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Nonlinear Effect for Anisotropy of Charged Particle Pitch Angle Distribution at Geostationary Orbit

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Abstract. The new phenomenological model of a prediction of perpendicular anisotropy index of charged particle pitch angle distribution at geostationary (geosynchronous) orbit (GEO) in the Earth's magnetosphere, and also in any circular orbit depending from the local time LT in an orbit and the geomagnetic activity index Kp is offered. Comparison of model with the numerous experimental data is lead. It is proved, that the general analytical dependence of perpendicular anisotropy index of charged particle pitch angle distribution on GEO received as a first approximation can be used for conditions of magnetically quiet time for quantitative forecasts and comparisons with experimental data on GEO. The nonlinear effect is theoretically predicted for a difference between the maximal value of perpendicular anisotropy index of charged particle pitch angle distribution and the minimal value of perpendicular anisotropy index (in local midnight LT = 0 h) on GEO from the Kp-index of geomagnetic activity. The nonlinear effect for anisotropy of charged particle pitch angle distribution will be, possibly, to some extent and on other radial distances from the Earth.

Keywords: geostationary orbit, new model, anisotropy dynamics of charged particles, data of the CRRES satellite, nonlinear effect.

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Introduction

The charged particle pitch angle distribution is dependence a differential flux of particles j from a local pitch angle of particles α in the range from $0°$ up to $180°$. It is the important characteristic for the charged particles in velocity space in the Earth's magnetosphere.

In the monography [1] for the description different meeting in the magnetosphere of pitch angle distributions was offered following distribution

$$j(\alpha) = j_{\perp} \sin^{\gamma(\alpha)} \alpha, \tag{1}$$

where $j_{\perp}$ is the perpendicular ($\alpha = 90°$) differential flux of charged particles.

The equation (1) differs from standard by that an anisotropy index (or a parameter) of pitch angle distribution not is a constant ($\gamma = const$), and it is function from α ($\gamma = \gamma(\alpha)$).

For the range of pitch angles $0° < \alpha < 90°$ $\gamma(\alpha)$ it is possible to find under the formula

$$\gamma(\alpha) = \frac{\ln j(\alpha) - \ln j_{\perp}}{\ln \sin \alpha}. \tag{2}$$

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For $\alpha = 90°$ the equation (2) gives the relation 0/0, therefore we find a limit for γ at $\alpha \to 90°$, using the rule of Lopitalya and considering, that $\left(\frac{dj}{d\alpha}\right)_{\perp} = 0$

$$\gamma_{\perp} = -\frac{1}{j_{\perp}}\left(\frac{d^2 j}{d\alpha^2}\right)_{\perp}. \tag{3}$$

The perpendicular anisotropy index (parameter) of pitch angle distribution $\gamma_{\perp}$, presented to a general view the formula (3), is the exact indicator of type of pitch angle distribution and in this its great value. Particularly, if pitch angle distributions are normal or type "head and shoulders" — $\gamma_{\perp} > 0$. If $\gamma_{\perp} = 0$, it will already correspond isotropic or "flattop" pitch angle distribution. And at last, pitch angle distributions of type "butterfly". In this case — $\gamma_{\perp} < 0$. Such representation (3) is exact at definition of the moment of occurrence of butterfly pitch angle distribution.

The literature on pitch angle distributions of the charged particles and anisotropy of pitch angle distributions is extensive, for example [1–13]. From the review for last years it is visible, that statistical and empirical models of anisotropy of charged particles pitch angle distributions are, and the analytical mathematical models based on the physics and describing a perpendicular anisotropy index of charged particles pitch angle distribution, possibly, no.

Therefore the purpose of the given work is mathematical modeling an anisotropy index of charged particles pitch angle distribution at geostationary (geosynchronous) orbit (GEO) in the Earth's magnetosphere in the form of: 1) the new mathematical model based on the physics and describing a perpendicular anisotropy index of charged particles pitch angle distribution on GEO depending from the local time LT on GEO and the Kp-index of geomagnetic activity, 2) the analysis of consequences of the offered analytical model and 3) nonlinear effect for anisotropy of charged particle pitch angle distribution.

1. Mathematical model

As a first approximation dependence of a perpendicular anisotropy index of charged particles pitch angle distribution from time $\gamma_{\perp}(t)$ we shall find from the equation

$$\frac{d\gamma_{\perp}}{dt} = \frac{d\gamma_{\perp}}{dL}\frac{dL}{dt}. \tag{4}$$

At carrying out of numerical calculations we shall assume in the equation (4), that $dL/dt \approx \langle dL/dt \rangle$. Then, the bounce-averaged radial drift velocity of charged particle motion in the Earth's magnetosphere can be determined, for example, for the Earth's dipole magnetic field, so [1]:

$$\left\langle \frac{dL}{dt} \right\rangle = -\Omega\frac{\phi_2}{\phi_0}L^4\cos\phi, \tag{5}$$

where L is the dimensionless McIlwain parameter; t is the time; ϕ is the azimuth angle (the local time LT = 0 h at midnight) or the geomagnetic east longitude in the magnetic equator plane; $\Omega = \frac{2\pi}{24}$ is the angular velocity of the Earth's rotation in 1/h; $\phi_0 = 92$ kV; and the dependence ϕ_2, measured in kV, from the index of geomagnetic activity $Kp \equiv Kp(t)$, is determined by the formula [14]

$$\phi_2 = \frac{0.045}{(1 - 0.16Kp + 0.01Kp^2)^3}. \tag{6}$$

Then the equation (4), taking (5) into account, is written as follows

$$\frac{d\gamma_{\perp}}{dt} + \frac{d\gamma_{\perp}}{dL}\frac{\Omega\phi_2(t)L^4(t)\cos\phi(t)}{\phi_0} = 0. \tag{7}$$

Now, we can add equations that describe the trajectory of the spacecraft (SC) in the gravitational field of the Earth to equation (7). It will be easier if one specifies the trajectory (SC) in a parametric form. In this case, for a GEO (for a circular orbit) we get

$$L(t) = 6.6; \quad \phi(t) = \Omega t + \varphi_m, \tag{8}$$

where $\varphi_m = \text{const}$ will be determined from a comparison with experimental data in a GEO and t is already the local time LT along the GEO in hours.

In the future for a spacecraft with any circular orbit or with an elliptical orbit, such a replacement in (7) can be done similarly to (8).

The equations (7), (8) represent the general formulation of new phenomenological model of a prediction of perpendicular anisotropy index dynamics of charged particle pitch angle distribution on GEO in the Earth's magnetosphere.

As a result, using (6)–(8), we receive for $E = \text{const}$, $Kp = \text{const}$, $L = \text{const}$

$$\frac{d\gamma_\perp}{dt} = -C\Omega\cos(\Omega t + \varphi_m), \tag{9}$$

where

$$C = \left|\frac{d\gamma_\perp}{dL}\right| \frac{\phi_2}{\phi_0} L^4 \tag{10}$$

and at such definition (10) $C > 0$ always.

If in (10) $\frac{d\gamma_\perp}{dL} < 0$, the equation (9) will be transformed to the equation

$$\frac{d\gamma_\perp}{dt} = C\Omega\cos(\Omega t + \varphi_m^-). \tag{11}$$

Then the analytical solution of the differential equation (11), (10) will look like

$$\gamma_\perp(t) = C[\sin(\Omega t + \varphi_m^-) - \sin\varphi_m^-] + \gamma_{\perp 0}, \tag{12}$$

where $\gamma_{\perp 0}$ is the perpendicular anisotropy index of charged particle pitch angle distribution at $t = 0$, i.e., when the local time along the GEO is LT $= 0$ h at midnight.

If in (10) $\frac{d\gamma_\perp}{dL} > 0$ then the analytical solution of the differential equation (9), (10) looks like

$$\gamma_\perp(t) = -C[\sin(\Omega t + \varphi_m^+) - \sin\varphi_m^+] + \gamma_{\perp 0}. \tag{13}$$

If $Kp(t) \neq \text{const}$ and (6) $\phi_2(t) \neq \text{const}$ (a dependence from time t can be complex), we shall receive, using (10), value C^*

$$C^* = \frac{L^4}{\phi_0} \tag{14}$$

and the following general formula for modeling (predicting) calculations $\gamma_\perp(t)$ on GEO and in any circular orbit SC

$$\gamma_\perp(t) = \pm C^*\Omega \int_0^t \frac{d\gamma_\perp}{dL} \phi_2(t) \cos(\Omega t + \varphi_m^\mp) dt + \gamma_{\perp 0}, \tag{15}$$

as generally a gradient $\frac{d\gamma_\perp}{dL} = f(L, LT, E, Kp, t)$.

2. Experimental data and calculations

For definition at geostationary (geosynchronous) orbit of concrete value $\varphi_m^+ = \text{const}$ we use comparison with the numerous experimental data [5], received on GEO. For $\dfrac{d\gamma_\perp}{dL} > 0$ on these experimental data the maximum of perpendicular anisotropy index of charged particle pitch angle distribution $\gamma_\perp(t_m)$ (13), when $t_m = 13$ h LT on GEO. Thus, in the further t_m will designate the moment of time in hours LT, when the perpendicular anisotropy index of charged particle pitch angle distribution on GEO has the maximal value.

Further in a point of a maximum the first derivative (9) $\dfrac{d\gamma_\perp}{dt}(t = t_m) = 0$, therefore the condition should be satisfied

$$\cos(\Omega t_m + \varphi_m^+) = 0. \tag{16}$$

Considering, that $t_m = 13$ h, the condition (16) is carried out, when $\Omega t_m + \varphi_m^+ = 3\pi/2$. Whence follows, that

$$\varphi_m^+ = \frac{\pi(18 - t_m)}{12}. \tag{17}$$

Under the formula (17) for experimental data [5] value $\varphi_m^+ = 5\pi/12$ rad, $\gamma_{\perp 0} = 0.1860$, $\gamma_\perp(t_m) = 1.5059$ for $Kp = 0$ (Fig. 1), and for $Kp = 3$ $\gamma_{\perp 0} = 0.0$, $\gamma_\perp(t_m) = 1.8353$ (Fig. 2).

For the moment of time $t = t_m$ following analytical dependence turns out $\gamma_\perp(t_m)$ (13) for perpendicular anisotropy index of charged particle pitch angle distribution on GEO, when $Kp \approx \text{const}$, for example, within one day

$$\gamma_\perp(t_m) = -C[\sin(\Omega t_m + \varphi_m^+) - \sin\varphi_m^+] + \gamma_{\perp 0} = C[1 + \sin\varphi_m^+] + \gamma_{\perp 0}, \tag{18}$$

and for the intermediate moment of time $t = t_p$ it is received following dependence $\gamma_\perp(t_p)$

$$\gamma_\perp(t_p) = -C[\sin(\Omega t_p + \varphi_m^+) - \sin\varphi_m^+] + \gamma_{\perp 0}. \tag{19}$$

Further we shall find the useful formula in the form of a difference of two equations (18) and (19)

$$\gamma_\perp(t_m) - \gamma_\perp(t_p) = C[1 + \sin(\Omega t_p + \varphi_m^+)]. \tag{20}$$

The formula (20) is useful to a finding of a constant C, when at $t = 0$ value $\gamma_{\perp 0}$ unknown, and other values in (20) known from experimental data. Then knowing C, value $\gamma_{\perp 0}$ can find from the equation (18). On the other hand at $t_p = 0$ equation (20) passes in the equation (18) as it and should be. If value $\gamma_{\perp 0}$ known, the constant C can be found at once from the equation (18).

Thus, determining φ_m^+ (17), $\gamma_{\perp 0}$ and C, for $\dfrac{d\gamma_\perp}{dL} > 0$ final analytical dependence turns out (13) for perpendicular anisotropy index of charged particle pitch angle distribution on GEO, when $Kp \approx$ const, for example, within one day.

To compare dependence from local time $\gamma_\perp(t)$ (13), (17) with averaged on LT and energy E experimental data for electrons [5] on GEO, let's make modeling calculations (as a first approximation) for $Kp=0$ and $Kp=3$. Results of calculations are presented on Fig. 1 and Fig. 2.

For the index of geomagnetic activity $Kp = 0$ (Fig. 1) at comparison (13) with experimental data good conformity is received, as on numerous data [5] the averaged values of an anisotropy index $\gamma_\perp(t)$ for $0 \leqslant Kp \leqslant 1$ have the maximal values approximately in the moment of time $t_m = 13$ h LT on GEO and they rather are not sensitive to value of kinetic energy E. For the geomagnetic activity index $Kp = 3$ (Fig. 2) at comparison more good conformity with numerous data [5] of the averaged values of an anisotropy index $\gamma_\perp(t)$ for $2 \leqslant Kp \leqslant 4$ is received.

The divergence is connected with experiment by that dependence $\gamma_\perp(t)$ (13) while is certain only as a first approximation. Thus, the general analytical dependence of perpendicular anisotropy index of charged particle pitch angle distribution on GEO $\gamma_\perp(t)$ (13), (17), received

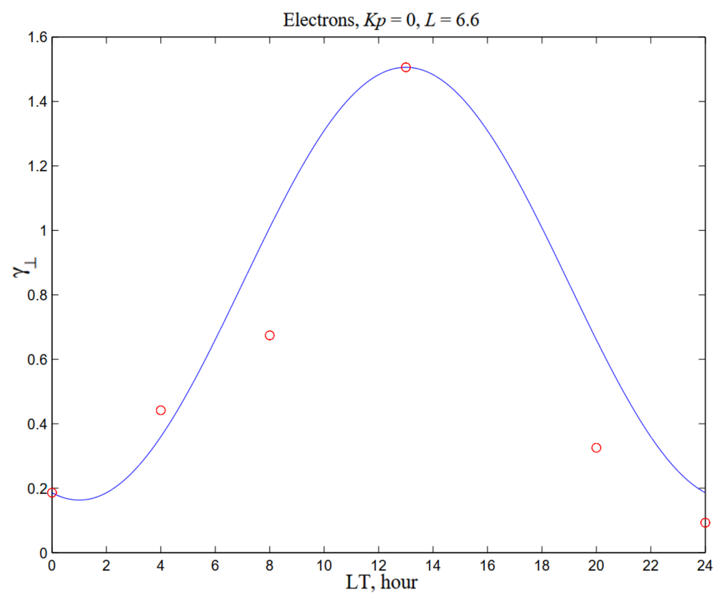

Fig. 1. The continuous line is modeling analytical dependence of perpendicular anisotropy index of electron pitch angle distribution on GEO $\gamma_\perp(t)$ (13) from the local time LT for the index of geomagnetic activity $Kp = 0$. Circles are designated the averaged experimental data [5]

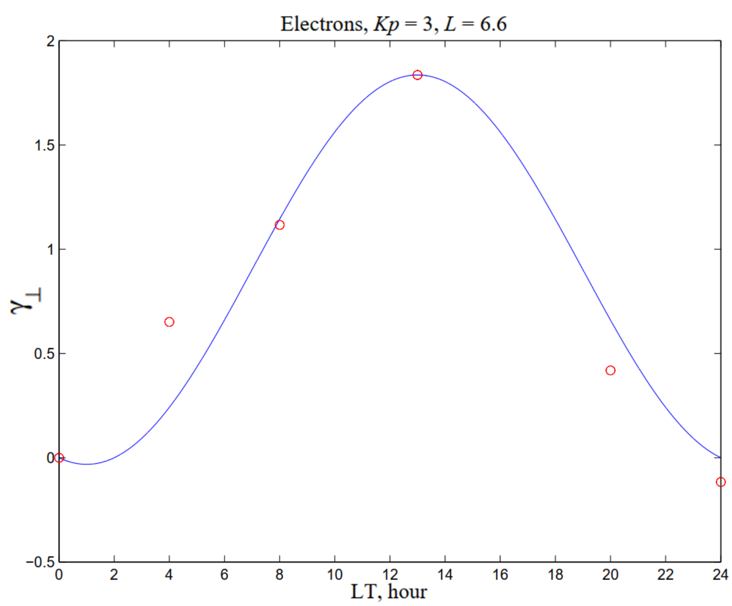

Fig. 2. The continuous line is modeling analytical dependence of perpendicular anisotropy index of electron pitch angle distribution on GEO $\gamma_\perp(t)$ (13) from the local time LT for the index of geomagnetic activity $Kp = 3$. Circles are designated the averaged experimental data [5]

as a first approximation, can be used for conditions of magnetically quiet time for quantitative forecasts and comparisons with experimental data on GEO.

Further, for $\dfrac{d\gamma_\perp}{dL} > 0$ and GEO we shall find a modeling (predicted) difference between the maximal value $\gamma_\perp(t_m)$ and the minimal value $\gamma_\perp(0) \equiv \gamma_{\perp 0}$ (at midnight) depending on Kp-index of geomagnetic activity, using for this purpose (18), (17), (10), (6). As a result the following simple equation turns out

$$\gamma_\perp(t_m) - \gamma_{\perp 0} = C[1 + \sin\varphi_m^+]. \tag{21}$$

The right part of the equation (21) should be more zero since on experimental data the difference between the maximal value $\gamma_\perp(t_m)$ and the minimal value $\gamma_{\perp 0}$ always is more than zero.

For an example we shall lead a modeling calculation for electrons on GEO. Thus, interesting nonlinear dependence (21), if approximately within one day $Kp =$ const or $Kp \approx$ const, which is presented on Fig. 3 turns out.

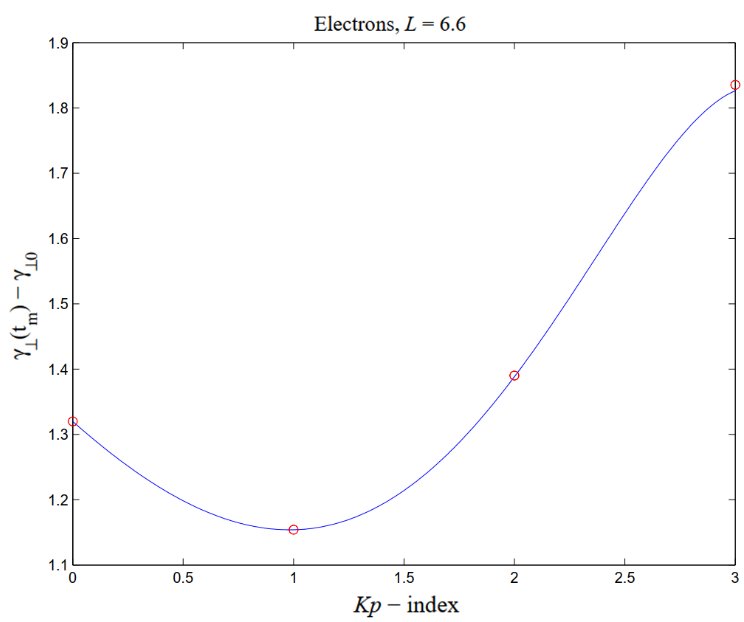

Fig. 3. A continuous line is modeling analytical dependence of a difference of perpendicular anisotropy indexes of electron pitch angle distribution on GEO $\gamma_\perp(t_m) - \gamma_{\perp 0}$ (21) from Kp-index of geomagnetic activity. Circles are designated the averaged experimental data [5]

Thus it is necessary to notice, that the right part of the equation (21), namely C (10), depends in this case as well from a gradient $\dfrac{d\gamma_\perp}{dL}(Kp)$. Therefore, to receive the best consent (21) with the averaged experimental data on GEO [5], we shall make the following. First, we shall believe dependence ϕ_2 from Kp-index of geomagnetic activity under the formula (6) [14] fair. Secondly, using the method of the least squares, we shall find dependence of a gradient $\dfrac{d\gamma_\perp}{dL}$ from Kp-index of geomagnetic activity for conditions of magnetically quiet time ($0 \leqslant Kp \leqslant 3$) in the following form

$$\frac{d\gamma_\perp}{dL}(L = 6.6, Kp) = 0.7234 - 0.5113Kp + 0.2068Kp^2 - 0.0305Kp^3. \tag{22}$$

From here follows, that the offered technique allows to predict (to forecast) very important dependence of a gradient $\dfrac{d\gamma_\perp}{dL}$ from Kp-index of geomagnetic activity. With dependence (22) very good consent (21) with the averaged experimental data [5] has turned out, that evidently proves to be true Fig. 3. Still it is necessary to add, that concrete dependence (22) on GEO is received (forecast) for the first time. The received dependence (22) also shows, that on GEO for a range ($0 \leqslant Kp \leqslant 3$) the gradient $\frac{d\gamma_\perp}{dL}(L = 6.6, Kp)$ is more than zero that corresponds to a prospective condition on a gradient prior to the beginning of calculations. And for other distances ($1 < L \leqslant 6.1$) dependence of a gradient $\frac{d\gamma_\perp}{dL}$ from Kp-index is visually presented, for example in [11].

Such theoretical prediction (21), in general, it is necessary to check in the further on corresponding experimental data. And still, in the equation (21) value C (10) is certain while as a first approximation, but in the future C can specify on experimental data, using the equation (21).

Thus, the received nonlinear dependence (21) can be considered as a theoretical prediction of nonlinear effect for a difference between the maximal value of a perpendicular anisotropy index of charged particle pitch angle distribution $\gamma_\perp(t_m)$ and the minimal value of a perpendicular anisotropy index $\gamma_\perp(0) \equiv \gamma_{\perp 0}$ (at midnight) on GEO ($L = 6.6$) from Kp-index of geomagnetic activity.

The presented nonlinear effect for anisotropy of charged particle pitch angle distribution (21) will be, possibly, to some extent and on other radial distances from the Earth, i.e. at other values of the McIlwain parameter L.

For some experimental data when a gradient $\frac{d\gamma_\perp}{dL} < 0$, the following (similar previous) formulas and the equations are received. In this case for $t_m = 13$ h the condition (16) is carried out, when $\Omega t_m + \varphi_m^- = 5\pi/2$. Whence follows, that

$$\varphi_m^- = \frac{\pi(30 - t_m)}{12}. \tag{23}$$

Under the formula (23) value $\varphi_m^- = 17\pi/12$ rad. For the moment of time $t = t_m$ following analytical dependence $\gamma_\perp(t_m)$ (12) turns out for perpendicular anisotropy index of charged particle pitch angle distribution on GEO, when $Kp \approx$ const, for example, within one day

$$\gamma_\perp(t_m) = C[\sin(\Omega t_m + \varphi_m^-) - \sin\varphi_m^-] + \gamma_{\perp 0} = C[1 - \sin\varphi_m^-] + \gamma_{\perp 0}. \tag{24}$$

The first composant in the right part of the equation (24) should be more zero since on experimental data the difference between the maximal value $\gamma_\perp(t_m)$ and the minimal value $\gamma_{\perp 0}$ always is more than zero.

For the intermediate moment of time $t = t_p$ it is received following dependence $\gamma_\perp(t_p)$

$$\gamma_\perp(t_p) = C[\sin(\Omega t_p + \varphi_m^-) - \sin\varphi_m^-] + \gamma_{\perp 0}. \tag{25}$$

Further we find the useful formula in the form of a difference of two equations (24) and (25)

$$\gamma_\perp(t_m) - \gamma_\perp(t_p) = C[1 - \sin(\Omega t_p + \varphi_m^-)]. \tag{26}$$

The formula (26) is useful for a finding of a constant C, when at $t = 0$ value $\gamma_{\perp 0}$ unknown, and other values in (26) known from experimental data. Then knowing C, the value $\gamma_{\perp 0}$ can be found from the equation (24). On the other hand at $t_p = 0$ the equation (26) passes in the equation (24) as it and should be. If value $\gamma_{\perp 0}$ known the constant C can be found at once from the equation (24).

Thus, determining φ_m^- (23), $\gamma_{\perp 0}$ and C, for $\frac{d\gamma_\perp}{dL} < 0$ final analytical dependence turns out (12) for perpendicular anisotropy index of charged particle pitch angle distribution on GEO, when $Kp \approx$ const, for example, within one day.

Formulas and the equations for gradients $\frac{d\gamma_\perp}{dL} < 0$ and $\frac{d\gamma_\perp}{dL} > 0$ are interconnected. In particular, the equation (23) can be presented so

$$\varphi_m^- = \frac{\pi(30 - t_m)}{12} = \frac{\pi(12 + 18 - t_m)}{12} = \pi + \frac{\pi(18 - t_m)}{12} = \pi + \varphi_m^+. \tag{27}$$

Then, using (27) and the reduction formulas in the trigonometry, it is possible to make following transitions of the equations: (24) in (18), (25) in (19), and (26) in (20) and thus to prove interrelation of the equations.

For an example when $\frac{d\gamma_\perp}{dL} < 0$, and $t_m = 12$ LT, in work [13] for the first time have been used special cases of the formulas and the equations (12), (23)–(26) for comparison to numerous experimental data, received with 1999 on 2007 on GEO. Comparison was made only at a qualitative physical level. As in [8] the pitch angle anisotropy (in the form of the relation of two average values) of the Earth's external radiation belt in the field of GEO in another way was quantitatively determined, but for very plenty of experimental data. In this work [13] it has been found, that at $t_m = 12$ LT the value $\varphi_m^- = 3\pi/2$ (23). Thus following final analytical dependence (the special case (12)) has been received for perpendicular anisotropy index of charged particle pitch angle distribution on GEO, when $Kp \approx \text{const}$, for example, within one day

$$\gamma_\perp(t) = C\left[\sin\left(\Omega t + \frac{3\pi}{2}\right) + 1\right] + \gamma_{\perp 0} \equiv 2C\sin^2\left(\frac{\Omega}{2}t\right) + \gamma_{\perp 0}. \tag{28}$$

And for a modeling (predicted) difference between the maximal value $\gamma_\perp(12)$ and the minimal value $\gamma_\perp(0) \equiv \gamma_{\perp 0}$ (at midnight) depending on Kp-index of geomagnetic activity more simple equation (the special case (24)) has been received at $\varphi_m^- = 3\pi/2$

$$\gamma_\perp(12) - \gamma_{\perp 0} = 2C. \tag{29}$$

To compare dependence from local time $\gamma_\perp(t)$ (28) with experimental data [8], test (modeling) calculations have been made [13], for example, for protons with energy $E = 120$ keV on GEO for $Kp = 3$- и $Kp = 5$.

In the same work [13] for the first time nonlinear effect (29) has been theoretically predicted for a difference between the maximal value of perpendicular anisotropy index of charged particle pitch angle distribution (in local midday LT = 12 h) and the minimal value of perpendicular anisotropy index (at midnight LT = 0 h) on GEO depending on Kp-index of geomagnetic activity.

On the whole, results of all calculations for $\frac{d\gamma_\perp}{dL} < 0$ are very in detail presented in [13].

3. Conclusion

1. The new phenomenological model of a prediction of perpendicular anisotropy index of charged particle pitch angle distribution at geostationary (geosynchronous) orbit (GEO) in the Earth's magnetosphere, and also in any circular orbit depending from the local time LT in an orbit and the geomagnetic activity index Kp is offered.

2. Comparison of model with the numerous experimental data is lead. It is proved, that the general analytical dependence of perpendicular anisotropy index of charged particle pitch angle distribution on GEO received as a first approximation can be used for conditions of magnetically quiet time for quantitative forecasts and comparisons with experimental data on GEO.

3. The nonlinear effect is theoretically predicted for a difference between the maximal value of perpendicular anisotropy index of charged particle pitch angle distribution and the minimal value of perpendicular anisotropy index (in local midnight LT = 0 h) on GEO from Kp-index of geomagnetic activity.

4. The technique is offered which allows to predict (to forecast) very important dependence of a gradient $\frac{d\gamma_\perp}{dL}$ from Kp-index of geomagnetic activity. For the first time concrete dependence of a gradient $\frac{d\gamma_\perp}{dL}$ from Kp-index on GEO for conditions of magnetically quiet time is received (forecast).

5. The nonlinear effect for anisotropy of charged particle pitch angle distribution will be, possibly, to some extent and on other radial distances from the Earth.

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Нелинейный эффект для анизотропии питч-углового распределения заряженных частиц на геостационарной орбите

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Аннотация. Предложена новая феноменологическая модель предсказания перпендикулярного индекса анизотропии питч-углового распределения заряженных частиц на геостационарной (геосинхронной) орбите (ГСО) в магнитосфере Земли, а также на любой круговой орбите в зависимости от местного времени LT на орбите и Kp–индекса геомагнитной активности. Проведено сравнение модели с многочисленными экспериментальными данными. Доказано, что общая аналитическая зависимость перпендикулярного индекса анизотропии питч-углового распределения заряженных частиц на ГСО, полученная в первом приближении, может быть использована для магнитоспокойных условий для количественных прогнозов и сравнений с экспериментальными данными на ГСО. Теоретически предсказан нелинейный эффект для разности между максимальным значением перпендикулярного индекса анизотропии питч-углового распределения заряженных частиц и минимальным значением перпендикулярного индекса анизотропии (в местную полночь LT = 0 ч) на ГСО от Kp–индекса геомагнитной активности. Нелинейный эффект для анизотропии питч-углового распределения заряженных частиц будет, вероятно, в той или иной степени и на других радиальных расстояниях от Земли.

Ключевые слова: геостационарная орбита, новая модель, динамика анизотропии заряженных частиц, данные спутника CRRES, нелинейный эффект.