$$

where

$$\alpha = -\frac{\mathcal{H}_p + \ln p(\varnothing)}{\langle \mathcal{V} \rangle - \mathcal{V}(\varnothing)} \tag{6}$$

[‖] In the eventological theory of a supply and demand $\mathcal{V}(X)$ is interpreted as the price of set of events-purchases-sales $X \subseteq \mathfrak{X}$.

is a parameter of E-distributions. For negative values of this parameter E-distributions (5) belong to the class of the Boltzmann E-distributions, and for positive values — to the class of the opposite Boltzmann E-distributions.

Theorem 2 (eventological H-theorem on E-distributions of the Gibbs types). *Let $(\Omega, \mathcal{F}, \mathbf{P})$ is an E-space, $\mathfrak{X} \subseteq \mathcal{F}$ is a finite set of events, $\mathcal{V}(X)$ is a non-negative bounded function on $2^{\mathfrak{X}}$, $p_*(X)$ is some fixed E-distribution on $2^{\mathfrak{X}}$, and E-distributions $p(X)$ on $2^{\mathfrak{X}}$ provide the mean value of the function of price $\mathcal{V}(X)$ at the given level*

$$\langle \mathcal{V} \rangle = \sum_{X \subseteq \mathfrak{X}} p(X) \mathcal{V}(X.) \tag{$\mathcal{V}$}$$

Then a minimum of relative entropy

$$\mathcal{H}_{\frac{p}{p_*}} = \sum_{X \subseteq \mathfrak{X}} p(X) \ln \frac{p(X)}{p_*(X)} \to \min_p$$

among E-distributions p, satisfying to restriction ($\mathcal{V}$), is achieved for $X \subseteq \mathfrak{X}$ on E-distributions of two Gibbs types ($\alpha < 0, \alpha > 0$):

$$p(X) = \frac{1}{\mathcal{Z}} \exp \Big\{ \alpha \mathcal{V}(X) \Big\} p_*(X), \tag{7}$$

which without normalizing multiplier $\mathcal{Z} = \sum_{X \subseteq \mathfrak{X}} \exp \Big\{ \alpha \mathcal{V}(X) \Big\} p_*(X)$ is written in the equivalent kind:

$$\frac{p(X)}{p(\varnothing)} = \exp \Big\{ \alpha (\mathcal{V}(X) - \mathcal{V}(\varnothing)) \Big\} \frac{p_*(X)}{p_*(\varnothing)}, \tag{7$'$}$$

where

$$\alpha = \frac{\mathcal{H}_{\frac{p}{p^*}} - \ln \frac{p(\varnothing)}{p_*(\varnothing)}}{\langle \mathcal{V} \rangle - \mathcal{V}(\varnothing)}. \tag{8}$$

is a parameter of E-distributions. For negative values of this parameter E-distributions (7) belong to the class of the Gibbs distributions if this parameter has a negative value and belong to the class of the opposite Gibbs distributions if this parameter has a positive value; and

$$\mathcal{H}_p = - \sum_{X \subseteq \mathfrak{X}} p(X) \ln p(X)$$

is an entropy of E-distribution p.

7. Interpretations of the Eventological H-Theorem

7.1. Interpretation of the Theorem 1

In the theorem 1 it affirms, that among the given E-distributions an entropy $\mathcal{H}_p$ achieves the minimum on E-distributions of two Boltzmann types ($\alpha < 0, \alpha > 0$) of a kind (5). The entropy "measures" deviation of E-distributions from equiprobable and achieves the maximum for equiprobable E-distributions. Therefore, the eventological H-theorem actually approves, that from all E-distributions, "keeping" at the given level the mean value of function of price $\mathcal{V}$, the E-distributions of two Boltzmann types ($\alpha < 0, \alpha > 0$) lay "most closer" to the equiprobable E-distribution.

Thus for these E-distributions maximizing entropy the parameter α appears connected by the formula (6) to the entropy and to the mean value $\langle \mathcal{V} \rangle$ of the function of price $\mathcal{V}$. It is possible to tell that parameter α is proportional to the entropy with an opposite sign and inversely proportional to mean value $\langle \mathcal{V} \rangle$. It characterizes the mean inverse activity in the given market of the total buyer and the total seller accordingly.

7.2. Interpretation of the Theorem 2

In the theorem 2 it affirms, that among the given E-distributions a relative entropy $\mathcal{H}_{\frac{p}{p^*}}$ achieves the minimum on E-distributions of two Gibbs types ($\alpha < 0, \alpha > 0$) of a kind (7). The relative entropy "measures" a deviation of one E-distribution from another and achieves the minimum equal to zero when these E-distributions coincide. Therefore, the eventological H-theorem actually approves, that from all E-distributions, "keeping" at the given level the mean value of function of price $\mathcal{V}$, E-distributions of two Gibbs types ($\alpha < 0, \alpha > 0$) lay "most closer" to the fixed E-distribution p^*.

Thus for these E-distributions, minimizing a relative entropy, the parameter α appears connected by the formula (8) to the relative entropy of these E-distributions relatively p^* and to the mean value $\langle \mathcal{V} \rangle$ of the function of price $\mathcal{V}$. It is possible to tell, that parameter α is proportional to the relative entropy and inversely proportional to the mean value $\langle \mathcal{V} \rangle$. It characterizes the mean inverse activity in the given market of the total buyer and the total seller accordingly.

8. Discussion

When a relative entropy $\mathcal{H}_{\frac{p}{p_*}}$ of some physical system (with distribution p) concerning an environment (with distribution p_*) has the minimal value, in statistical thermodynamics it is considered, that the system is in balance with an environment, and a decrease of relative entropy in due course corresponds to approach of system to an equilibrium with the given environment**.

The given physical analogy stated in eventological language, is used by us for construction the more general eventological model of market behaviour of the total buyer and the total seller based on the idea of their equilibrium choice between *supply and demand*. When a relative entropy $\mathcal{H}_{\frac{p}{p_*}}$ of the total buyer and of the total seller (with distribution p) relatively market demand and supply (with distribution p_*) has the minimal value, in eventology it is considered, that the total buyer and the total seller are in equilibrium with the market demand and supply, and *decrease of relative entropy in due course corresponds to an aspiration of the total buyer and the total seller to equilibrium with the market.*

The eventological H-theorem serves as the mathematical justification of use of the eventological market model [1] which opens some new concepts of modelling supply and demand (*eventological cross of supply and demand, equilibrium interval of prices and of subsets of goods*) and specifies on the *exponential character of functional dependence of curves of supply and demand for subsets of goods from the price of these subsets.*

References

[1] O.Yu.Vorobyev, Eventology, Krasnoyarsk, Sib. Fed. Univ., 2007 (in Russian).

[2] M.M.Bakhtin, Toward a Philosophy of the Act, Austin, University of Texas Press, 1993; St.Petersburg, 1920.

[3] B.A.W.Russell, History of Western Philosophy and its Connections with Political and Social Circumstances from the Earlist Times to the Present Day, London, George Allen & Unwin, 1946.

[4] V.A.Lefebvre, Algebra of Conscience, Boston, Kluwer Academic Publishers, 2003.

**It is the *principle of a minimum of relative entropy of systems at the fixed level of entropy of environments*, which is equivalent to the *principle of a maximum entropy*: "increase of entropy of system at its approach to an equilibrium". "Change of a maximum with a minimum" speaks formal differences in a sign between traditional definitions of entropy: $-\sum_{X \subseteq \mathfrak{X}} p(X) \ln p(X)$ and of relative entropy: $\sum_{X \subseteq \mathfrak{X}} p(X) \ln(p(X)/p_*(X))$.

[5] A.N.Kolmogorov, Grundbegriffe der Wahrscheinlichkeitrechnung, Berlin, Ergebnisse der Mathematik, 1933.

[6] L.Boltzmann, Weitere studien uber das warmegleichgewicht unter gasmolekulen, *Wiener Berichte*, **66**(1872), 275–370.

[7] R.L.Stratonovich, Information theory, Moscow, Sovietskoe radio, 1975 (in Russian).

[8] A.Isihara, Statistical Physics, New York, London, Academic Press, 1971.

[9] O.Yu. Vorobyev, On elements of axiomatizing eventology, *Journal of Siberian Federal University. Mathematics & Physics*, **3**(2010), №2, 57–164,

Многособытийные модели рынка, следующие из эвентологической H-теоремы

Олег Ю. Воробьев
Елена Е. Голденок

Рассматривается многособытийная и многоагентная эвентологическая модель предложения и спроса, в рамках которой вводится новое понятие равновесного интервала цен и подмножеств товаров. Эта модель следует из эвентологической H-теоремы (эвентологического обобщения больцмановской H-теоремы), которая служит теоретическим обоснованием применения эвентологических распределений предложения и спроса в математическом описании многоагентных и многособытийных рыночных систем.

Ключевые слова: спрос, предложение, рынок, многоагентные системы, многособытийные системы, равновесный интервал цен, равновесный интервал множеств товаров, H-теорема Больцмана, событие, вероятность, эвентология.