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Existence of Bianchi type Cosmological Phantom universe with Polytropic EoS Parameter in Lyra's Geometry

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Abstract. This study explores the Bianchi type-V cosmological model with in the frame work of general relativity, featuring a perfect fluid governed by the polytropic equation in Lyra's Geometry, expressed $p = \alpha\rho + k\rho^n$, as proposed at [1]. We considered the case representing phantom universe for$(1 + \alpha + \mathrm{k}\rho^{n-1}) \leqslant 0$, $k < 0$, where ρ increases with the radius $a(t)$. The role of Lyra;s Geometry has been discussed/ The solution to Einsteins field equation have been derived, providing insights into the physical and cosmological attributes of this particular model.

Keywords: accelerated expansion, polytropic Eequation of state, perfect fluid, phantom universe, Lyra's geometry.

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Introduction

In modern cosmology, the present combination of the universe's energy is delineated as around 5% ordinary matter, 20% dark matter, and 75% dark energy [1] The universe's expansion initiated with a remarkable inflationary surge propelled by vacuum energy. Between 10^{-35} and 10^{-33} seconds after the onset of the Big Bang, the universe underwent a staggering expansion by a factor of 10^{30} [2–4]. However, inflation fails to provide an explanation for the time preceding the origin of the universe. As a result, the universe transitioned into the radiation era and, as the temperature decreased less than 10^3 K, the matter era advanced [5]. Currently, the universe undergoes accelerated expansion [6], attributed to either the cosmological constant or a form of dark energy with negative pressure violating the strong energy condition [7]. This phase represents a second inflationary period, which is distinct from the initial one. However, the nature of the pre-radiation era (very early universe), dark matter and dark energy has remained enigmatic and mysterious, prompting speculation. The nature of the early universe in the context of the Big Bang theory is not understood [8]. The characteristic s of the universe are not consistent

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with weather the universe was hot or cold state. The investigation of cosmic microwave background radiation (CMB) was achieved via the hot universe theory [9–11] therefore, we accepted the Big Bang theory. According to that theory, the early universe was combined with an ultra-relativistic classical gas can present a photon, electrons, positrons, quarks, antiquarks, etc. It is clearly observed that at the early stage of the universe, the scale factor vanishes while the energy density and temperature become infinite. This situation reveals the initial cosmological singularity commonly known as the Big Bang. The up-to-date studies on a cosmological model based on Bose–Einstein condensate dark matter (BEC DM). The Bose–Einstein condensate EoS [12–15] can be generalized as $p = \alpha\rho + k\rho^2$. If k $>$ 0 the model represents repulsive, if k $<$ 0, the model represents attractive self-interaction and when k = 0, the standard linear equation of state $p = \alpha\rho c^2$.

This paper delves into the examination of the modified equation of state, as proposed by [1].

$$p = \alpha\rho + k\rho^n \tag{1}$$

The author of [16, 17] reviewed different available sources and studied the likeness between the polytropic equation of state and a cosmological model, where the fluid that fills the universe has an effective bulk viscosity. The author of [18] suggested that a polytropic gas model. The authors of [19] used the available information from different sources and studied the polytropic inflationary model in brane world scenario. The author [20] studied Bianchi-I Dark with polytropic DE. The authors of [21] studied the Kantowski universe with the Polytropic EoS parameter. The authors of [22] conducted a remarkable study on anisotropic and homogeneous cosmological models with the polytropic EoS parameter. The authors of [23–25] performed an intended study on Bianchi-type-V cosmological models in modified theories of gravitation.

Einstein's field equations, integral to comprehending the uniformity and static model of the universe, impose constraints allowing only dynamic cosmological models for non-zero energy density. Consequently, Einstein's general theory of relativity becomes interpretable in terms of geometry. Following the advent of general relativity, Weyl [26] in 1918 expanded Riemannian geometry, applying it to physical contexts to formulate the initial unified theory encompassing gravity and electromagnetism. However, Einstein vehemently opposed Weyl's unified theory, leading to its neglect for over several years. Subsequently, Lyra [27] introduced a gauge function, effectively modifying Riemannian geometry and rendering it a structureless manifold. This alteration naturally gave rise to cosmological constants within the fabric of the universe's geometry. Since we have performed remarkable research, we are interested in continuing our research with this paper on the Bianchi type-V model and the energy-momentum tensor consisting of a perfect fluid with polytropic equation of state in Lyra's Geometry. In the case $(1 + \alpha + k\rho^{n-1}) \leqslant 0$, $k < 0$ the universe is considered to represent the phantom universe.

This document is structured as subsequent sections: In Section 2 Field equations of these metrics are obtained by Bianchi type-V metric in the existence of a perfect fluid with the polytropic EoS parameter in Lyra's Geometry. Section 3 is dedicated to solution of the model. Section 4 is devoted to the physical and geometric properties of the model. Section 5 — exact solution of the model.

1. Metric and field equations

Since Bianchi models are spatially homogeneous and anisotropic and are more appropriate for describing the universe because it has less symmetry than do standard FRW models. So, we considered here Bianchi type-V cosmological model.

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2x}dy^2 - C^2e^{-2x}dz^2 \tag{2}$$

where A(t), B(t) and C(t) are the three anisotropic directions of expansion in normal three-dimensional space.
The average scale factor a(t), the spatial volume V and the average Hubble's parameter are

$$a\left(t\right)=\left(ABC\right)^{\frac{1}{3}} \tag{3}$$

$$V=a^{3}=ABC \tag{4}$$

$$H=\frac{1}{3}(H_{1}+H_{2}+H_{3} \tag{5}$$

Here, $H_1=\frac{\acute{A}}{A}$, $H_2=\frac{\acute{B}}{B}$ and $H_1=\frac{\acute{C}}{C}$.

$$\frac{\acute{V}}{V}=\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=3H \tag{6}$$

The Einstein modified the filed equation in normal gauge for Lyra's modified equation is given by (where $8\pi G=1$, $C=1$)

$$R_{i}^{j}-\frac{1}{2}g_{i}^{j}R+\left[\frac{3}{2}\phi_{i}\phi^{j}-\frac{3}{4}\phi_{k}\phi^{k}g_{i}^{j}\right]=-T_{i}^{j} \tag{7}$$

$$\phi_{i}=\left(\beta\left(t\right),0,0,0\right) \tag{8}$$

where ϕ_i is the displacement vector Let T_{ij} be the energy- momentum tensor of the matter [9]. Additionally,

$$T_{;j}^{ij}\equiv 0 \tag{9}$$

The energy momentum tensor T_{ij}

$$T_{ij}=\left(p+\rho\right)u_{i}u_{j}-pg_{ij} \tag{10}$$

where ρ is the energy density, p is the pressure and u^i is the four velocity vectors satisfying $g_{ij}u^iu^j=1$. The above perfect fluid obeys the polytropic equation of state.

$$p=\alpha\rho+k\rho^{n}\text{ with }\ k<0 \tag{11}$$

The conservation equation $T_{;j}^{ij}\equiv 0$ leads to

$$\dot{\rho}+\left(p+\rho\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{12}$$

Using Eq. (11) and Eq. (12), we have

$$\dot{\rho}+3H\rho(1+\alpha+k\rho^{n-1})=0 \tag{13}$$

We considered the case here for the phantom universe is $(1+\alpha+k\rho^{n-1})\leqslant 0$, $k<0$ [23], where $-1\leqslant\alpha\leqslant 1$, k is the polytropic constant and n is the polytropic index. Moreover conservation of the L.H.S of Eq. (7) leads to

$$R_{i}^{j}-\frac{1}{2}g_{i}^{j}R+\left[\frac{3}{2}\phi_{i}\phi^{j}-\frac{3}{4}\phi_{k}\phi^{k}g_{i}^{j}\right]=0 \tag{14}$$

$$\frac{3}{2}\phi_{i}\left[\frac{\partial\phi^{j}}{\partial x^{j}}+\phi^{l}\Gamma_{lj}^{j}\right]+\frac{3}{2}\phi^{j}\left[\frac{\partial\phi_{i}}{\partial x^{j}}-\phi_{l}\Gamma_{ij}^{l}\right]-\frac{3}{4}g_{i}^{j}\phi_{k}\left[\frac{\partial\phi^{k}}{\partial x^{j}}+\phi^{l}\Gamma_{lj}^{k}\right]-\frac{3}{4}g_{i}^{j}\phi^{k}\left[\frac{\partial\phi_{k}}{\partial x^{j}}-\phi_{l}\Gamma_{kj}^{l}\right]=0 \tag{15}$$

Eq. (10) leads to

$$\frac{3}{2}\beta\dot{\beta}+\frac{3}{2}\beta^{2}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{16}$$

2. Solution and model

In a co-moving coordinate system, by Eqs. (7) and (10) we have

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{1}{A^2}+\frac{3}{4}\beta^2=p \tag{17}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{A^2}+\frac{3}{4}\beta^2=p \tag{18}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}+\frac{3}{4}\beta^2=p \tag{19}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{3}{A^2}-\frac{3}{4}\beta^2=-\rho \tag{20}$$

$$\frac{2\dot{A}}{A}-\frac{\dot{B}}{B}+\frac{\dot{C}}{C}=0 \tag{21}$$

Integrating Eq. (21), we obtain

$$A^2=BC \tag{22}$$

Without loss of generality, subtracting (17) from (18) and (18) from (19) and taking the second integral of each, we obtain the following three relations.

$$\frac{A}{B}=c_1e^{\int\frac{d_1}{a^3}dt},\quad \frac{A}{C}=c_2e^{\int\frac{d_2}{a^3}dt}\quad \text{and}\quad \frac{B}{C}=c_3e^{\int\frac{d_3}{a^3}dt} \tag{23}$$

where c_1, c_2, c_3, d_1, d_2 and d_3 are constants of integration. Since, $a\left(t\right)=\left(ABC\right)^{\frac{1}{3}}$, by Eq. (17) and Eqs. (23) we obtain

$$A(t)=l_1ae^{\left(m_1\int a^{-3}dt\right)},\quad B(t)=l_2ae^{\left(m_2\int a^{-3}dt\right)},\quad C(t)=l_3ae^{\left(m_3\int a^{-3}dt\right)} \tag{24}$$

where $l_1=(d_1d_2)^{\frac{1}{3}}$, $m_1=\dfrac{k_1+k_2}{3}$, $l_2=\left(\dfrac{d_3}{d_1}\right)^{\frac{1}{3}}$, $m_2=\dfrac{k_3-k_1}{3}$, $l_3=(d_3d_2)^{-\frac{1}{3}}$, $m_3=\dfrac{-\left(k_3+k_2\right)}{3}$
The constants m_1, m_2, m_3 and l_1, l_2, l_3 will satisfy these two conditions:

$$m_1+m_2+m_3=0\ \text{ and }\ l_1l_2l_3=1 \tag{25}$$

Using Eq. (22) in Eqs. (24), we obtain

$$l_1=1,\quad l_2=l_3^{-1}=k_1(say),\quad m_1=0,\quad m_2=-m_3=k_2(say) \tag{26}$$

By using Eq. (26) in Eqs. (25), we have

$$A\left(t\right)=a\left(t\right),\quad B\left(t\right)=k_1a(t).e^{\left(k_2\int(a(t))^{-3}dt\right)},\quad C(t)=\frac{a(t)}{k_1}.e^{\left(-k_2\int(a(t))^{-3}dt\right)} \tag{27}$$

By using Eq. (27) in Eqs. (2) we have

$$ds^2=dt^2-a^2dx^2-\left[k_1a(t).e^{\left(k_2\int(a(t))^{-3}dt\right)}\right]^2e^{-2x}dy^2-\left[\frac{a(t)}{k_1}.e^{\left(-k_2\int(a(t))^{-3}dt\right)}\right]^2e^{-2x}dz^2 \tag{28}$$

This represents the Bianchi type-V cosmological model with an average scale factor.

3. Physical and geometric properties

Solve the differential equation Eq. (13) in case of $(1+\alpha+k\rho^{n-1}) \leqslant 0$, $k<0$, represents the phantom universe, where the density increases with the radius. We obtain density, average scale factor and pressure as follows

$$\rho = \left[\frac{1+\alpha}{a^{-3(1+\alpha)(n-1)}-k}\right]^{\frac{1}{n-1}} \text{ for } (1+\alpha)>0 \tag{29}$$

$$a\left(t\right) = \left[\frac{1+\alpha}{\rho^{n-1}}+k\right]^{3(1+\alpha)(n-1)} \tag{30}$$

$$p = \alpha\left[\frac{1+\alpha}{a^{-3(1+\alpha)(n-1)}-k}\right]^{\frac{1}{n-1}} + k\left[\frac{1+\alpha}{a^{-3(1+\alpha)(n-1)}-k}\right]^{\frac{n}{n-1}} \tag{31}$$

Clearly, $\rho\left(t\right) \propto \left[a(t)\right]^{3}$ indicates that the universe is considered to be the phantom universe.

$$\omega\left(t\right) = \frac{p}{\rho} = \alpha + k\left[\frac{1+\alpha}{a^{-3(1+\alpha)(n-1)}-k}\right] \tag{32}$$

When $(1+\alpha)>0$ or $\alpha>-1$ and $k<0$, the EoS parameter $\omega\left(t\right)<-1$ representing the model is the phantom universe, which leads to no Big Rip singularity. Solving Eq. (16) for the displacement vector $\beta(t)$, we have

$$\beta(t) = ca^{-3} \tag{33}$$

c is the integration constant Considering c=1 and substituting $a(t)=\beta^{-1/3}(t)$ in eqs. (28), (31) and (32), we have

$$\rho = \left[\frac{1+\alpha}{\beta^{(1+\alpha)(n-1)}-k}\right]^{\left(\frac{1}{n-1}\right)} \tag{34}$$

$$p = \alpha\left[\frac{1+\alpha}{\beta^{(1+\alpha)(n-1)}-k}\right]^{\left(\frac{1}{n-1}\right)} + k\left[\frac{1+\alpha}{\beta^{(1+\alpha)(n-1)}-k}\right]^{\left(\frac{n}{n-1}\right)} \tag{35}$$

$$\omega\left(t\right) = \alpha + k\left[\frac{1+\alpha}{\beta^{(1+\alpha)(n-1)}-k}\right. \tag{36}$$

$$ds^2 = dt^2 - \frac{dx^2}{\beta^{\frac{2}{3}}\left(t\right)} - \left[k_1.\frac{e^{\left(k_2\int\beta(t)dt\right)}}{\beta^{\frac{1}{3}}\left(t\right)}\right]^2 e^{-2x}dy^2 - \left[\frac{e^{\left(-k_2\int\beta(t)dt\right)}}{k_1\beta^{\frac{1}{3}}\left(t\right)}\right]^2 e^{-2x}dz^2 \tag{37}$$

Since, $\rho(t) \propto \left[a(t)\right]^3$ and $\rho(t) \propto \dfrac{1}{\beta(t)}$, it can be described with this displacement vector $\beta(t)$ in the late universe. Since the ρ increases with respect to a(t) the universe accelerating rapidly. At $n=-1$, the analytical model of phantom universe represents bouncing universe "disappearing" at t=0.

4. Exact solutions of the model

The significance of the hyperbolic function as an exponential component is that the derived deceleration parameter demonstrates time dependence, highlighting that the universe's is in the acceleration phase. Therefore, opting for this average scale factor is deemed physically acceptable. Consider an average scale factor $a(t)$ as by [26]

$$a\left(t\right) = \left(\sin ht\right)^{\frac{1}{3}} \tag{38}$$

Therefore, by Eqs. (32) and (35)

$$A(t) = (\sin ht)^{\frac{1}{3}}. \tag{39}$$

$$B(t) = k_1(\sin ht)^{\frac{1}{3}} (\cot ht - \operatorname{cosec} ht)^{k_2} \tag{40}$$

$$C(t) = \frac{(\sin ht)^{\frac{1}{3}} (\cot ht - \operatorname{cosec} ht)^{k_2}}{k_1}. \tag{41}$$

Substituting Eqs. (38)–(41) in Eq. (2) we get

$$\begin{aligned} ds^2 = dt^2 - t^{\frac{2}{3}} dx^2 - \left[(\sin ht)^{\frac{1}{3}} (\cot ht - \operatorname{cosec} ht)^{k_2}\right]^2 e^{-2x} dy^2 - \\ - \left[\frac{(\sin ht)^{\frac{1}{3}} (\cot ht - \operatorname{cosec} ht)^{k_2}}{k_1}\right]^2 e^{-2x} dz^2. \end{aligned} \tag{42}$$

This represents the field equation of the Bianchi type-V cosmological model with polytropic EoS parameter in Lyra's geometry which is in phantom era.
We obtain the spatial volume V(t), density (ρ), pressure (p), EoS parameter $\omega(t)$, Hubble's parameter H(t), energy density parameter $\Omega(t)$, scalar expansion $\theta(t)$, deceleration parameter $q(t)$, anisotropy parameter, and shear scalar for the model are

$$V(t) = a^3 = \sin ht. \tag{43}$$

$$\rho = \left[\frac{1+\alpha}{(\sin ht)^{(1+\alpha)(n-1)} - k}\right]^{\left(\frac{1}{n-1}\right)} \tag{44}$$

$$p = \alpha \left[\frac{1+\alpha}{(\sin ht)^{(1+\alpha)(n-1)} - k}\right]^{\left(\frac{1}{n-1}\right)} + k \cdot \left[\frac{1+\alpha}{(\sin ht)^{(1+\alpha)(n-1)} - k}\right]^{\left(\frac{n}{n-1}\right)} \tag{45}$$

$$\omega(t) = \alpha + k \cdot \left[\frac{1+\alpha}{(\sin ht)^{(1+\alpha)(n-1)} - k}\right]^{\left(\frac{-1}{n-1}\right)} \tag{46}$$

$$H(t) = \cot ht \tag{47}$$

$$\Omega(t) = \frac{\rho}{3H} = \frac{\left[\frac{1+\alpha}{(\sin ht)^{(1+\alpha)(n-1)} - k}\right]^{\frac{1}{n-1}}}{3 \cot ht} \tag{48}$$

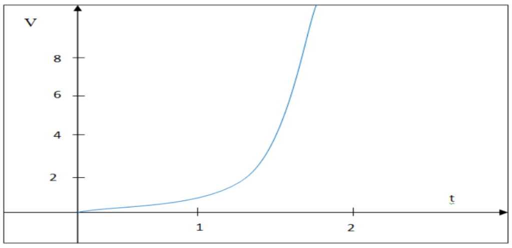
$$\theta(t) = 3H = 3 \cot ht \tag{49}$$

$$q(t) = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \sec h^2 t - 1 \tag{50}$$

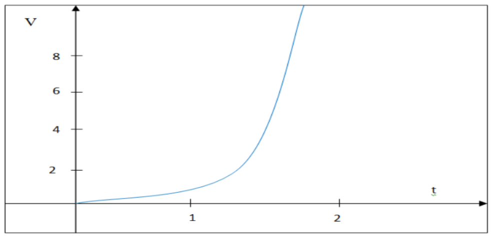
$$A_m = \frac{1}{3}\left(\frac{3}{\cot ht \cdot \sin ht}\right)^2. \tag{51}$$

$$\sigma^2 = \left(\frac{1}{\sin ht}\right)^2 \tag{52}$$

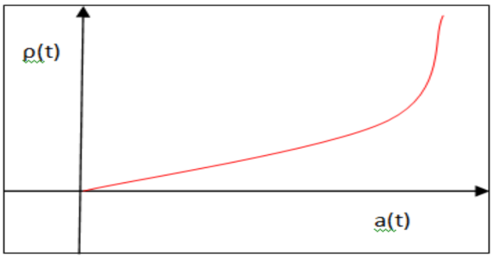
Here, we can note that spatial volume is zero for t=0 and that the scalar expansion is infinite, which is the big bang scenario. Additionally pressure (p), density (ρ), Hubble's(H) and shear scalar (σ) diverge in the early universe. As $t \to \infty$ increases the, volume becomes infinity where the pressure (p), density (ρ), Hubble parameter (H) and shear scalar (σ) approach zero. Since, $\lim_{t\to\infty} \frac{\rho}{\theta^2} = constant$ represents the anisotropic nature of the model.



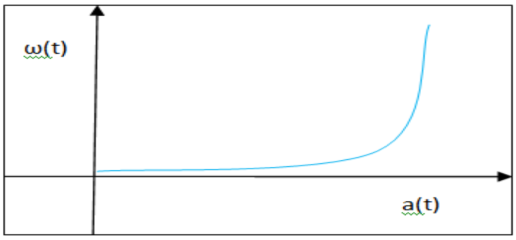
Volume V v/s Cosmic time t



displacement vector $\beta(t)$ v/s average scale factor a(t)



density ρ v/s average scale factor a(t)



EoS parameter $\omega\,(t)$ ρ v/s average scale factor a(t)

Conclusion

In this paper, we studied the Bianchi type-V cosmological model in the presence of a perfect fluid with polytropic equation of state. We considered a this case $(1 + \alpha + k\rho^{n-1}) < 0$, $k < 0$ representing that the model of the universe is phantom stage with increasing density with respect to average scale factor. We observe that the spatial volume is zero for t=0 and that the scalar expansion is infinite, which shows that the universe starts evolving with zero volume at t=0 which is the big bang scenario. We also, observed in this model that as time increases, the volume increases and becomes infinitely large $t \to \infty$. Clearly, $\rho\,(t) \propto [a\,(t)]^3$ and $\rho(t) \propto \dfrac{1}{\beta(t)}$ $ indicates that the universe is considered to be the phantom universe, where the density increases with the radius (average scale factor). Moreover pressure increases with respect to the scale factor. The EoS parameter $\omega\,(t) < -1$ representing the model is the phantom universe and leads to no Big Rip singularity. When $n = -1$ this is the analytical model of the phantom bouncing universe "disappearing" at t=0. $\lim_{t\to\infty} \dfrac{\rho}{\theta^2} = constant$ indicates that this model is an anisotropic model.

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Существование космологического фантома типа Бьянки с политропным параметром уравнения состояния в геометрии Лиры

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Аннотация. Эта работа исследует космологическую модель Bianchi Type-V с в рамках общей теории относительности, в которой есть идеальная жидкость, управляемая политропическим уравнением в геометрии, выраженной $p = \alpha\rho + k\rho^n$, как предложено в citens. Мы рассмотрели случай, представляющий фантомную вселенную для $(1 + \alpha + \text{k } \rho^{n-1}) \leqslant 0$, $k < 0$, где ρ увеличивается с радиусом $a(t)$. Была получена роль геометрии Лиры;

Ключевые слова: ускоренное расширение, политропическое уравнение состояния, идеальная жидкость, фантомная вселенная, геометрия Лиры.