A numerical technique for symbolic formulae simplification

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The technique, which at simplification brings the formula with a minimum possible amount of parameters, is offered. It is based on the IIR-filter synthesis algorithm. The technique effectiveness doesn't depend on a form of original formula. It depends only on form of formula to which the initial one may be actually simplified.

The technique, which at simplification brings the formula with a minimum possible amount of parameters, is offered. Therefore this number will be guaranteed not greater then one in initial formula. The technique effectiveness doesn't depend on a form of original formula. It depends only on form of formula to which the initial one may be actually simplified.

The technique consists of following basic steps.

Let we have an algebraic notation of one-variable function $f(x)$. It is necessary to find a function $g(x)$, which is identical to $f(x)$, but noted in more simple form. I.e. the notation of $g(x)$ contains the not greater amount of elementary functions than $f(x)$.

1) Firstly we calculate a sequence of $g(x)$ values with some constant step k: $y_0 = f(0)$, $y_1 = f(k)$, $y_2 = f(2k)$, ... The length of sequence will be talk about later.
2) Then the received sequence is accepted as a pulse response of IIR-filter, and we execute a synthesis of this filter in accordance with algorithm described in [1, 2].
3) When analyzing the received filter coefficients, firstly we determine the kind of $g(x)$, and then all constants ingressed in $g(x)$ (including absolute term) are computed. The function kind is defined with the feedback coefficients, and the function parameters are evaluated using either remaining coefficients or all filter coefficients [3].

Let's describe more in detail the second step of technique. The mentioned algorithm performs synthesis successfully just in that cases when the pulse response is described by some algebraic function. For the present the efficiency of algorithm is proved not for any function, but for set Φ of functions stated below:

1) polynomials: $g(x) = p_n(x) = \alpha_0 \cdot x^n + \alpha_1 \cdot x^{n-1} + ... + \alpha_n$, $\alpha_0 \neq 0$;
2) exponential functions: $g(x) = k \cdot a^{bx} + c$, $a \neq 0$, $a \neq 1$, $b \neq 0$, $k \neq 0$;
3) any sine and cosine functions: $g(x) = a \cdot \sin(b \cdot x + c)$, $a \neq 0$, $b \neq \pi k$, $k \in Z$, $d \neq 0$;
4) any periodic functions with the assumption that period is multiple to step with which f(x) values was taken.

The proved theorems statements can be found in [4]. A minimum amount of pulse response counts necessary for calculation is talked about there too. In the large it linearly depends on an amount of parameters of function g(x) which describes the filter pulse response.

In addition, a hypothesis that algorithm is efficient for any function which is the linear combination of any functions from Φ is put forward. This hypothesis is corroborated by number of computational experiments.

References

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