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Antenna Arrays on Surfaces of Revolution

Alexey A. Erokhin[a], Vasily S. Panko[a]*,
Yury P. Salomatov[a] and Mikhail I. Sugak[b]
[a]Siberian Federal University
79 Svobodny, Krasnoyarsk, 660041, Russia
[b]Saint Petersburg Electrotechnical University "LETI"
5 Professora Popova Str, St. Petersburg, 197376, Russia

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In this paper explicit analytical expressions are derived which describe radiation pattern of multi ring antenna array and multi ring continuous radiator allocated on an arbitrary surface of revolution. Radiation patterns of semi-spherical, conical and hyperbolical antenna arrays are shown.

Keywords: scanning, ring antenna, antenna array, surface of revolution.

Антенные решетки на поверхностях вращения

А.А. Ерохин[а], В.С. Панько[а], Ю.П. Саломатов[а], М.И. Сугак[б]
[а]Сибирский федеральный университет
Россия, 660041, Красноярск, пр. Свободный, 79
[б]Санкт-Петербургский электротехнический университет "ЛЭТИ"
Россия, 197376, Санкт-Петербург, ул. Профессора Попова, 5

В данной статье приведены явные аналитические выражения, описывающие диаграммы направленности многокольцевой антенной решетки и многокольцевого непрерывного излучателя, расположенных на произвольной поверхности вращения. Показаны диаграммы направленности полусферической, конической и гиперболической антенных решеток.

Ключевые слова: сканирование, кольцевая антенна, антенная решетка, поверхность вращения.

1. Introduction

Directional properties of antenna arrays of various kinds are studied in the literature in detail [1-3]. In [4] and [5] main characteristics for certain class of arrays – ring and multi ring having a planar shape are described. These results can be generalized to the case of antenna arrays which are located not in the plane but on an arbitrary rotational surface (sphere, cone and similar). In this paper we will

* Corresponding author E-mail address: vpanko@sfu-kras.ru

derive an expression for radiation pattern of such arrays and will present results for base surfaces of different kind.

2. Radiation pattern of antenna array allocated on a surface of revolution

Let us consider antenna array consisting of N identical elements allocated in $z = z_0$ plane. Radiation pattern of such array is [1]:

$$\vec{f}(\theta,\varphi) = \vec{F}_э(\theta,\varphi) e^{ikz_0\cos\theta} \sum_{n=1}^{N} \dot{I}_n e^{ik(x_n \sin\theta\cos\varphi + y_n \sin\theta\sin\varphi)}, \qquad (1)$$

where $\vec{F}_э(\theta,\varphi)$ – radiation pattern of array element; $\dot{I}_n$ – amplitude and phase of n-th element excitation; x_n, y_n – coordinates of n-th element, k – free space wave number.

From this expression it can be seen that location of array plane z_0 makes an impact on phase characteristics of array only. Considering a ring array we obtain:

$$x_n \sin\theta\cos\varphi + y_n \sin\theta\sin\varphi = R\sin\theta\cos\left[\varphi - \frac{2\pi(n-1)}{N} - \alpha\right],$$

where R – array radius; α – angular location of “first” element.

As a rule, for ring array it is reasonable to assign:

$$\dot{I}_n = \left|\dot{I}_n\right| e^{i\varphi_n} = I_0 e^{i\varphi_n},$$

where I_0 – equal amplitude of array elements excitation, φ_n – excitation phase.

Phase distribution in ring array with maximum radiation in direction (θ_0, φ_0) can be expressed as

$$\varphi_n = -kR\sin\theta_0 \cos\left[\varphi_0 - \frac{2\pi(n-1)}{N} - \alpha\right].$$

To avoid an influence of ring plane location on phase radiation pattern while consideration of multi rings antennas, it is necessary to set a phase distribution as following:

$$\varphi_n = -\left\{kz_0\cos\theta_0 + kR\sin\theta_0\cos\left[\varphi_0 - \frac{2\pi(n-1)}{N} - \alpha\right]\right\}.$$

Now the expression for radiation pattern of ring array allocated in $z = z_0$ plane can be written as

$$\vec{f}(\theta,\varphi) = \vec{F}_э(\theta,\varphi) e^{ikz_0(\cos\theta - \cos\theta_0)} I_0 \sum_{n=1}^{N} e^{ikR\left\{\sin\theta\cos\left[\varphi - \frac{2\pi(n-1)}{N} - \alpha\right] - \sin\theta_0\cos\left[\varphi_0 - \frac{2\pi(n-1)}{N} - \alpha\right]\right\}}.$$

Radiation pattern of ring array element can be expressed as

$$\vec{F}_э(\theta,\varphi) = \vec{p}(\theta,\varphi) F(\theta,\varphi) e^{i\Phi(\theta,\varphi)},$$

i.e., as the product of three factors. Let is suppose that array elements have identical polarization and phase characteristics $\vec{p}(\theta,\varphi)$, $\Phi(\theta,\varphi)$ but different amplitude patterns – $F_n(\theta,\varphi)$. Therefore, we can write:

$$\vec{f}(\theta,\varphi)=\vec{p}(\theta,\varphi)e^{i\left[kz_0(\cos\theta-\cos\theta_0)+\Phi(\theta,\varphi)\right]}I_0\sum_{n=1}^{N}F_n(\theta,\varphi)e^{ikR\left\{\sin\theta\cos\left[\varphi-\frac{2\pi(n-1)}{N}-\alpha\right]-\sin\theta_0\cos\left[\varphi_0-\frac{2\pi(n-1)}{N}-\alpha\right]\right\}}.$$

If array element possesses phase center, then we can assign $\Phi(\theta,\varphi)=\text{const}$ in expression for $\vec{F}_э(\theta,\varphi)$ and will not pay attention to the influence of element phase characteristic. The same can be said about polarization characteristic. Finally, we obtain

$$\dot{f}(\theta,\varphi)=e^{i\left[kz_0(\cos\theta-\cos\theta_0)\right]}I_0\sum_{n=1}^{N}F_n(\theta,\varphi)e^{ikR\left\{\sin\theta\cos\left[\varphi-\frac{2\pi(n-1)}{N}-\alpha\right]-\sin\theta_0\cos\left[\varphi_0-\frac{2\pi(n-1)}{N}-\alpha\right]\right\}}.$$

Then we will examine antenna array consisting of *M* rings whose centers is situated on *0Z* axis. For radiation pattern of such multi ring array we can write:

$$\dot{f}(\theta,\varphi)=\sum_{m=1}^{M}I_m e^{ikz_m(\cos\theta-\cos\theta_0)}\sum_{n=1}^{N_m}F_n^m(\theta,\varphi)e^{ikR_m\left\{\sin\theta\cos\left[\varphi-\frac{2\pi(n-1)}{N_m}-\alpha_m\right]-\sin\theta_0\cos\left[\varphi_0-\frac{2\pi(n-1)}{N_m}-\alpha_m\right]\right\}}, \quad (2)$$

where the following variables describe an every *m*-th ring array: I_m – amplitude of elements excitation (within every ring amplitude is supposed to be uniform); z_m – location of array center along *0Z* axis; $F_n^m(\theta,\varphi)$ – amplitude radiation pattern of *n*-th element; R_m – radius; N_m – number of elements; α_m – angular position of "first" element.

This expression allows us to explore the effect of the amplitude distribution for individual ring array I_m, rings location along *0Z* axis z_m, various orientations of elements $F_n^m(\theta,\varphi)$ and ring radii (R_m) on radiation pattern of multi ring array allocated on surface of revolution. The simplest case is $I_m = I_0$, $z_m = 0$, $F_n^m(\theta,\varphi)=F_0(\theta,\varphi)$.

3. Continuous circular radiators on a surface of revolution

Expression for normalized radiation pattern of a continuous ring radiator with an arbitrary direction of radiation maximum (θ_0, φ_0) can be obtained from (1) in the limit when $N\to\infty$:

$$F(\theta,\varphi)=\frac{1}{2\pi}e^{ikz_0\cos\theta}\int_0^{2\pi}e^{ikR_0\left[\sin\theta\cos(\varphi-\psi)-\sin\theta_0\cos(\varphi_0-\psi)\right]}d\psi,$$

where R_0 – ring radius; φ_0, θ_0 – maximum radiation direction; k – free space wave number.

Then after transformation described in [4] we will derive:

$$F(\theta,\varphi)=J_0\left(kR_0\sqrt{\sin^2\theta+\sin^2\theta_0-2\sin\theta\sin\theta_0\cos(\varphi-\varphi_0)}\right).$$

where J_0 – Bessel function of first kind of zero order.

Phase distribution $\Phi(\varphi)$ in continuous ring radiator with radius R_0, situated in $z = 0$ plane and having radiation maximum in θ_0, φ_0 direction:

$$\Phi(\varphi)=-kR_0\sin\theta_0\cos(\varphi_0-\varphi).$$

If the ring is located in plane $z = z_0$, then phase distribution becomes:

$$\Phi(\varphi)=-k\left[z_0\cos\theta_0+R_0\sin\theta_0\cos(\varphi_0-\varphi)\right].$$

Let us suppose that radiating system consists of a number of continuous ring with different radius, and radiation maximums coincide with each other:

$$\Phi = -kz\cos\theta_0 .$$

Here we assume that the centers of the rings are located on the axis *0Z*. Radiation pattern of such a system can be written as:

$$\dot{f}_\Sigma(\theta,\varphi) = 2\pi\sum_{m=1}^{M} R_m \dot{I}_m e^{ikz_m\cos\theta} J_0\left[kR_m\delta(\theta,\varphi,\theta_0,\varphi_0)\right],$$

where $\delta(\theta,\varphi,\theta_0,\varphi_0) = \sqrt{\sin^2\theta + \sin^2\theta_0 - 2\sin\theta\sin\theta_0\cos(\varphi-\varphi_0)}$, R_m – ring antennas radii; M – number of rings.

To get all the rings radiating in the same direction (θ_0, φ_0), it is necessary to set

$$\dot{I}_m = I_m e^{-ik\left[z_m\cos\theta_0 + R_m\sin\theta_0\cos(\varphi_0-\varphi)\right]}.$$

Now it is possible to write the expression for normalized radiation pattern in that case:

$$F_\Sigma(\theta,\varphi) = \frac{1}{\sum_{m=1}^{M} I_m R_m}\sum_{m=1}^{M} R_m I_m e^{ikz_m(\cos\theta-\cos\theta_0)} J_0\left[kR_m\delta(\theta,\varphi,\theta_0,\varphi_0)\right].$$

If we assume all rings are located on certain surface of revolution:

$$\dot{f}(\theta,\varphi) = \int_0^{R_0} I\left[z(r)\right] e^{ikz(r)(\cos\theta-\cos\theta_0)} r J_0\left[kr\delta(\theta,\varphi,\theta_0,\varphi_0)\right]dr,$$

where $I(z)$ – amplitude distribution; R_0 – antenna radius; $z(r)$ – function describing the antenna surface.

For direction of radiation maximum ($\theta = \theta_0$, $\varphi = \varphi_0$), this expression can be written in the following form:

$$\dot{f}(\theta_0,\varphi_0) = \int_0^{R_0} I(r) r J_0\left[kr\delta(\theta_0,\varphi_0,\theta_0,\varphi_0)\right]dr.$$

This integral does not depend on θ_0, φ_0 and is fully determined by antenna radius and amplitude distribution. Its maximum equals to:

$$\dot{f}_{\max} = \int_0^{R_0} rI(r)dr.$$

This expression determines maximum field of antenna allocated on a surface of revolution. As can be seen in this case, field amplitude is fully determined by antenna aperture R_0 (not taking into account shape of amplitude distribution). This fact that antenna can be lengthy does not affect to field amplitude in direction of radiation maximum, because integral does not depend on $z(r)$.

Antenna can have arbitrary shape of surface of revolution. Maximum power is radiated in the direction of surface axis of revolution, and does not depend on surface shape.

4. Examples of antenna arrays on surfaces of revolution

Let us consider various kinds of antenna arrays allocated on surfaces of revolution. Fig. 1-3 demonstrates semi-spherical, conical, hyperbolical array and radiation patterns of such arrays for three scan angles and $N \to \infty$.

5. Conclusion

In this paper we derived analytical expression which can be used for calculation of radiation patterns of multi ring antenna arrays and multi ring continuous radiators allocated on an arbitrary surface of revolution.

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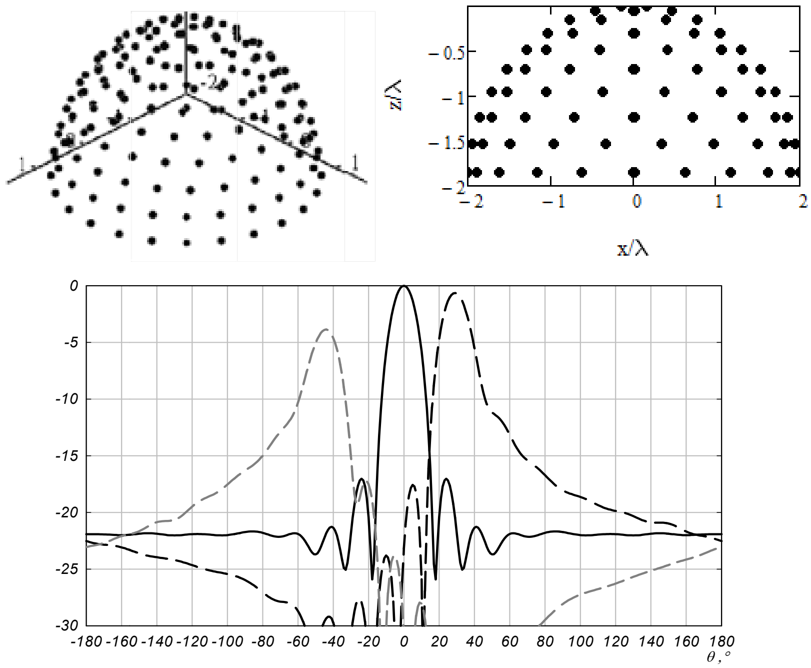

Fig. 1. Semi-spherical array (top), radiation pattern of semi-spherical array (bottom)

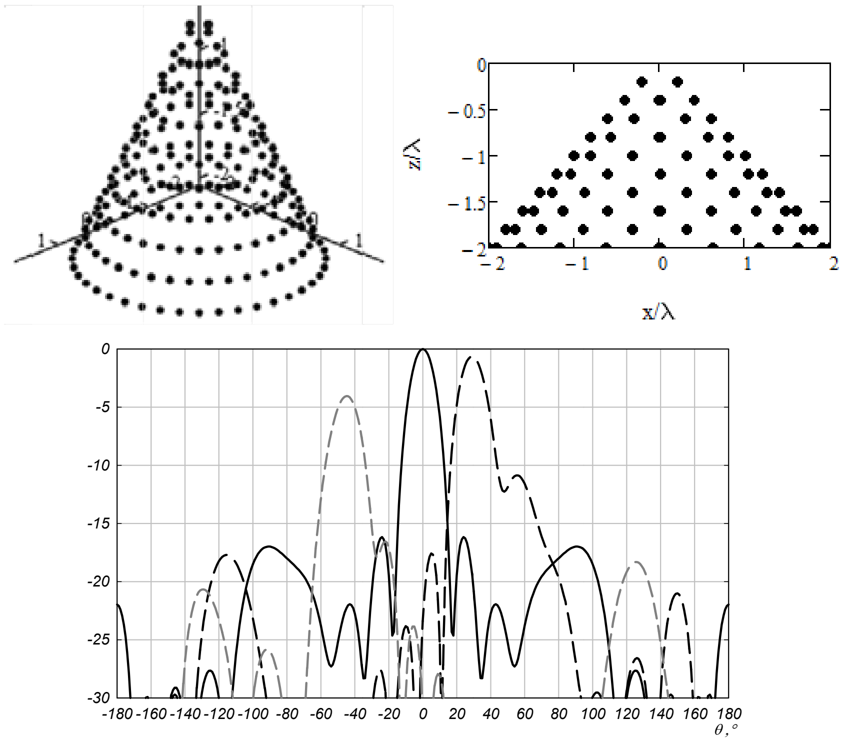

Fig. 2. Conical array (top), radiation pattern of conical array (bottom)

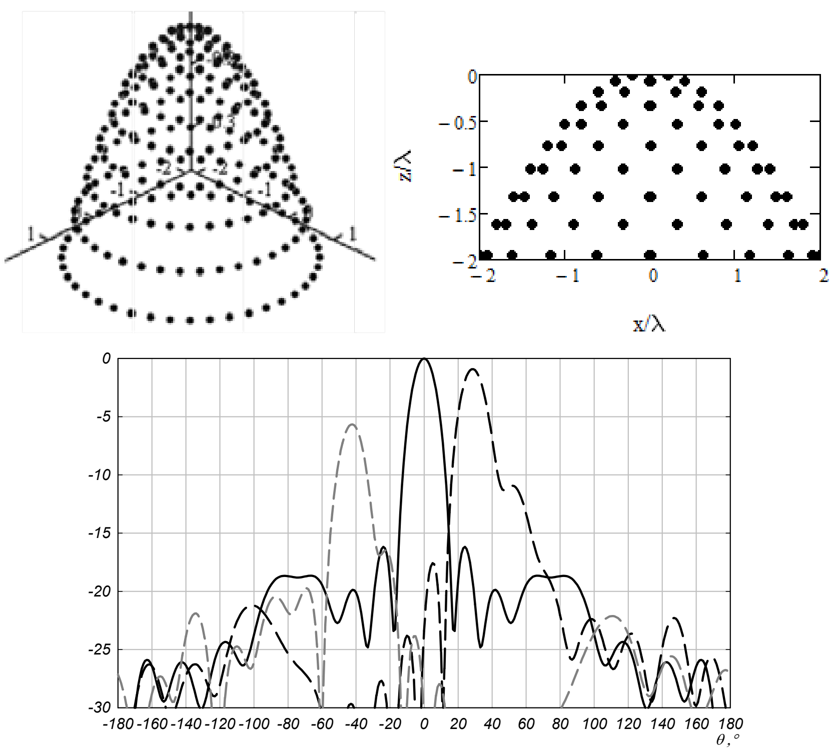

Fig. 3. Hyperbolical array (top), radiation pattern of hyperbolical array (bottom)

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