EDN: JSPTAR
УДК 519.635.1, 519.635.4

On Calculation of Bending of a Thin Orthotropic Plate Using Legendre and Chebyshev Polynomials of the First Kind

Oksana V. Germider*
Vasily N. Popov†
Northern (Arctic) Federal University named after M. V. Lomonosov
Arkhangelsk, Russian Federation

Received 10.04.2024, received in revised form 05.05.2024, accepted 24.06.2024

Abstract. The problem of bending of a thin orthotropic rectangular plate clamped at the edges is considered in the paper. The solution is obtained using the Legendre and Chebyshev polynomials of the first kind. The function that approximates the solution of the biharmonic equation for an orthotropic plate is presented in the form of a double series expansion in these polynomials. Matrix transformations and properties of the Legendre and Chebyshev polynomials are also used. Roots of these polynomials are used as collocation points, and boundary value problem is reduced to a system of linear algebraic equations with respect to coefficients of the expansion. The problem of bending of a plate caused by the action of a distributed transverse load of constant intensity that corresponds to hydrostatic pressure is considered. This boundary value problem has analytical solution. The results of calculations for various ratios of the lengths of sides of the plate are presented. The values of deviation of solutions constructed using Legendre and Chebyshev polynomials from the analytical solution of the problem are presented in terms of the infinite norm and the finite norm in the space of square-integrable functions.

Keywords: bending a thin orthotropic plate, collocation method, Chebyshev polynomials of the first kind, Legendre polynomials.

Citation: O.V. Germider, V.N. Popov, On Calculation of Bending of a Thin Orthotropic Plate Using Legendre and Chebyshev Polynomials of the First Kind, J. Sib. Fed. Univ. Math. Phys., 2024, 17(5), 586–598. EDN: JSPTAR.

Study of bending of a thin rectangular plate is essential in modeling thin-walled spatial structures. Structures made of orthotropic materials unlike structures made of isotropic materials have high load-bearing capacity. Then one can reduce their weight with an increase in their strength. In this regard, the development of methods for modelling of such plates under the action of various types of loads is one of the main tasks of mechanics of thin-walled structures. Solution of the problem of bending the median plane of a square orthotropic plate pinched on all sides is constructed [1]. The method of initial functions using an exponential series with unknown coefficients was employed. Distributions of bending moments and shearing forces were found. The results of calculation of bending of a rectangular plate based on integral transformations under the action of constant intensity load, hydrostatic pressure and point load concentrated in the center of the plate were presented [2], [3]. Bending of the orthotropic plate under various boundary conditions was studied [4]. Numerical solution of the problem of bending of a rectangular plate consisting

*o.germider@narfu.ru https://orcid.org/0000-0002-2112-805X
†v.popov@narfu.ru https://orcid.org/0000-0003-0803-4419

of orthotropic layers arbitrarily oriented in the plane of the plate which are rigidly fixed to each other was obtained by the method of collocation and least residuals [5]. Equilibrium models of plates with rigid inclusions were considered [6]. A dynamic stiffness matrix was constructed for plane vibrations of a free orthotropic plate [7]. The results of analysis of frequencies of these vibrations were presented [8]. Deflections of a structural element representing a plate with a contour attachment the points of which are located on a rigid base when an acceleration pulse is transmitted in the direction perpendicular to the plane of the plate were calculated [9]. Study was conducted on bending of rectangular orthotropic thin plates with rotationally fixed edges under the action of arbitrary transverse loads [10]. The procedure for obtaining the stress distribution over the plate thickness for a strongly orthotropic material for three approximate models was described [11]. The first approximate model is the classical Kirchhoff-Love theory. The second model allows one to find transverse shear deformations and stresses. The third approximation is the Ambartsumian theory. It allows one to find transverse shear and normal stresses. In the presented work, to construct a solution of the problem of bending of a thin rectangular orthotropic plate with pinched edges systems of Legendre and Chebyshev polynomials of the first kind orthogonal on the segment $[-1, 1]$ are used. They play an important role both in the general theory of special functions and in the theory of orthogonal polynomials. Function that approximates the solution of the biharmonic equation for an orthotropic plate is represented as a double series expansion over these polynomials in combination with matrix transformations. In this case, the boundary value problem is written in dimensionless form. To find the coefficients in this decomposition approach proposed in [12] is used. It is based on the properties of Legendre and Chebyshev polynomials. The problem of bending of the plate due to the action of a distributed transverse load of constant intensity that corresponds to hydrostatic pressure is considered. This problem has analytical solution. The results of numerical solution of the problem are presented. Following [13], the obtained values of deviation of the constructed solutions using Legendre and Chebyshev polynomials from the analytical solution of the problem are given in terms of the infinite norm [14] and the finite norm in the space of functions integrable with the square [14, 15]. To discretize the integral norm the decomposition of the integrand function into the Chebyshev series is used. Coefficients of this decomposition can be found using values of this function calculated in the roots of Chebyshev polynomials. The importance of sampling by function values at points is emphasized in [16]. Verification of the obtained values of the integral norm was carried out using algorithm from [17] in the Maple computer algebra system.

1. Derivation of basic equations

Let us consider a thin orthotropic rectangular plate ($0 \leqslant x \leqslant d_1$, $0 \leqslant y \leqslant d_2$, $-h/2 \leqslant z \leqslant h/2$) which is under the action of a transverse load of intensity $q(x, y)$. Let us take the median plane of the undeformed plate for the xy plane, and z axis is directed towards the unloaded outer plane. Volumetric forces are neglected. In this case, the partial differential equation to determine bending of the plate has the form [18]:

$$D_x \frac{\partial^4 \omega}{\partial x^4} + 2H \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 \omega}{\partial y^4} = q, \tag{1}$$

where $\omega(x, y)$ is the bending of the median surface of the plate, $D_x = E'_x h^3/12$, $D_y = E'_y h^3/12$, $H = D_1 + 2D_{xy}$, $D_1 = E" h^3/12$, $D_{xy} = Gh^3/1$, G is the shear modulus, h is the thickness of the plate. Bending stiffnesses D_x and D_y are [19]

$$D_x = \frac{E_x h^3}{12(1-\nu_1\nu_2)}, \quad D_y = \frac{E_y h^3}{12(1-\nu_1\nu_2)}, \quad D_1 = \nu_1 D_y = \nu_2 D_x, \tag{2}$$

where E_x and E_y are Young's modules for the main directions of elasticity, ν_1, ν_2 are Poisson's coefficients.

For a plate clamped along the contour, i.e., for $x = 0$, d_1 and $y = 0$, d_2, boundary conditions have the form [18]

$$\omega = 0, \quad \frac{\partial \omega}{\partial x} = 0, \quad x = 0,\, d_1, \tag{3}$$

$$\omega = 0, \quad \frac{\partial \omega}{\partial y} = 0, \quad y = 0,\, d_2. \tag{4}$$

Let us rewrite equation (1) and boundary conditions (3) and (4) in new dimensionless variables $x^* = x/d_1$ and $y^* = y/d_1$:

$$\frac{\partial^4 \omega^*}{\partial x^{*4}} + \frac{2H}{D_x}\frac{\partial^4 \omega^*}{\partial x^{*2}\partial y^{*2}} + \frac{D_y}{D_x}\frac{\partial^4 \omega^*}{\partial y^{*4}} = q^*, \tag{5}$$

$$\omega^* = 0, \quad \frac{\partial \omega^*}{\partial x^*} = 0, \quad x^* = 0,\, 1, \tag{6}$$

$$\omega^* = 0, \quad \frac{\partial \omega^*}{\partial y^*} = 0, \quad y^* = 0,\, d_2^*; \quad d_2^* = \frac{d_2}{d_1}, \tag{7}$$

where $q = q_0 q^*$, $\omega = \dfrac{\omega^* q_0 d_1^4}{D_x}$, q_0 is the intensity of some constant load.

Let us construct a solution of boundary value problem (5)–(7) by the collocation method using Chebyshev polynomials of the first kind and the roots of these polynomials as collocation points.

2. Construction of a solution of boundary value problem using Chebyshev polynomials of the first kind

Let us present function ω^* as a double Chebyshev series. For this purpose, let $x_1 = 2x^* - 1$, $x_2 = 2y^*/d_2^* - 1$, where x_1, $x_2 \in [-1, 1]$ since Chebyshev polynomials of the first kind are defined on the segment $[-1, 1]$. In this case, problem (5)–(7) has the following form in variables x_1 and x_2

$$\kappa_1 \frac{\partial^4 \omega^*}{\partial x_1^4} + \kappa_2 \frac{\partial^4 \omega^*}{\partial x_1^2 \partial x_2^2} + \kappa_3 \frac{\partial^4 \omega^*}{\partial x_2^4} = q^*, \tag{8}$$

$$\omega^* = 0, \quad \kappa_4 \frac{\partial \omega^*}{\partial x_1} = 0, \quad x_1 = -1,\, 1, \tag{9}$$

$$\omega^* = 0, \quad \kappa_5 \frac{\partial \omega^*}{\partial x_2} = 0, \quad x_2 = -1,\, 1, \tag{10}$$

where $\kappa_1 = 16$, $\kappa_2 = \dfrac{32H}{D_x d_2^{*2}}$, $\kappa_3 = \dfrac{16 D_y}{D_x d_2^{*4}}$, $\kappa_4 = 2$, $\kappa_5 = \dfrac{2}{d_2^*}$.

Limiting the expansion of ω^* to the terms of the series with numbers $k_i \leqslant n_i$ for x_i $(i = 1, 2)$, one can write

$$\omega^*(x_1, x_2) = \sum_{\substack{k_i=0 \\ i=1,2}}^{n_i} a_{k_1 k_2} T_{k_1}(x_1) T_{k_2}(x_2) = \mathbf{T_1}(x_1) \otimes \mathbf{T_2}(x_2)\mathbf{A}, \tag{11}$$

where $\mathbf{T_i}(x_i) = (T_0(x_i)\, T_1(x_i) \ldots T_{n_i-1}(x_i)\, T_{n_i}(x_i))$ is a matrix of size $1 \times n'_i$ $(n'_i = n_i + 1,\ i = 1,\ 2)$, the elements of which are Chebyshev polynomials of the first kind $T_{j_i}(x_i) = \cos(j_i \arccos x_i)$ $(j_i = \overline{0, n_i},\ i = 1,\ 2)$ [20], $\mathbf{A}$ is the matrix with size $n'_1 n'_2 \times 1$ with elements $a_{k_1 k_2}$: $\mathbf{A} = (a_{00}\, a_{01} \ldots a_{n_1 n_2 - 1}\, a_{n_1 n_2})^T$. The sign $\otimes$ in (11) is used to denote the Kronecker tensor product of two matrices [21]. The elements of the matrix are found by collocation. Let us choose the roots of polynomials T_{n_1+1} and T_{n_2+1} as collocation points in (8) for x_1 и x_2:

$$x_{i,k_i} = \cos\left(\frac{\pi(2n_i - 2k_i + 1)}{2(n_i + 1)}\right), \quad k_i = \overline{0, n_i},\ i = 1, 2. \tag{12}$$

Then

$$T_{j_i}(x_{i,k_i}) = \cos\left(\frac{\pi j_i(2n_i - 2k_i + 1)}{2(n_i + 1)}\right), \quad j_i, k_i = \overline{0, n_i},\ i = 1, 2. \tag{13}$$

Moreover, if n_i is odd then $x_{i,m_i} = -x_{i,n_i - m_i}$ and $T_{j_i}(x_{i,m_i}) = (-1)^{j_i} T_{j_i}(x_{i,n_i-m_i})$, $(m_i = \overline{0,\ (n_i-1)/2};\ j_i = \overline{0,\ n_i};\ i = 1, 2)$. If n_i is even then $x_{i,n_i/2} = 0$, $x_{i,m_i} = -x_{i,n_i-m_i}$ and $T_{j_i}(x_{i,m_i}) = (-1)^{j_i} T_{j_i}(x_{i,n_i-m_i})$, $(m_i = \overline{0,\ n_i/2 - 1};\ j_i = \overline{0,\ n_i};\ i = 1, 2)$. The value of $T_{j_i}(0)$ is found using the following representation [20]

$$T_{j_i}(x_i) = \sum_{k=0}^{[j_i/2]} \varsigma_k x_i^{j_i - 2k}, \quad \varsigma_k = \frac{(-1)^k 2^{j_i - 2k - 1} j_i (j_i - k - 1)!}{(j_i - 2k)!k!},$$

where $[j_i/2]$ is the integer part of the number $j_i/2$. If j_i is even then $T_{j_i}(0) = \varsigma_{j_i/2} = (-1)^{j_i/2}$, otherwise $T_{j_i}(0) = 0$ $(i = 1, 2)$.

The derivative of $\mathbf{T_i}(x_i)$ with respect to x_i is represented as a product of $\mathbf{T_i J_i}$ as follows [22]

$$\frac{\mathrm{d}T_{j_i}}{\mathrm{d}x_i} = j_i \sum_{\substack{k_i=0 \\ j_i + k_i - \text{nech.}}}^{j_i - 1} c_{k_i} T_{k_i}(x_i), \quad j_i \geqslant 1,$$

where $c_0 = 1$ and $c_{k_i} = 2$ $(k_i > 0)$, and $\mathbf{J_i}$ is an upper-triangular matrix with nonzero elements $J_{i,0\,j_i} = j_i$ (j_i is odd, $j_i = \overline{1,\ n_i}$) and $J_{i,k_i\,j_i} = 2j_i$ ($j_i - k_i > 0$ and $j_i + k_i$ – odd, $j_i,\ k_i = \overline{1,\ n_i}$, $i = 1, 2$). Here and below, numbering of rows and columns in matrices is started from scratch.

For the second and fourth derivatives of $\mathbf{T_i}(x_i)$ with respect to x_i one can write

$$\frac{\mathrm{d}^j \mathbf{T_i}}{\mathrm{d}x_i^j} = \mathbf{T_i} \mathbf{J_i}^j, \quad j = 2, 4;\ i = 1, 2. \tag{14}$$

Substituting collocation points (12) into equation (8), a system of linear algebraic equations is obtained. Then equations at points $x_i = x_{i,0}$ and $x_i = x_{i,n_i}$ are excluded, and equations corresponding to boundary conditions $\omega^*(\pm 1, x_{2,k_2}) = 0$ and $\omega^*(x_{1,k_1}, \pm 1) = 0$ are introduced:

$$\mathbf{T_1}(-1) \otimes \mathbf{T_2}(x_{2,k_2})\mathbf{A} = 0, \quad \mathbf{T_1}(1) \otimes \mathbf{T_2}(x_{2,k_2})\mathbf{A} = 0, \quad k_2 = \overline{0,\ n_2}, \tag{15}$$

$$\mathbf{T_1}(x_{1,k_1}) \otimes \mathbf{T_2}(-1)\mathbf{A} = 0, \quad \mathbf{T_1}(x_{1,k_1}) \otimes \mathbf{T_2}(1)\mathbf{A} = 0, \quad k_1 = \overline{1,\ n_1 - 1}. \tag{16}$$

At points $x_i = x_{i,1}$ and $x_i = x_{i,n_i-1}$ equations satisfying conditions $\left.\dfrac{\partial \omega^*}{\partial x_i}\right|_{x_i=\pm 1} = 0$ $(i = 1,\, 2)$ are written

$$\mathbf{T_1}(-1)\mathbf{J_1} \otimes \mathbf{T_2}(x_{2,k_2})\mathbf{A} = 0, \quad \mathbf{T_1}(1)\mathbf{J_1} \otimes \mathbf{T_2}(x_{2,k_2})\mathbf{A} = 0, \quad k_2 = \overline{0,\, n_2}, \tag{17}$$

$$\mathbf{T_1}(x_{1,k_1}) \otimes (\mathbf{T_2}(-1)\mathbf{J_2})\,\mathbf{A} = 0, \quad \mathbf{T_1}(x_{1,k_1}) \otimes (\mathbf{T_2}(1)\mathbf{J_2})\,\mathbf{A} = 0, \quad k_1 = \overline{1,\, n_1 - 1}. \tag{18}$$

As a result, using (11), (14)-(18), one can obtain

$$\mathbf{BA} = \mathbf{F}, \quad \mathbf{B} = \sum_{m=1}^{5} \mathbf{B_m}, \tag{19}$$

where $\mathbf{F} = (f_{00}\, f_{01}\, \ldots\, f_{n_1\, n_2})^T$ with elements $f_{k_1\, k_2} = q^*(x_{1,k_1}, x_{2,k_2})$, $(k_i = \overline{2,\, n_i - 2},\, i = 1, 2)$, square matrices $\mathbf{B_m}$ $(m = \overline{1,5})$ of size $n_1' n_2' \times n_1' n_2'$ defined as

$$\mathbf{B_1} = \kappa_1 \mathbf{G_1''}\mathbf{J_1}^4 \otimes \mathbf{G_2''}, \quad \mathbf{B_2} = \kappa_2 \mathbf{G_1''}\mathbf{J_1}^2 \otimes \left(\mathbf{G_2''}\mathbf{J_2}^2\right), \quad \mathbf{B_3} = \kappa_3 \mathbf{G_1''} \otimes \left(\mathbf{G_2''}\mathbf{J_2}^4\right)$$

$$\mathbf{B_4} = \mathbf{G_3} \otimes \mathbf{G_2} + \mathbf{G_1''} \otimes \mathbf{G_4}, \quad \mathbf{B_5} = \kappa_4 \mathbf{G_5}\mathbf{J_1} \otimes \mathbf{G_2} + \kappa_5 \mathbf{G_1''} \otimes (\mathbf{G_6}\mathbf{J_2}).$$

Here $\mathbf{G_i}$, $\mathbf{G"_i}$, $\mathbf{G_{3+i}}$ and $\mathbf{G_{4+i}}$ are square matrices with sizes $n_i' \times n_i'$ $(i = 1, 2)$:

$$\mathbf{G_i} = \begin{bmatrix} \mathbf{T_i}(x_{i,0}) \\ \mathbf{T_i}(x_{i,1}) \\ \mathbf{T_i}(x_{i,2}) \\ \ldots \\ \mathbf{T_i}(x_{i,n_i-2}) \\ \mathbf{T_i}(x_{i,n_i-1}) \\ \mathbf{T_i}(x_{i,n_i}) \end{bmatrix}, \quad \mathbf{G_i''} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{T_i}(x_{i,2}) \\ \ldots \\ \mathbf{T_i}(x_{i,n_i-2}) \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{G_{2+i}} = \begin{bmatrix} \mathbf{T_i}(-1) \\ \mathbf{0} \\ \mathbf{0} \\ \ldots \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{T_i}(1) \end{bmatrix}, \quad \mathbf{G_{4+i}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{T_i}(-1) \\ \mathbf{0} \\ \ldots \\ \mathbf{0} \\ \mathbf{T_i}(1) \\ \mathbf{0} \end{bmatrix}.$$

To find values $\mathbf{T_i}(-1)$ and $\mathbf{T_i}(1)$ relations $T_{i,j_i}(-1) = (-1)^{j_i}$, $T_{i,j_i}(1) = 1$, $(j_i = \overline{0,\, n_i},\, i = 1, 2)$ are used citebibGer3.

The elements of matrix $\mathbf{A}$ are obtained from equation (19). Function ω^* is restored the using (11).

3. Constructing a solution of boundary value problem using Legendre polynomials

Let us represent function ω^* as a finite sum of a double Legendre series:

$$\omega^*(x_1, x_2) = \sum_{\substack{k_i=0 \\ i=1,2}}^{n_i} a_{k_1 k_2} P_{k_1}(x_1) P_{k_2}(x_2) = \mathbf{P_1}(x_1) \otimes \mathbf{P_2}(x_2)\mathbf{A}, \tag{20}$$

where $\mathbf{P_i}(x_i) = (P_0(x_i)\, P_1(x_i) \ldots P_{n_i-1}(x_i)\, P_{n_i}(x_i))$ $(i = 1, 2)$, and Legendre polynomials $P_{j_i}(x_i)$ are defined as follows

$$P_0(x_i) = 1, \quad P_1(x_i) = x_i, \quad (j_i + 1)P_{j_i+1}(x_i) = (2j_i + 1)x_i P_{j_i}(x_i) - j_i P_{j_i-1}(x_i), \quad j \geqslant 1.$$

As collocation points x_{i,k_i} for x_i in equation (8) the roots of polynomial P_{n_i+1} $(i = 1, 2)$ are used. According to [22], these roots x_{i,k_i} are eigenvalues of a symmetric matrix $\mathbf{L_i}$ of size

$n_i' \times n_i'$ with nonzero elements $L_{i,k_i+1\,k_i} = L_{i,k_i\,k_i+1} = (k_i+1)/\sqrt{4(k_i+1)^2-1}$ $(k_i = \overline{0,\,n_i-1}$, $i = 1,2)$. Moreover, if n_i is odd then $x_{i,m_i} = -x_{i,n_i-m_i}$ and $P_{j_i}(x_{i,m_i}) = (-1)^{j_i} P_{j_i}(x_{i,n_i-m_i})$, $(m_i = \overline{0,\,(n_i-1)/2};\ j_i = \overline{0,\,n_i};\ i = 1,2)$. If n_i is even then $x_{i,n_i/2} = 0$, $x_{i,m_i} = -x_{i,n_i-m_i}$ and $P_{j_i}(x_{i,m_i}) = (-1)^{j_i} P_{j_i}(x_{i,n_i-m_i})$, $(m_i = \overline{0,\,n_i/2-1};\ j_i = \overline{0,\,n_i};\ i = 1,2)$. The value $P_{j_i}(0)$ is found using the following representation [22]

$$P_{j_i}(x_i) = \sum_{k=0}^{[j_i/2]} \varsigma_k x_i^{j_i-2k}, \quad \varsigma_k = \frac{(-1)^k (2j_i-2k)!}{2^{j_i}(j_i-2k)!(j_i-k)!k!}.$$

Thus, if j_i is even then

$$P_{j_i}(0) = \varsigma_{j_i/2} = \frac{(-1)^{j_i/2} j_i!}{2^{j_i}\left(\dfrac{j_i}{2}\right)!^2},$$

otherwise, $P_{j_i}(0) = 0$ $(i = 1,2)$.

The derivative of $\mathbf{P_i}(x_i)$ with respect to x_i is represented as a product of $\mathbf{P_i J_i}$ using [22]

$$\frac{\mathrm{d}P_{j_i}}{\mathrm{d}x_i} = \sum_{\substack{k_i=0 \\ j_i+k_i-\text{nech.}}}^{j_i-1} (2k_i+1) P_{k_i}(x_i), \quad j_i \geqslant 1,$$

where $\mathbf{J_i}$ is an upper-triangular matrix of size $n_i' \times n_i'$ with nonzero elements $J_{i,k_i\,j_i} = 2k_i + 1$ $(j_i - k_i > 0$ and $j_i + k_i$ – odd, $j_i,\ k_i = \overline{0,\,n_i}$, $i = 1,2$ For the second and fourth derivatives of $\mathbf{P_i}(x_i)$ with respect to x_i one can write

$$\frac{\mathrm{d}^{\mathrm{j}}\mathbf{P_i}}{\mathrm{d}x_i^j} = \mathbf{P_i J_i}^j, \quad j = 2,4;\ i = 1,2. \tag{21}$$

Using the selected collocation points for equation (8), equalities (20) and (21)and taking into account boundary conditions (9) and (10), system of equations (19) is obtained, where matrices $\mathbf{G_i}$, $\mathbf{G"_i}$, $\mathbf{G_{3+i}}$, $\mathbf{G_{4+i}}$ and $\mathbf{G"_i}$ are defined by $\mathbf{P_i}$ $(i = 1,2)$. In this case, the values $\mathbf{P_i}(-1)$ and $\mathbf{P_i}(1)$ are found using as follows $P_{i,j_i}(-1) = (-1)^{j_i}$, $P_{i,j_i}(1) = 1$, $(j_i = \overline{0,\,n_i}$, $i = 1,2)$. Restoring elements of matrix $\mathbf{A}$ from (19), one can obtain function $(\omega^*(x_1,x_2)$ from (20).

4. Numerical results and their analysis

As an example, let us consider the problem of bending of a rectangular orthotropic plate under the action of a transverse load which is defined as

$$q^*(x^*,y^*) = \cos(\pi(2x^*-1))\left(1+\cos\left(\pi\left(\frac{2y^*}{d_2^*}-1\right)\right)\right) + \\ + \cos\left(\pi\left(\frac{2y^*}{d_2^*}-1\right)\right)\left(\frac{2H}{D_x d_2^{*2}}\cos(\pi(2x^*-1)) + \frac{\nu_2}{\nu_1 d_2^{*4}}(1+\cos(\pi(2x^*-1)))\right). \tag{22}$$

In this case, the analytical solution of boundary value problem (5)–(7) has the form

$$\omega_a^*(x^*,y^*) = \frac{1}{16\pi^4}(1+\cos(\pi(2x^*-1)))\left(1+\cos\left(\pi\left(\frac{2y^*}{d_2^*}-1\right)\right)\right). \tag{23}$$

The values of the physical parameters from [1] and [18] are used in calculations: $E_x' = 131 \cdot 10^7$ kg/m^2, $E_y' = 42 \cdot 10^7$ kg/m^2, $E' = 5.1 \cdot 10^7$ kg/m^2, $G = 11.1 \cdot 10^7$ kg/m^2. Deviations of constructed solutions (11) and (20) from analytical solution (23) are found by the infinite norm [14]:

$$\|\omega^* - \omega_a^*\|_\infty = \max_{(x^*,y^*)\in\Omega} |\omega^*(x^*,y^*) - \omega_a^*(x^*,y^*)|, \tag{24}$$

where $\Omega = [0,\,1]\times[0,\,d_2^*]$, and the finite norm in the space of square integrable functions [14] and [15]:

$$\|\omega^* - \omega_a^*\|_2 = \left(\int_0^1 \int_0^{d_2^*} (\omega^*(x^*,y^*) - \omega_a^*(x^*,y^*))^2 \mathrm{d}x^* \mathrm{d}y^*\right)^{1/2}. \tag{25}$$

Evaluation of expression (24) is carried out in term of the infinite norm of the difference between vectors $\mathbf{W}$ and $\mathbf{W_a}$ with elements equal to the values of functions ω^* and ω_a^* at uniformly distributed points $(x_{k_1}^*, y_{k_2}^*)$ from Ω domain:

$$e_\infty = \|\mathbf{W} - \mathbf{W_a}\|_\infty = \max_{\substack{0\leqslant k_i\leqslant m_i\\ i=1,2}} |\omega^*(x_{k_1}^*, y_{k_2}^*) - \omega_a^*(x_{k_1}^*, y_{k_2}^*)|,$$

where $\mathbf{W} = (w_{00}\, w_{01}\, \ldots\, w_{m_1\, m_2})^T$, $w_{k_1k_2} = \omega^*(x_{k_1}^*, y_{k_2}^*)$, $\mathbf{W_a} = (w_{a,00}\, w_{a,01}\, \ldots\, w_{a,m_1\, m_2})^T$ and $w_{a,k_1k_2} = \omega_a^*(x_{k_1}^*, y_{k_2}^*)$ $(k_i = \overline{0, m_i},\ i = 1, 2)$. The obtained values of the deviation estimate for the infinite norm e_∞ are presented in the Tab. 1 for $n_1 = n_2 = n$ and $m_1 = m_2 = 100$ for $d_2^* = \dfrac{d_2}{d_1}$ from [2,3] and [23]. The notation $e_{T,\infty}$ is used in the case of Chebyshev polynomials, and notation $e_{P,\infty}$ is used for Legendre polynomials. The degree of 10 is indicated in parentheses. For values d_2^* shown in Tab. 1 the maximum deflection value in the center of the plate is $0.002566496\ 10^{-9}$. The third and sixth columns of this table present estimates of the deviations of solutions (11) and (20) between successive iterations of $n-1$ and n according to the infinite norm

$$e_{n,\infty} = \max_{\substack{0\leqslant k_i\leqslant m_i\\ i=1,2}} |\omega_n^*(x_{k_1}^*, y_{k_2}^*) - \omega_{n-1}^*(x_{k_1}^*, y_{k_2}^*)|,$$

where $m_1 = m_2 =$100. The fourth column of Tab. 1 contains the values of the infinite norm $\tilde{e}_{T,\infty}$ of the difference between $\mathbf{W_a}$ and the vector with elements obtained as a result of interpolation of function (23) by Chebyshev polynomials. The corresponding values of the norm $\tilde{e}_{P,\infty}$ in the case of Legendre polynomials are given in the seventh column of this table. It can be seen from the results presented in Tab. 1 that solutions obtained using Legendre and Chebyshev polynomials of the first kind coincide with the analytical solution with high accuracy (23) for relatively small values of n. The obtained values of deviation for the infinite norm $e_{T,\infty}$ and $e_{P,\infty}$ of these solutions approach the corresponding values of deviation norms $\tilde{e}_{T,\infty}$ and $\tilde{e}_{P,\infty}$ for polynomial interpolations of function (23). It indicates good approximation properties of the method. The values of $e_{T,n,\infty}$ and $e_{P,n,\infty}$ can be used as an estimate of the error of the constructed solutions.

To discretize norm (25), integrand function $(\omega^*(x^*,y^*) - \omega_a^*(x^*,y^*))^2$ is represented the in the form of a partial sum of a double series according to Chebyshev polynomials

$$(\omega^*(x_1,x_2) - \omega_a^*(x_1,x_2))^2 = \sum_{\substack{k_i=0\\ i=1,2}}^{q_i} a_{q,k_1k_2} T_{k_1}(x_1) T_{k_2}(x_2) = \mathbf{T_{1,q}}(x_1) \otimes \mathbf{T_{2,q}}(x_2) \mathbf{A_q}, \tag{26}$$

where $\mathbf{T_{q,i}}(x_i) = (T_0(x_i)\, T_1(x_i) \ldots T_{q_i-1}(x_i)\, T_{q_i}(x_i))$. Matrix elements $\mathbf{A_q} = (a_{q,00}\, a_{q,01} \ldots a_{q,q_1q_2-1}\, a_{q_1q_2})^T$ are determined using roots x_{i,k_i} of polynomials T_{q_i+1} $(i = 1, 2)$:

$$\mathbf{A} = \mathbf{G_{1,q}}^{-1} \otimes \mathbf{G_{2,q}}^{-1} \mathbf{S_q}, \tag{27}$$

Table 1. Values of deviations in term of the infinite norm e_∞, $e_{n,\infty}$ and $\tilde{e}_\infty$ versus n for $q^*(x^*, y^*)$ given in (22)

n	$e_{T,\infty}$	$e_{T,n,\infty}$	$\tilde{e}_{T,\infty}$	$e_{P,\infty}$	$e_{P,n,\infty}$	$\tilde{e}_{P,\infty}$
$d_2^* = 0.5$						
9	5.5(–6)	7.2(-6)	1.1(–7)	8.1(–6)	9.1(–6)	1.4(–7)
12	7.9(–9)	1.8(–8)	2.7(–11)	1.3(–8)	2.9(–7)	8.9(–11)
15	5.2(–11)	6.2(–11)	3.0(–13)	9.5(–11)	1.1(–10)	5.1(–13)
18	1.0(–14)	5.7(–13)	2.7(–17)	2.0(–14)	1.1(–12)	6.1(–17)
$d_2^* = 1.0$						
9	5.4(–6)	7.1(–6)	1.1(–7)	7.9(–6)	8.9(–6)	1.4(–7)
12	7.8(–9)	1.8(–8)	2.7(–11)	1.3(–8)	2.9(–7)	8.9(–11)
15	5.1(–11)	6.1(–11)	3.0(–13)	9.3(–11)	1.0(–10)	5.1(–13)
18	1.0(–14)	5.6(–13)	2.7(–17)	2.0(–14)	1.1(–12)	6.1(–17)
$d_2^* = 1.5$						
9	5.8(–6)	7.7(–6)	1.1(–7)	8.6(–6)	9.6(–6)	1.4(–7)
12	8.4(–9)	1.9(–7)	2.7(–11)	1.4(-8)	3.1(-7)	8.9(-11)
15	5.5(–11)	6.5(–11)	3.0(–13)	1.0(–10)	1.1(–10)	5.1(–13)
18	1.0(–14)	6.0(–13)	3.1(–17)	2.2(–14)	1.2(–12)	6.1(–17)

where $\mathbf{S_q} = (s_{00}\, s_{01}\, \ldots\, s_{q_1\, q_2})^T$ with elements: $s_{k_1\, k_2} = (\omega^*(x_{1,k_1}, x_{2,k_2}) - \omega_a^*(x_{1,k_1}, x_{2,k_2}))^2$, ($k_i = \overline{0,\, q_i}$, $i = 1, 2$), square matrix $\mathbf{G_{i,q}}$ has size $(q_i + 1) \times (q_i + 1)$ and it is defined similarly to $\mathbf{G_i}$ ($i = 1, 2$). The inverse to $\mathbf{G_{i,q}}$ matrix $\mathbf{G_{i,q}}^{-1}$ is obtained by transposing $\mathbf{G_{i,q}}$ then multiplying $\mathbf{G_{i,q}}^T$ by $2/(q_i + 1)$ and dividing elements of the first row of this matrix by 2 ($i = 1, 2$). It follows from the equality [20]

$$\frac{2}{q_i + 1} \sum_{k_i=0}^{q_i} T_{j_1}(x_{i,k_i}) T_{j_2}(x_{i,k_i}) = \gamma_{T,j_1} \delta_{j_1,j_2},$$

where δ_{j_1,j_2} is the Kronecker symbol, $\gamma_{T,0} = 2$, $\gamma_{T,j_1} = 1$ ($j_1 > 0$, $i = 1, 2$).

Using representation (26), one can obtain for the double integral in (25)

$$e_2^2 = \int_0^1 \int_0^{d_2^*} (\omega^*(x^*, y^*) - \omega_a^*(x^*, y^*))^2 \mathrm{d}x^* \mathrm{d}y^* = \frac{d_2^*}{4} \int_{-1}^1 \int_{-1}^1 (\omega^*(x_1, x_2) - \omega_a^*(x_1, x_2))^2 \mathrm{d}x_1 \mathrm{d}x_2 =$$
$$= \frac{d_2^*}{4} \int_{-1}^1 \mathbf{T_{1,q}}(x_1) \mathrm{d}x_1 \otimes \int_{-1}^1 \mathbf{T_{2,q}}(x_2) \mathrm{d}x_2 \mathbf{A_q}. \quad (28)$$

According to [20], there is the following relation for $j_i = 0$ and even j_i

$$\int_{-1}^1 T_{j_i}(x_i) \mathrm{d}x_i = \frac{2}{1 - j_i^2}, \quad j_i \geqslant 0,\ i = 1, 2,$$

otherwise, the value of the integral is zero.

Then

$$\int_{-1}^1 \mathbf{T_{i,q}}(x_i) \mathrm{d}x_i = \mathbf{R_i}, \quad (29)$$

where $\mathbf{R_i}$ is a matrix of size $1 \times (q_i + 1)$ with elements $R_{i,0\, j_i} = 2/(1 - j_i^2)$ (j_i — even, $j_i = \overline{0,\, q_i}$, $i = 1, 2$).

Substituting (27) and (29) into (28), one can obtain

$$e_2^2 = \frac{d_2^*}{4}\mathbf{R_1} \otimes \mathbf{R_i}\left(\mathbf{G_{1,q}}^{-1} \otimes \mathbf{G_{2,q}}^{-1}\mathbf{S_q}\right). \tag{30}$$

The Tab. 2 shows the values of deviations $e_{T,2}$ and $e_{P,2}$ of constructed solutions (11) and (20) from the analytical solution of problem (23) in term of the norm (25) based on (30) at $q_1 = q_2 = 10$. The degree of 10 is indicated in parentheses.

Table 2. Values of deviations in term of the integral norm e_2, $e_{n,2}$ and $\tilde{e}_2$ versus n for $q^*(x^*, y^*)$ given in (22)

n	$e_{T,2}$	$e_{T,n,2}$	$\tilde{e}_{T,2}$	$e_{P,2}$	$e_{P,n,2}$	$\tilde{e}_{P,2}$
$d_2^* = 0.5$						
9	2.0(–6)	2.5(–6)	2.6(–8)	3.1(–6)	2.8(–6)	2.5(–8)
12	2.9(–9)	6.6(–8)	8.2(–12)	4.7(–9)	1.0(–7)	6.3(–12)
15	1.9(–11)	2.3(–11)	6.4(–14)	3.4(–11)	3.8(–11)	5.4(–14)
18	3.8(–15)	2.1(-13)	7.6(-18)	7.5(–15)	4.1(–13)	8.2(–18)
$d_2^* = 1.0$						
9	2.8(–6)	3.5(–6)	3.7(–8)	4.1(–6)	4.4(–6)	3.6(–8)
12	4.1(–9)	9.2(–8)	1.1(–11)	6.6(–9)	1.5(–7)	8.9(–12)
15	2.7(–11)	6.2(–11)	9.0(–14)	4.9(–11)	5.4(–11)	7.6(–14)
18	5.3(–15)	3.0(–13)	1.1(–17)	1.1(–14)	5.8(–13)	1.1(–17)
$d_2^* = 1.5$						
9	3.5(–6)	4.4(–6)	4.5(–8)	5.1(–6)	5.5(–6)	4.4(–8)
12	5.0(–9)	1.1(–7)	1.4(–11)	8.2(–9)	1.8(–7)	1.1(–11)
15	3.3(–11)	4.0(–11)	1.1(–13)	6.1(–11)	6.7(–11)	9.3(–14)
18	6.7(–15)	3.7(–13)	1.5(–17)	1.3(–14)	7.1(–13)	1.5(–17)

Verification of the obtained values of $e_{T,2}$ and $e_{P,2}$ was carried out using algorithm from [17] in the Maple computer algebra system. The third and sixth columns of Tab. 2 show the values of deviations of obtained solutions (11) and (20) between successive iterations of $n-1$ and n in term of the integral norm calculated similarly to (30). The fourth column of this table shows the values of norm $\tilde{e}_{T,2}$ in the case of interpolation of function (23) by Chebyshev polynomials. The corresponding values of norm $\tilde{e}_{P,2}$ when using Legendre polynomials are given in the seventh column of this table.

Tables 3 and 4 show the values of $\omega^*(x^*, y^*)$ in the center of the plate, as well as the norms $e_{n,\infty}$ and $e_{n,2}$ versus n for $q^*(x^*, y^*) = 1$ and $q^*(x^*, y^*) = x^*$, respectively. For the square orthotropic plate ($d_2^* = 1$) under the action of a load with dimensionless intensity $q^*(x^*, y^*) = 1$, comparison with the results obtained in [1] is presented. The maximum bending value in the center of the plate $\omega^*(x^*, y^*)$ is equal to 0.00225757 for $n = 12$, and the value of 0.002257679 is reached for $n = 44$, where n is the number of members of the exponential series [1]. The results presented in Tables 1–4 show that solutions obtained using Legendre and Chebyshev polynomials of the first kind have sufficiently fast convergence, and the obtained norm estimates can be used as an estimate of the error of the constructed solutions in the corresponding function spaces.

Conclusion

Solution of the bending problem of a thin orthotropic rectangular plate clamped along the contour is constructed using the collocation method in the matrix implementation. Chebyshev

Table 3. Values of $\omega^*(x^*, y^*)$ in the center of the plate and the norms $e_{n,\infty}$ and $e_{n,2}$ versus n for $q^*(x^*, y^*) = 1$

n	$\omega^*_T\left(\frac{1}{2}, \frac{d^*_2}{2}\right)$	$e_{T,n,\infty}$	$\omega^*_P\left(\frac{1}{2}, \frac{d^*_2}{2}\right)$	$e_{P,n,\infty}$	$e_{T,n,2}$	$e_{P,n,2}$
			$d^*_2 = 0.5$			
9	0.000485168	6.4(−7)	0.000485279	7.2(−7)	1.9(−7)	2.2(−7)
12	0.000484932	4.2(−8)	0.000484931	5.5(−8)	1.1(−8)	1.5(−8)
15	0.000484933	2.1(−9)	0.000484933	2.7(−9)	2.7(−10)	3.6(−10)
18	0.000484933	8.2(-10)	0.000484933	1.1(−9)	6.6(−11)	1.1(−10)
			$d^*_2 = 1$			
9	0.002258721	1.8(−6)	0.002259184	2.1(−6)	8.5(−7)	9.9(−7)
12	0.002257672	1.5(−7)	0.002257670	2.1(−7)	4.7(−8)	6.6(−8)
15	0.002257679	6.7(−9)	0.002257678	8.2(−9)	1.1(−9)	1.4(−9)
18	0.002257679	2.7(−9)	0.002257679	3.5(−9)	2.8(−10)	4.0(−10)
			$d^*_2 = 1.5$			
9	0.002710060	7.4(−6)	0.002710432	7.5(−6)	3.0(−6)	3.1(−6)
12	0.002709799	1.2(−6)	0.002709800	1.9(−6)	5.4(−7)	8.1(−7)
15	0.002709801	4.6(−8)	0.002709801	6.3(−8)	1.3(−8)	1.8(−8)
18	0.002709802	1.3(−8)	0.002709803	1.9(−8)	2.7(−9)	4.3(−9)

Table 4. Values of $\omega^*(x^*, y^*)$ in the center of the plate and the norms $e_{n,\infty}$ and $e_{n,2}$ versus n for $q^*(x^*, y^*) = x^*$

n	$\omega^*_T\left(\frac{1}{2}, \frac{d^*_2}{2}\right)$	$e_{T,n,\infty}$	$\omega^*_P\left(\frac{1}{2}, \frac{d^*_2}{2}\right)$	$e_{P,n,\infty}$	$e_{T,n,2}$	$e_{P,n,2}$
			$d^*_2 = 0.5$			
9	0.000242584	6.0(−7)	0.000242640	6.6(−7)	1.1(−7)	1.3(−7)
12	0.000242466	3.4(−8)	0.000242466	4.3(−8)	5.7(−9)	8.1(−9)
15	0.000242466	3.6(−9)	0.000242466	5.1(−9)	4.5(-10)	6.8(−10)
18	0.000242466	6.1(-10)	0.000242466	8.5(−10)	4.2(−11)	6.6(−11)
			$d^*_2 = 1$			
9	0.001129361	9.1(-7)	0.001129592	1.1(−6)	4.4(−7)	5.1(−7)
12	0.001128836	1.5(−7)	0.001128835	2.1(−7)	3.6(−8)	5.2(−8)
15	0.001128839	8.9(−9)	0.001128839	1.2(−8)	1.2(-9)	1.7(-9)
18	0.001128840	2.2(−9)	0.001128840	3.2(−9)	2.9(−10)	4.7(−10)
			$d^*_2 = 1.5$			
9	0.001355030	4.1(−6)	0.001355215	4.2(−6)	1.7(−6)	1.7(−6)
12	0.001354899	7.8(−7)	0.001354900	1.1(−6)	2.9(−7)	4.2(−7)
15	0.001354901	5.0(−8)	0.001354900	6.6(−8)	9.3(−9)	1.3(−8)
18	0.001354901	1.5(−8)	0.001354901	2.3(−8)	2.5(−9)	4.1(−9)

polynomials of the first kind and Legendre polynomials are used as the basic system of functions. The results of modeling the bending of the median plane of the plate under consideration for various ratios of the lengths of the sides of the plate and types of transverse load using the roots of the Chebyshev and Legendre polynomials are presented. It is shown that the constructed solution of the boundary value problem converges quickly enough to the analytical solution given in the work. Estimates of the errors of the constructed solutions for the infinite norm and the finite norm in the space of functions integrable with the square are obtained.

The research was carried out at the expense of a grant from the Russian Science Foundation, project no. 24-21-00381.

References

[1] D.P.Goloskokov, A.V.Matroso, Method of initial functions in analyses of the bending of a thin orthotropic plate clamped along the contour, *Vestnik of Saint Petersburg University. Applied Mathematics. Computer Science. Control Processes*, **17**(2021), no. 4, 330–344 (in Russian). DOI: 10.21638/11701/spbu10.2021.402

[2] R.Li, Y.Zhong, B.Tian, Y.Liu, On the finite integral transform method for exact bending solutions of fully clamped orthotropic rectangular thin plates, *Applied Mathematics Letters*, **22**(2009), 1821–1827. DOI: 10.1016/j.aml.2009.07.003

[3] V.R.Lakshmi Gorty, N.Gupt, Bending of fully clamped orthotropic rectangular thin plates using finite continuous ridgelet transform, *Materials Today: Proceedings*, **47**(2021), 4199–4205. DOI: 10.1016/j.matpr.2021.04.458

[4] Q.Xu, Z.Yang, S.Ullah, Z.Jinghui, Y.Gao, Analytical bending solutions of orthotropic rectangular thin plates with two adjacent edges free and the others clamped or simply supported using finite integral transform method, *Advances in Civil Engineering*, (2020), no. 8848879, 1–11. DOI: 10.1155/2020/8848879

[5] S.K.Golushko, S.V.Idimeshev, V.P.Shapeyev, Razrabotka i primeneniye metoda kollokatsiy i naimen'shikh nevyazok dlya resheniya zadach mekhaniki anizotropnykh sloyev plastin, *Vychislitel'nyye tekhnologii*, **19**(2014), no. 5, 24–36 (in Russian).

[6] P.Lazarev, G.M.Semenova, N.A.Romanova, On a Limiting Passage as the Thickness of a Rigid Inclusions in an Equilibrium Problem for a Kirchhoff-Love Plate with a Crack, *J. Sib. Fed. Univ. Math. Phys.*, **14**(2021), no 1, 28–41. DOI: 10.17516/1997-1397-2021-14-1-1-14

[7] O.Ghorbel, J.B.Casimir, L.Hammami, I.Tawfiq, M.Haddar, In-plane dynamic stiffness matrix for a free orthotropic plate, *Journal of Sound and Vibration*, **364**(2016), 234–246. DOI: 10.1016/j.jsv.2015.11.028

[8] A.Deutsch, M.Eisenberger Benchmark analytic in-plane vibration frequencies of orthotropic rectangular plates, *Journal of Sound and Vibration*, **541**(2022), 117248. DOI: 10.1016/j.jsv.2022.117248

[9] I.V.Shcherbakov, B.A.Lyukshin, Modelirovaniye povedeniya otklika ortotropnoy plastiny pri vozdeystvii dinamicheskoy nagruzki, *Vestnik Tomskogo gosudarstvennogo universiteta. Matematika i mekhanika*, **61**(2019), 111–118. (in Russian). DOI: 10.17223/19988621/61/10

[10] S.Zhang, L.Xu, Bending of rectangular orthotropic thin plates with rotationally restrained edges: A finite integral transform solution, *Applied Mathematical Modelling*, **46**(2017), 48–62. DOI: 10.1016/j.apm.2017.01.053.

[11] S.V.Sheshenin, R.R.Muradkhanov, Asimptoticheskoye issledovaniye izgiba plastiny dlya sil'no ortotropnogo materiala, *Izvestiya Rossiyskoy akademii nauk. Mekhanika tverdogo tela*, **3**(2023), 36–57 (in Russian). DOI: 10.31857/S0572329922600608

[12] O.V.Germider, V.N.Popov, On the calculation of the Poiseuille number in the annular region for non-isothermal gas flow, *Journal of Siberian Federal University Mathematics and Physics*, **16**(2023), no. 3, 330–339. EDN: NHSPJB

[13] X.Hu, Z.Wang, B.Hu, A collocation method based on roots of chebyshev polynomial for solving Volterra integral equations of the second kind, *Applied Mathematics Letters*, **29**(2023), no. 108804. DOI: 10.1016/j.aml.2023.108804

[14] E.Kosov, V.Temlyakov, Sampling discretization of the uniform norm and applications, *J. Math. Anal. Appl.*, (2024), no. 128431. DOI: 10.1016/j.jmaa.2024.128431

[15] S.I.Novikov, Ob odnoy zadache interpolyatsii so znachitel'nym znacheniyem L_2-normy operatora Laplasa, *Tr. IMM UrO RAN*, **28**(2022), no. 4, 143–153 (in Russian). DOI: 10.21538/0134-4889-2022-28-4-143-153

[16] V.N.Temlyakov, Diskretizatsiya integral'nykh norm polinomov s garmonikami iz giperbolicheskogo kresta po znacheniyam v tochkakh, *Trudy MIAN*, **312**(2021), 282–293 (in Russian). DOI: 10.4213/tm4133

[17] J.Berntsen, T.O.Espelid, A.Genz, Algorithm 698: DCUHRE: an adaptive multidemensional integration routine for a vector of integrals, *ACM Trans. Math. Softw*, **17**(1991), no 4, 452–456. DOI: 10.1145/210232.210234

[18] S.P.Timoshenko, S.Woinowsky-Kriege, Theory of plates and shells, New York, McGraw-Hill, 1959.

[19] E.Ventsel, Th.Krauthammer, Thin Plates and Shells. Theory: Analysis and Applications, Boca Raton, CRC Press, 2001.

[20] J.Mason, D.Handscomb, Chebyshev polynomials, Florida. CRC Press, 2003.

[21] S.Liu, G.Trenkler, Hadamard, Khatri-Rao, Kronecker and other matrix products, *International Journal of Information and Systems Sciences*, **4**(2008), no. 1, 160–177.

[22] J.Shen, T.Tang, L.Wang, Spectral Methods, Berlin, Springer Berlin Heidelberg, 2011.

[23] R.W.Laureano, J.L.Mantari, J.Yarasca, A.S.Oktem, J.Monge, X.Zhou, Boundary Discontinuous Fourier analysis of clamped isotropic and cross-ply laminated plates via Unified Formulation, *Composite Structures*, **328**(2024), no. 117736. DOI: 10.1016/j.compstruct.2023.117736

Вычисление изгиба тонкой ортотропной пластины с использованием многочленов Лежандра и Чебышева первого рода

Оксана В. Гермидер
Василий Н. Попов
Северный (Арктический) федеральный университет имени М. В. Ломоносова
Архангельск, Российская Федерация

Аннотация. В работе получено решение задачи об изгибе тонкой ортотропной прямоугольной пластины, защемленной по краям, с использованием многочленов Лежандра и Чебышева первого рода. Функция, аппроксимирующая решение бигармонического уравнения для ортотропной пластины, представлена в виде разложения в двойной ряд по этим многочленам в комбинации с матричными преобразованиями и свойствами многочленов Лежандра и Чебышева. С использованием корней этих многочленов в качестве точек коллокации краевая задача приведена к решению системы линейных алгебраических уравнений относительно коэффициентов в разложении искомой функции по этим многочленам. Представлены результаты вычисления изгиба пластины, обусловленного действием распределенной поперечной нагрузки постоянной интенсивности, нагрузки вида, допускающего аналитическое решение краевой задачи, и с интенсивностью, соответствующей гидростатическому давлению, для различных отношений длин сторон пластины. Полученные значения отклонений построенных решений с использованием многочленов Лежандра и Чебышева от аналитического решения задачи приведены по бесконечной норме и конечной норме в пространстве интегрируемых с квадратом функций.

Ключевые слова: изгиб тонкой ортотропной пластины, метод коллокации, многочлены Чебышева первого рода, многочлены Лежандра.