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Shunkov Groups with the Minimal Condition for Noncomplemented Abelian Subgroups

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In the present paper, we give a complete exhaustive description of the pointed out Shunkov groups.

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Introduction

A great many deep and bright results are connected with groups, satisfying various minimal conditions, and with groups, having wide systems of complemented subgroups (see, for instance, [1–7]).

The present paper is devoted to the Shunkov groups with the minimal condition above.

Below p and q are always primes; $min - ab$, $min - ab\bar{c}$, $min - p$ and $min - p'$ are the minimal conditions respectively for abelian, abelian noncomplemented, for p- and p'-subgroups. All other notations are standard.

Remind that the group G is called Shunkov, if for any its finite subgroup K, every subgroup of the factor group $N_G(K)/K$, generated by two conjugate elements of prime order, is finite (V. D. Mazurov, 1998). The class of Shunkov groups is wide and includes, for instance, binary finite groups, 2-groups. The known Suchkova–Shunkov Theorem [8] (see also [4, Theorem 4.5.1]) asserts: The Shunkov group with $min - ab$ is Chernikov.

Further, remind that the subgroup H of the group G is called complemented in G, if for some subgroup K of G, $G = HK$ and $H \cap K = 1$; K is called a complement of H in G. The group G is called completely factorizable, if every its subgroup is complemented in it (N. V. Chernikova [9]). The fundamental N. V. Chernikova's Theorem [9, 10] (see also, for instance, [1, Theorem 7.2]) gives an exhaustive description of completely factorizable groups and asserts: The group $G \neq 1$ is completely factorizable iff $G = A \leftthreetimes B$ where A is a direct product of normal subgroups of prime orders of G and B is a direct product of subgroups of prime orders or $B = 1$; in particular, the p-group G is completely factorizable iff it is elementary abelian. The known Kargapolov [11]–Gorchakov [12] Theorem asserts: The group is completely factorizable iff all its abelian subgroups are complemented.

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It is natural to consider groups, having infinite abelian subgroups, in which all such subgroups are complemented. B. I. Mishchenko [13] has described the infinite solvable and the infinite radical in the sense of B. I. Plotkin groups with complemented infinite abelian subgroups (see Theorem 1 [13] and Corollary [13, p. 158]). Since all such groups are locally finite, it is natural to consider the locally finite groups with $min - ab\overline{c}$. N. S. Chernikov [14, 15] has described these locally finite groups (see Theorem [15] and Corollary 3.5 [15]). N. S. Chernikov [14, 16] has established that binary finite groups with $min - ab\overline{c}$ are locally finite (see Theorem 3 [16]).

1. The main result and some corollaries

The author succeeded in proving the following general theorem, which is the main result of the present paper.

Theorem. *For the Shunkov group G the following statements are equivalent:*

(i) *G satisfies the minimal condition for abelian noncomplemented subgroups.*

(ii) *G is a Chernikov group or a non-Chernikov group with complemented infinite abelian subgroups.*

(iii) *G is a Chernikov group or G is a completely factorizable group, or $G = A \rtimes B$ where A is infinite and A is a direct product of normal in G subgroups of prime orders, $B = C \times D$ is finite, C is a direct product of subgroups of prime orders or $C = 1$, D is cyclic $\neq 1$ and for every $p \in \pi(D)$, $p^2 || D|$, and also for every $g \in D \setminus \{1\}$, $C_A(g)$ is finite.*

(In view of O.Yu. Shmidt's Theorem (see, for instance, [20, Theorem 1.45]), in (iii) G is locally finite.)

Theorem is equivalent to the author's Theorem [17].

Theorem implies the following proposition.

Proposition ([17]). *The Shunkov p-group G (in particular, the 2-group G) satisfies the minimal condition for abelian noncomplemented subgroups iff it is Chernikov or elementary abelian.*

Note that Theorem [17] and Proposition [17] are exactly all results of [17].

The following new author's assertions are the immediate consequences of Proposition.

Corollary 1. *For the 2-group G the following statements are equivalent:*

(i) *G satisfies the minimal condition for abelian noncomplemented subgroups.*

(ii) *G satisfies the minimal condition for noncomplemented subgroups.*

(iii) *G is Chernikov or elementary abelian.*

Corollary 2. *For the Shunkov p-group G the following statements are equivalent:*

(i) *G satisfies the minimal condition for abelian noncomplemented subgroups.*

(ii) *G satisfies the minimal condition for noncomplemented subgroups.*

(iii) *G is Chernikov or elementary abelian.*

In connection with the results above, note that for every $p > 665$, there exists the non-solvable group of exponent p containing an infinite abelian subgroup, in which every abelian subgroup of order $> p$ is complemented (N. S. Chernikov [18]). Thus the above requirements: "G is a 2-group", "G is Shunkov" are essential.

2. Proof of the main result

A. *Show that* **(i)** *implies* **(iii)**.

Let **(i)** hold. The subsequent proof will be accomplished in a series of steps.

(1) *G is periodic.*

Proof. Let G have some element g of infinite order. Then some subgroup $< g^{2^n} >$ of the infinite chain $< g^2 >\supset< g^4 >\supset ... \supset< g^{2^k} >\supset< g^{2^{k+1}} >\supset ...$ has a complement D in G. But $1 < |D\cap < g > | < \infty$, which is a contradiction. $\square$

(2) *If G has a normal infinite locally finite subgroup H, then the statement* **(iii)** *is valid.*

Proof. First, let H be Chernikov. Now remind the following S. N. Chernikov's Proposition (see, for instance, [1, Proposition 1.13, p. 62]): A periodic group of automorphisms of the group, which is a direct product of finitely many quasicyclic subgroups, is finite. Further, H contains the characteristic subgroups R of finite index, which is such product. Since G is periodic (see **(1)**), in view of the last Proposition, $|G : C_G(R)| < \infty$. In accordance with Lemma 1.1 [15], an abelian group with $min - ab\overline{c}$ is precisely Chernikov or a direct product of groups of prime orders. Every maximal abelian subgroup of $C_G(R)$ satisfies $min - ab\overline{c}$ and is not such product and so is Chernikov. Hence follows: G satisfies $min - ab$. Therefore in virtue of Suchkova–Shunkov Theorem [8] (see above), G is Chernikov and, at the same time, **(iii)** is valid.

Now let H be non-Chernikov. Remind the following N. S. Chernikov's Theorem (see [15, Theorem]): The locally finite group with $min - ab\overline{c}$ is the same as in **(iii)**. Consequently, with regard to N. V. Chernikova's Theorem (see, above), $H = K \leftthreetimes L$, where K is a direct product of normal in H subgroups of prime orders, L is abelian without quasicyclic subgroups. Let F be the Fitting subgroup of H. Then F is locally nilpotent and $F = K \leftthreetimes (F \cap L) \trianglelefteq G$. Since H is solvable, in view of Proposition 5.4.4 (ii) [19, (see p. 144)] , $C_H(F) = Z(F)$. Therefore because of H is infinite, F is infinite too. Obviously, F is non-Chernikov. Further, every mentioned direct multiplier of K belongs to $Z(F)$ (for instance, in view of Proposition 1.16 [1, (see p. 70)]). So F is abelian. In accordance with Lemma 1.9 [15], the group, satisfying $min - ab\overline{c}$ and having a normal abelian non-Chernikov subgroup, is the same as in **(iii)**. Thus **(iii)** is valid. $\square$

(3) *Either the statement* **(iii)** *is valid, or the product L of all normal locally finite subgroups of G is finite and also G includes some normal infinite subgroup M, which does not satisfy $min - ab$ and has no subnormal locally finite subgroups $\neq 1$.*

Proof. Assume that **(iii)** is not valid. Then G is infinite. In consequence of O.Yu. Shmidt's Theorem (see, for instance, [20, Theorem 1.45]), L is locally finite. By virtue of the assertion **(2)**, L is finite. So $|G : C_G(L)| < \infty$. Again by virtue of **(2)**, $C_G(L)$ is not locally finite. Therefore, with regard to Suchkova–Shunkov Theorem [8] (see above), $C_G(L)$ does not satisfy $min - ab$. So some maximal abelian subgroup A of $C_G(L)$ is not Artinian. Clearly, $L \cap C_G(L) \subseteq Z(C_G(L))$ and so $L \cap C_G(L) \subseteq A$. Further, A has some infinite descending series

$$A = A_0 \supset A_1 \supset A_2 \supset \ldots \supseteq \cap_{n=1}^{\infty} A_n \supseteq L \cap C_G(L) \supseteq 1.$$

Some A_n has a complement D in G. Put $M =< (D \cap A)^G >$. In view of Chunikhin's Lemma (see, for instance, [21, Lemma 1.36]), $M \subseteq D$. Also $M \subseteq C_G(L)$ and $D \cap L \cap C_G(L) = 1$. So $M \cap L \subseteq (D \cap C_G(L)) \cap L = 1$. In consequence of Theorem 1.1 in §2 of Chapter 5 [22] (see [22, p. 345]), every subnormal locally finite subgroup of M belongs to L. Consequently, M has no subnormal locally nontrivial subgroups. Also with regard to Suchkova–Shunkov Theorem, M does not satisfy $min - ab$. $\square$

(4) *If G is a p-group, then* **(iii)** *is valid and, at the same time, G is Chernikov or elementary abelian.*

Proof. Let G be a p-group. It is easy to see, with regard to N. V. Chernikova's Theorem above: G is Chernikov or elementary abelian iff **(iii)** is valid.

Assume that **(iii)** is not valid. Now define the finite subgroup H of G in the following way.

First, if G has an element g of order p^2, then put $H =< g >$.

Suppose that G is of exponent p. Then G is non-abelian. If for some g, $h \in G$, $[g, g^h] \neq 1$, then we put $H =< g, g^h >$. Since G is Shunkov, H is finite. Further, assume that also for every $g \in G$ and $h \in G$, $[g, g^h] = 1$. Take a, $b \in G$ such that $[a, b] \neq 1$. Since $< a^h : h \in G >$ and $< b^h : h \in G >$ are normal abelian subgroups of G, the subgroup $< a^h : h \in G >< b^h : h \in G >$is metabelian and non-abelian. Further, the known S. N. Chernikov's Theorem (see, for instance, [1, Proposition 1.1]) asserts: Periodic locally solvable groups are locally finite. Then $< a, b >$ is finite non-abelian. Now put $H =< a, b >$.

Let A be any abelian subgroup of $C_G(H)$. Then AH is a nilpotent non-(elementary abelian) group with $min - ab\bar{c}$. In accordance with Lemmas 2.2 [15] and 1.1 [15]: Every non-Chernikov locally nilpotent p-group with $min - ab\bar{c}$ is elementary abelian. Thus, AH is Chernikov. So $C_G(H)$ satisfies $min - ab$. Now remind Shunkov's Theorem [23]: The 2-group with $min - ab$ is Chernikov. Remind An. Ostilovskiy's Theorem [24] (see also [4, Theorem 4.4.1]): The Shunkov $2'$-group with $min - ab$ is Chernikov. In view of these theorems, $C_G(H)$ is Chernikov.

Let F be a subgroup of maximal order among all $X \lhd H$, for which $C_G(X)$ is non-Chernikov. Take $u \in H \setminus F$ such that $u^p \in F$ and also $uF \in Z(H/F)$. Since $< u > F \trianglelefteq H$ and also $| < u > F| > |F|$, the $C_G(< u > F)$ is Chernikov.

Put $T =< u > C_G(F)$. If $| < u > | = p$, then $u^p = 1 \in Z(T)$. If $| < u > | \neq p$, then H and, at the same time, F are cyclic. Therefore in this case we have: $u^p \in F \subseteq Z(C_G(F))$. Consequently, $[u^p, T] = [u^p, < u > C_G(F)] = 1$, i.e. $u^p \in Z(T)$.

In view of S. N. Chernikov's Lemma (see, for instance, [1, Lemma 3.7, p. 151]), $C_T(u) =< u > (C_T(u) \cap C_G(F))$. Then $|C_T(u) : C_T(u) \cap C_G(F)| < \infty$. Since $C_T(u) \cap C_G(F) \subseteq C_G(< u > F)$ and $C_G(< u > F)$ is Chernikov (see above), the subgroup $C_T(u) \cap C_G(F)$ is Chernikov too. Therefore $C_T(u)$ is also Chernikov.

Further, it is easy to see: the statement **(iii)** of Theorem with T in the character of G is not valid. Therefore in view of the assertion **(3)**, T contains some normal subgroup M that does not satisfy $min - ab$ and has no normal locally finite subgroups $\neq 1$.

Let K be a normal subgroup of T, having some abelian non-Chernikov subgroup B. In view of Lemma 1.2 [15], K contains some subgroup $L \lhd T$ with infinite $B/L \cap B$ and non-Chernikov $L \cap B$. Taking this into account it is easy to see: M has some infinite descending series

$$M = M_0 \supset M_1 \supset M_2 \supset \ldots \supset M_\alpha \supset M_{\alpha+1} \supset \ldots \supset M_\gamma = \cap_{\alpha<\gamma} M_\alpha \supseteq 1$$

of normal subgroups of T such that all M_α, $\alpha < \gamma$, do not satisfy $min - ab$ and M_γ satisfies $min - ab$. In view of mentioned Shunkov's and An. Ostilovskiy's Theorems, M_γ is Chernikov. So $M_\gamma = 1$. Further, since $C_M(u)$ is Chernikov, for some β such that $0 < \beta < \gamma$, we have: $C_{M_\beta}(u) = 1$. Take $v \in M_\beta \setminus \{1\}$. Then, because of $u^p \in Z(T)$, we have: u, $v \in C_G(u^p)$ and $< u > \cap < u^v >=< u^p >$. Since G is Shunkov, $<< u >, < u^v >>$ is a finite p-group. So $<< u >, < u^v >> \cap M_\beta$ has some element $\neq 1$ centralizing u, which is a contradiction.

Thus **(iii)** is valid. □

(5) *If for some element $g \in G$ of prime order and for some infinite normal subgroup H of G we have: $H \cap C_G(g) = 1$, then* **(iii)** *is valid.*

Proof. First, give the following Popov–Sozutov–Shunkov Theorem (see Lemmas 2.7, 5.24 [25], Theorem 5.11 [25], Lemma 5.20 [25]): Let $X = U \rightthreetimes < v >$ be an infinite group with $| < v > | = p$, $C_X(v) =< v >$ and $| < v, v^u > | < \infty$, $u \in U$. Then: X is periodic; all divisible abelian subgroups of U belongs to $Z(U)$; every finite subgroup of U, normalized by v belongs to some infinite locally finite subgroup of U, normalized by v. Further, if for some $u \in U$, all subgroups $U \cap < u, fv >$ with $f \in U$ are abelian, then the normal closure $< u^X >$ of u in X is abelian.

Now give some comments. Since $< v >$ is obviously a Sylow p-subgroup of $< v, v^u >$ and $< v, v^u >= (U \cap < v, v^u >) < v >$, for some $w \in U \cap < v, v^u >$ we have: $< v >^u =< v >^w$, i.e. $u = w$ and $u \in U \cap < v, v^u >$. Obviously, for some $a \in < uv >$ and $x \in U \cap < v, v^u >$, $| < a > | = p$ and $< a >=< v >^x$. So $< uv >=< v >^x$. Thus, $(X \setminus U) \cup \{1\} = \cup_{u \in U} < v^u >$. Hence follows: for $y, z \in X \setminus U$, $| < y, z > | < \infty$.

Now return directly to the present assertion **(5)**. Since G is Shunkov, for any x, $y \in G$, we have: $| < g^x, g^y > | < \infty$.

If H contains a quasicyclic subgroup, then in view of Popov–Sozutov–Shunkov Theorem above, $Z(H)$ contains all such subgroups. Then every maximal abelian subgroup of H contains a quasicyclic subgroup. Consequently in view of Lemma 1.1 [15], all maximal abelian subgroups of H are Chernikov and so H satisfies $min - ab$. Therefore in view of Suchkova–Shunkov Theorem mentioned above, H is Chernikov. So in accordance with the assertion **(2)**, the statement **(iii)** is valid.

Now let H have no quasicyclic subgroups. Take u, $f \in H$. For some $h \in H$, $fg = g^h$ (see comments above). Also $H \cap < g^h, g^{hu} >$ is a finite subgroup, normalized by g^h, and $u \in H \cap < g^h, g^{hu} >$ (see comments above). Then $H \cap < u, fg > \subseteq H \cap < g^h, g^{hu} >$. Further, in view of Popov–Sozutov–Shunkov Theorem above, $H \cap < g^h, g^{hu} >$ belongs to some infinite locally finite subgroup R of H, normalized by g^h. By virtue of J. G. Thompson Theorem [26], R is locally nilpotent. Since R has no quasicyclic subgroups, R is also non-Chernikov. Therefore in view of Lemma 2.2 [15], R is abelian. At the same, $H \cap < u, fg >$ is abelian. Consequently, in view of Popov–Sozutov–Shunkov Theorem above, $< u^{H<g>} >$ is abelian. Thus H is the product of normal locally finite subgroups $< u^{H<g>} >$, taking by all $u \in H$. Then in consequence of O. Yu. Shmidt's Theorem, H is locally finite. Therefore **(iii)** is valid (see **(2)**). □

(6) *If for $g \in G$ of prime order the centralizer $C_G(g)$ satisfies $min - ab$, then* **(iii)** *is valid.*

Proof. Let $C_G(g)$ satisfy $min - ab$. In view of Suchkova–Shunkov Theorem, $C_G(g)$ is Chernikov. Assume that **(iii)** is not valid. Let M be such as in **(3)**. Then M has some descending series

$$M = M_0 \supset M_1 \supset M_2 \supset \ldots \supset M_\gamma = \cap_{\alpha<\gamma} M_\alpha$$

such that $M_\gamma \lhd G$ and M_γ satisfies $min - ab$, and for $\alpha < \gamma$, $M_\alpha \lhd G$ and M_α does not satisfy $min - ab$ (see above the proof of the assertion **(4)**). In view of Suchkova–Shunkov Theorem [8], M_γ is Chernikov. Consequently $M_\gamma = 1$. Therefore because of $C_G(g)$ is Chernikov, for some $\beta < \gamma$ we have: $C_G(g) \cap M_\beta = 1$. But then, with regard to **(5)**, **(iii)** is valid, which is a contradiction. □

Remind: the group with a normal abelian subgroup of finite index is called almost abelian.

(7) *If for $g \in G$ of prime order the $C_G(g)$ is almost abelian, then* **(iii)** *is valid.*

Proof. First, **(iii)** is valid, if $C_G(g)$ is Chernikov (see **(6)**). Let $C_G(g)$ be almost abelian non-Chernikov and A be its abelian subgroup of finite index. Since A is non-Chernikov, it is a direct product of groups of prime orders (see Lemma 1.1 [15]). Therefore, obviously, A has an infinite chain $A_1 \supset A_2 \supset \ldots \supset A_n \supset A_{n+1} \supset \ldots$ with factors of prime orders. Since G satisfies

$min - ab\bar{c}$, the set of all complemented in G terms of the chain is infinite. Let D_n complements some A_n in G. Then $A = A_n \times (A \cap D_n)$ (by S. N. Chernikov's Lemma). In view of Chunikhin's Lemma (see, for instance, [21, Lemma 1.36]), $< (A \cap D_n)^G > \subseteq D_n$. Since also $D_n \cap C_G(g)$ is finite, $< (A \cap D_n)^G > \cap C_G(g)$ is finite too. Therefore the centralizer of g in $< g >< (A \cap D_n)^G >$ is finite. Then in view of the assertion **(6)**, the statement **(iii)** with $< g >< (A \cap D_n)^G >$ in the character of G is valid. At the same time, $< (A \cap D_n)^G >$ is locally finite. Then in consequence of O. Yu. Shmidt's Theorem, the product of subgroups $< (A \cap D_n)^G >$, taken by all complemented in G subgroups A_n, is an infinite normal locally finite subgroup of G. Therefore in view of assertion **(2)**, the statement **(iii)** is valid. □

(8) *For $g \in G$ and $\pi = \pi(< g >)$ and $H =< g^G >$, all π'-subgroups of $C_H(g)$ are Chernikov.*

Proof. Assume that $C_H(g)$ has some non-Chernikov π'-subgroup. Then in view of Suchkova–Shunkov Theorem (mentioned above), this subgroup has some infinite chain $A \supset A_1 \supset A_2 \supset \ldots \supset A_n \supset A_{n+1} \supset \ldots$ of abelian subgroups. Some A_n has a complement D in G. Then, with regard to S.N. Chernikov's Lemma, we have:

$$A\times < g >= A_n \times (D \cap A\times < g >) = A_n \times (D \cap A) \times (D\cap < g >) = A \times (D\cap < g >).$$

Therefore, clearly, $< g >= D\cap < g >$, i.e. $< g > \subseteq D$. Since also $G = (A\times < g >)D$, by virtue of Chunikhin's Lemma (see, for instance, [21, Lemma 1.36]), $H \subseteq D$. But $A \subseteq H$ and $A \nsubseteq D$, which is a contradiction. □

(9) *If G satisfies $min - p'$ for some p, then* **(iii)** *is valid and also G is Chernikov or contains a normal elementary abelian p-subgroup of finite index.*

Proof. Assume that **(iii)** is not valid. Let M be from the assertion **(3)**. In view of the assertion **(4)**, every p-subgroup of G is abelian or Chernikov. Consequently, M has an element g of prime order $q \neq p$. Put $H =< g^M >$. In view of the assertion **(8)**, in $C_H(g)$ all q'-subgroup are Chernikov. Consequently $C_H(g)$ satisfies $min - p$. Also $C_H(g)$ satisfies $min - p'$.

Further, every abelian subgroup of $C_H(g)$ is a direct product of a p-subgroup and a p'-subgroup. Thus it is a direct product of two Artinian subgroups, and so it is Artinian. Thus, $C_H(g)$ satisfies $min - ab$. Then in view of the assertion **(6)**, the statement **(iii)** with H in the character of G is valid. Therefore H is a normal locally finite subgroup of M, which is a contradiction. Thus, **(iii)** is valid.

Now let G be non-Chernikov. Then, with regard to N. V.Chernikova's Theorem [9, 10] (see also Introduction), $G = U \leftthreetimes V$, U and V are abelian, U is a direct product of normal in G subgroups of prime orders and G has no quasicyclic subgroups. So $U = U_p \times U_{p'}$, $V = V_p \times V_{p'}$, where U_p and V_p are p-subgroups, $U_{p'}$ and $V_{p'}$ are p'-subgroups. Since $U_{p'}$ and $V_{p'}$ are Artinian abelian, by Kurosh'es Theorem (see, for instance, [19, Proposition 4.2.11, p. 101]), $U_{p'}$ and $V_{p'}$ are Chernikov. Since G has no quasicyclic subgroups, $U_{p'}$ and $V_{p'}$ are finite. Therefore $|G : U_p \leftthreetimes V_p| < \infty$. Since U_p is obviously a direct product of normal in G subgroups of order p, if $U_p \neq 1$, and V_p is a p-subgroup, $U_p \leftthreetimes V_p = U_p \times V_p$. In consequence of Lemma 1.1 [15], $U_p \times V_p$ is elementary abelian. □

(10) *The statement* **(iii)** *is necessarily valid.*

Proof. Assume that **(iii)** is not valid. Let M be from **(3)**. Further, let g be an element of some prime order p of M. Put $H =< g^M >$. Then $C_H(g)$ satisfies $min - p'$ (see **(8)**). Therefore in view of the assertion **(9)** (with $C_H(g)$ instead of G), $C_H(g)$ is almost abelian. Therefore by virtue of the assertion **(7)**, the statement **(iii)** with H in the character of G is valid. At the same time, H is locally finite, which is a contradiction. □

B. *Show that* **(iii)** *implies* **(ii)**.

Put $A^* = C_G(A)$. In view of S. N. Chernikov's Lemma, $A^* = A \leftthreetimes (A^* \cap B)$. Then because of A and $A^* \cap B$ are abelian, A^* is abelian too. Obviously, $A^* = C_G(A^*)$. Further, clearly, $D \cap A^* = 1$. Since C is a direct product of groups of prime orders or $C = 1$, we have for some subgroup $C^* \subseteq C$: $B = (A^* \cap B) \times (D \times C^*)$. Then $B = D \times (A^* \cap B) \times C^*$. So for $B^* = D \times C^*$ we have: $G = A^*B = A^* \leftthreetimes (D \times C^*) = A^* \leftthreetimes B^*$. Therefore in view of Proposition 2 [27], every infinite abelian subgroup of G is complemented in it.

Of course, **(ii)** implies **(i)**.

Theorem is proven.

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Шунковские группы с условием минимальности для недополняемых абелевых погрупп

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В настоящей работе мы даем полное исчерпывающее описание указанных шунковских групп.

Ключевые слова: шунковская, периодическая, локально конечная, вполне факторизуемая, черниковская группа, условия минимальности, дополняемые, абелевы подгруппы.