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Analysis of the Electric Current Distribution in a Three-Layer Conductive Structure

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Abstract. The paper presents an analytical model allowing to investigate the electric current distribution in a three-layer conductive structure. The proposed model takes into account the characteristics of the three conductive layers and the transient resistances between them. Expressions for the current distribution and electric potential variation along the structure, as well as its total resistance are obtained. In addition, quantitative estimates showing the features of the electric current redistribution between the layers with alteration of the layers parameters are presented.

Keywords: three-layer conductive structure, current distribution, resistance, specific contact resistivity, transmission line method.

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The study of the current flow processes in multilayer conductive structures is of great interest in a number of areas, both in scientific and applied terms. The papers [1–10] present an analysis of the electric current distribution in two-layer conductive structures as applied to semiconductor devices based on analytical one-dimensional models using the so-called transmission line method (TLM). In most of these works, the main objects of analysis are planar metal-semiconductor contacts and the static current distribution, and the dependence of the contact resistance on the geometric and electrical parameters of the structure are studied. One of the layers of the model is a metal, which is usually considered as an ideal conductor having zero resistance. The second layer is a semiconductor, which conductive properties are described by the specific volume resistance. These models also take into account the specific contact resistance between metal and semiconductor layers.

In [2] an attempt is made to take into account in the TLM model the contribution of capacitance between a metal and a semiconductor separated by an interface layer. In [3] a sufficiently detailed description of TLM models of semiconductor structures is given both in the region of planar metal-semiconductor contacts and a two-layer silicide-semiconductor structure in the interelectrode region. Model for integrated circuit contacts in [4] is built taking into account the resistance of the metal layer. In [5–10] a planar contact model taking into account the longitudinal resistance of the metal-semiconductor transition layer is presented. Accordingly, the resistance of the metal-transition layer and the transition layer-semiconductor are taken into account separately.

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Similar problems of constructing models of the electric current flow are also of interest in the study of processes in the human skin and muscle tissues in relation to electromyography and electrical stimulation [11–14]. In this case, the human skin is considered as a multilayer conductive structure. An attempt to build an analytical model of current flow for the skin, similar to the TLM models described above, is presented in [12, 13]. However, the results of the distribution of electric current in the human skin in [11–14] were obtained only on the basis of a numerical model.

In this paper, an analytical model is proposed that describes the flow of electric current in a three-layer conducting structure. Such analytical formulation has not been discussed previously and is suitable for solving research problems associated with any of the mentioned applied fields.

1. Problem Formulation

Consider the model of a three-layer structure shown in Fig. 1. Three conductive layers are highlighted in the figure. The indexes of the variables in the figure are assigned in accordance with the conditional numbers of the conductive layers: 1 is the top layer; 2 — the second (middle) layer; 3 — the third (the lowest) layer.

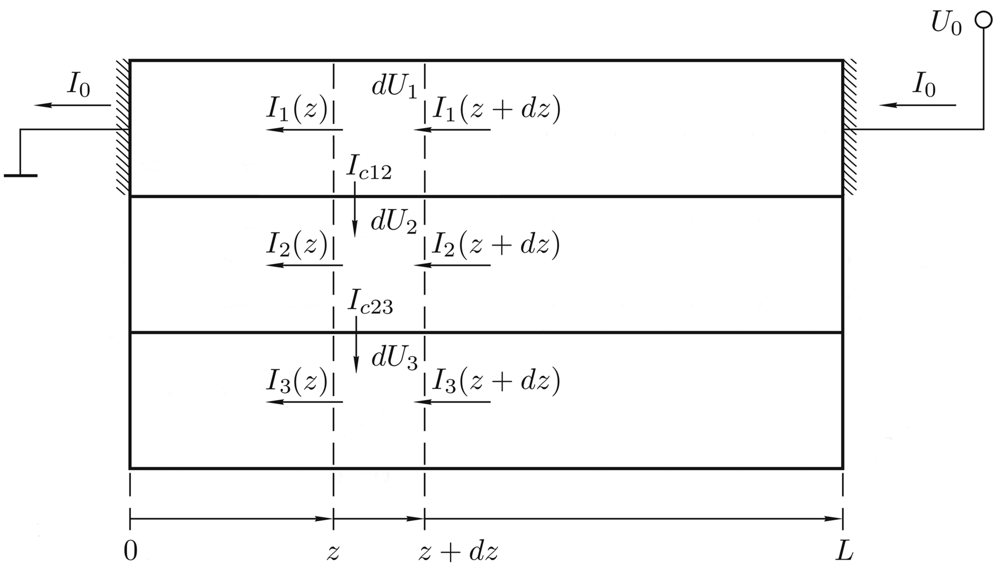

Fig. 1. Three-layer conductive structure

The layers of the structure are characterized by specific volume resistances ρ_1, ρ_2, ρ_3 and thicknesses h_1, h_2 and h_3 for the first, second and third layers, respectively. Layer parameters do not alter along the longitudinal coordinate z. The length of the structure is L. In the direction perpendicular to the plane of the figure, the three-layer structure is also homogeneous and its width is equal to W.

When considering the structure mentioned above, the following assumptions are used.

a) The length of the three-layer structure L much greater than the thicknesses of the layers h_1, h_2 and h_3. Taking into account the conditions $h_1 << L, h_2 << L, h_3 << L$ the transverse current distribution in each of the layers can be assumed to be uniform. Therefore, we will use a one-dimensional model, where all variables in each of the layers may vary along z axis only.

b) The interfaces between the layers are characterized by specific contact resistivities ρ_{c12}

between the first and second layers, and ρ_{c23} between the second and third layers. In practice, this assumption corresponds to the case when the thickness of the transition region between the resistive layers is much less than the values h_1, h_2 and h_3.

Without loss of generality, we assume that a constant voltage U_0 is applied to the upper layer of the structure. At the same time, on the left boundary (at $z = 0$) the electric potential is equal to zero, and on the right boundary (at $z = L$) the potential is positive and equal to U_0.

The boundary conditions for the considered model have the form:

$$\begin{gathered} I_1(z=0) = I_0, I_1(z=L) = I_0, \\ I_2(z=0) = 0, I_2(z=L) = 0, \\ I_3(z=0) = 0, I_3(z=L) = 0, \end{gathered} \tag{1}$$

where I_1, I_2, I_3 are the currents in the first, second and third layers, respectively. The total current I_0 flowing through a three-layer structure depends on the parameters of this structure and the applied voltage U_0. Obviously, $I_0 = I_1 + I_2 + I_3$.

2. Mathematical model

Equations for the currents flowing in the layers can be expressed as follows:

$$I_1(z) = \frac{Wh_1}{\rho_1}\frac{dU_1(z)}{dz}, \tag{2.1}$$

$$I_2(z) = \frac{Wh_2}{\rho_2}\frac{dU_2(z)}{dz}, \tag{2.2}$$

$$I_3(z) = \frac{Wh_3}{\rho_3}\frac{dU_3(z)}{dz}, \tag{2.3}$$

where dU_1, dU_2 and dU_3 are the voltage drops in the elementary sections dz in the first, second and third layers, respectively.

Part of the current $I_1(z)$ flowing in the upper layer of the structure branches off into the adjacent (second) layer, so that the current $I_1(z)$ in the section dz decreases by $dI_{c12}(z)$, where $dI_{c12}(z)$ is the current flowing through the interface between the layers. In this case, the current $I_2(z)$ in the second layer increases correspondingly by $dI_{c12}(z)$. Similarly, the current is redistributed between the second and third layers.

Therefore, it is correct to write the current balance ratios in the form:

$$I_1(z+dz) - I_1(z) = -dI_{c12}(z), \tag{3.1}$$

$$I_2(z+dz) - I_2(z) = dI_{c12}(z) - dI_{c23}(z), \tag{3.2}$$

$$I_3(z+dz) - I_3(z) = dI_{c23}(z). \tag{3.3}$$

On the other hand, the currents $dI_{c12}(z)$ and $dI_{c23}(z)$ flowing through the interface between the layers depend on the difference in electric potentials in adjacent layers

$$U_{c12}(z) = U_2(z) - U_1(z), \tag{4.1}$$

$$U_{c23}(z) = U_3(z) - U_2(z), \tag{4.2}$$

so

$$dI_{c12}(z) = \frac{W}{\rho_{c12}}U_{c12}(z)dz \quad \text{or} \quad \frac{dI_{c12}(z)}{dz} = \frac{W}{\rho_{c12}}U_{c12}(z), \tag{5.1}$$

$$dI_{c23}(z) = \frac{W}{\rho_{c23}}U_{c23}(z)dz \quad \text{or} \quad \frac{dI_{c23}(z)}{dz} = \frac{W}{\rho_{c23}}U_{c23}(z). \tag{5.2}$$

To find the currents flowing in the layers in the cross section $(z + dz)$, we write down the equations obtained by expanding expressions (2.1), (2.2) and (2.3) in a Taylor series, keeping the first two terms of the series:

$$I_1(z + dz) \approx \frac{Wh_1}{\rho_1}\left[\frac{dU_1(z)}{dz} + \frac{d^2U_1(z)}{dz^2}dz\right], \tag{6.1}$$

$$I_2(z + dz) \approx \frac{Wh_2}{\rho_2}\left[\frac{dU_2(z)}{dz} + \frac{d^2U_2(z)}{dz^2}dz\right], \tag{6.2}$$

$$I_3(z + dz) \approx \frac{Wh_3}{\rho_3}\left[\frac{dU_3(z)}{dz} + \frac{d^2U_3(z)}{dz^2}dz\right]. \tag{6.3}$$

Let's get the equation for distribution $U_1(z)$ in the first layer. To do this we substitute the right side of (6.1) in (3.1) instead of the first term $I_1(z+dz)$, and replace the second term $I_1(z)$ by (2.1), and the right side — $dI_{c12}(z)$ by (5.1) :

$$\frac{Wh_1}{\rho_1}\left[\frac{dU_1(z)}{dz} + \frac{d^2U_1(z)}{dz^2}dz\right] - \frac{Wh_1}{\rho_1}\frac{dU_1(z)}{dz} = -\frac{W}{\rho_{c12}}U_{c12}(z)dz \quad \text{or} \quad \frac{d^2U_1(z)}{dz^2} = -\frac{\rho_1}{h_1}\frac{U_{c12}(z)}{\rho_{c12}}.$$

Similarly, we obtain expressions for the second and third layers. Then the system of equations for all three layers has the form:

$$\frac{d^2U_1(z)}{dz^2} = -\frac{\rho_1}{h_1}\frac{U_{c12}(z)}{\rho_{c12}}, \tag{7.1}$$

$$\frac{d^2U_2(z)}{dz^2} = \frac{\rho_2}{h_2}\left[\frac{U_{c12}(z)}{\rho_{c12}} - \frac{U_{c23}(z)}{\rho_{c23}}\right], \tag{7.2}$$

$$\frac{d^2U_13z)}{dz^2} = \frac{\rho_3}{h_3}\frac{U_{c23}(z)}{\rho_{c23}}. \tag{7.3}$$

Let us take into account that $U_{c12}(z) = U_1(z) - U_2(z)$, whence, using (7.1) and (7.2), we obtain

$$\frac{d^2U_{c12}(z)}{dz^2} = \frac{d^2U_2(z)}{dz^2} - \frac{d^2U_1(z)}{dz^2} = \frac{\rho_2}{h_2}\left[\frac{U_{c12}(z)}{\rho_{c12}} - \frac{U_{c23}(z)}{\rho_{c23}}\right] + \frac{\rho_1}{h_1}\frac{U_{c12}(z)}{\rho_{c12}}.$$

Combining this relation with a similar expression for $U_{c23}(z)$, we write down the general system of equations

$$\begin{aligned} \frac{d^2U_{c12}(z)}{dz^2} &= \frac{1}{\rho_{c12}}\left[\frac{\rho_1}{h_1} + \frac{\rho_2}{h_2}\right]U_{c12}(z) - \frac{1}{\rho_{c23}}\frac{\rho_2}{h_2}U_{c23}(z), \\ \frac{d^2U_{c23}(z)}{dz^2} &= -\frac{1}{\rho_{c12}}\frac{\rho_2}{h_2}U_{c12}(z) + \frac{1}{\rho_{c23}}\left[\frac{\rho_2}{h_2} + \frac{\rho_3}{h_3}\right]U_{c23}(z). \end{aligned} \tag{8}$$

The solution of this system of equations makes it possible to determine the distribution of electric currents in a three-layer structure.

3. Analytical solution

To simplify the notations, we represent the system (8) in the following form:

$$\begin{aligned} \frac{d^2U_{c12}(z)}{dz^2} &= AU_{c12} + BU_{c23}, \\ \frac{d^2U_{c23}(z)}{dz^2} &= CU_{c12} + DU_{c23}. \end{aligned} \tag{9}$$

where $A=[(\rho_1/h_1)+(\rho_2/h_2)]/\rho_{c12}$, $B=-\rho_2/(\rho_{c23}h_2)$, $C=-\rho_2/(\rho_{c12}h_2)$, $D=[(\rho_2/h_2)+(\rho_3/h_3)]/\rho_{c23}$.

For the resulting system, the characteristic equation with respect to the parameter λ describing its particular solutions, has the form

$$\lambda^4-(A+D)\,\lambda^2+(AD-BC)=0. \tag{10}$$

This biquadratic equation has four roots λ_1, λ_2, λ_3, λ_4. Since the characteristic equation has two pairs of roots λ that differ in sign, and they are real simple, the solution of the system for U_{c12} can be written in the following form:

$$U_{c12}=C_1\exp(\lambda_1 z)+C_2\exp(\lambda_2 z)+C_3\exp(\lambda_3 z)+C_4\exp(\lambda_4 z), \tag{11}$$

where C_1, C_2, C_3, C_4 are constants, which values are determined by the boundary conditions (1). It is obvious that the terms containing $\lambda>0$ make an increasing contribution to (11) along the z axis, while the terms with $\lambda<0$ make a decreasing one.

Accordingly, the solution for U_{c23} (including $B=-\rho_2/(\rho_{c23}h_2)\neq 0$) will also contain four constants of integration:

$$U_{c23}=\frac{1}{B}\left(\frac{d^2U_{c23}(z)}{dz^2}-AU_{c12}\right)=\frac{1}{B}\sum_{i=1}^{4}C_i\lambda_i^2\exp(\lambda_i z)-\frac{A}{B}\sum_{i=1}^{4}C_i\exp(\lambda_i z). \tag{12}$$

To determine the constants C_1, C_2, C_3 and C_4 we use the boundary conditions. Using (1) and expressions (2.1), (2.2) and (2.3), we relate $I_1(z), I_2(z), I_3(z)$ and $U_{c12}(z), U_{c23}(z)$ on the boundaries of the structure $z=0$ and $z=L$ through $U_1(z), U_2(z), U_3(z)$ using the formulas

$$\frac{dU_{c12}}{dz}=\frac{dU_2}{dz}-\frac{dU_1}{dz}, \tag{13.1}$$

$$\frac{dU_{c23}}{dz}=\frac{dU_3}{dz}-\frac{dU_2}{dz}. \tag{13.2}$$

The derivatives of U_1, U_2, and U_3 on the right-hand sides of (13.1) and (13.2) are expressed using the boundary conditions (1).

On the left boundary of the structure at $z=0$, taking into account (2.1), (2.2), and (2.3), we have the relations

$$I_1(z=0)=\frac{Wh_1}{\rho_1}\frac{dU_1(z=0)}{dz}=I_0 \quad \text{or} \quad \frac{dU_1(z=0)}{dz}=I_0\frac{\rho_1}{Wh_1}, \tag{14.1}$$

$$I_2(z=0)=\frac{Wh_2}{\rho_2}\frac{dU_2(z=0)}{dz}=0 \quad \text{or} \quad \frac{dU_2(z=0)}{dz}=0, \tag{14.2}$$

$$I_3(z=0)=\frac{Wh_3}{\rho_3}\frac{dU_3(z=0)}{dz}=I_0 \quad \text{or} \quad \frac{dU_3(z=0)}{dz}=0. \tag{14.3}$$

Similarly, for the right boundary at $z=L$ we get :

$$I_1(z=L)=\frac{Wh_1}{\rho_1}\frac{dU_1(z=L)}{dz}=I_0 \quad \text{or} \quad \frac{dU_1(z=L)}{dz}=I_0\frac{\rho_1}{Wh_1}, \tag{15.1}$$

$$I_2(z=L)=\frac{Wh_2}{\rho_2}\frac{dU_2(z=L)}{dz}=0 \quad \text{or} \quad \frac{dU_2(z=L)}{dz}=0, \tag{15.2}$$

$$I_3(z=L)=\frac{Wh_3}{\rho_3}\frac{dU_3(z=L)}{dz}=I_0 \quad \text{or} \quad \frac{dU_3(z=L)}{dz}=0. \tag{15.3}$$

Using the obtained relations (14.1)–(14.3) and (15.1)–(15.3), we form a system of equations allowing us to find the constants C_i included in the solutions (11) and (12) for $U_{c12}(z)$ and $U_{c23}(z)$.

First equation of the system for U_{c12} at $z = 0$ (and, accordingly, taking into account $e^{\lambda,0} \equiv 0$) we obtain by substituting (14.1), (14.2) and dU_{c12}/dz, obtained by differentiation of (11), into (13.1). Similarly, we obtain the second equaton for U_{c23} at $z = 0$ by substituting (14.2), (14.3) and dU_{c12}/dz, obtained by differentiation of (12), into (13.2). Following the same logic, we get the third equation for U_{c12} at $z = L$ by substituting (15.1), (15.2) and dU_{c12}/dz into (13.1). Finally, the forth equation for U_{c23} at $z = L$ we obtain by substituting (15.2), (15.3) and dU_{c12}/dz into (13.2). Resulting four relations allow us to form a system of equations for the unknowns C_1, C_2, C_3, C_4:

$$\begin{aligned}
&C_1\lambda_1 + C_2\lambda_2 + C_3\lambda_3 + C_4\lambda_4 = -\frac{I_0\rho_1}{Wh_1},\\
&C_1\left(\lambda_1^3 - A\lambda_1\right) + C_2\left(\lambda_2^3 - A\lambda_2\right) + C_3\left(\lambda_3^3 - A\lambda_3\right) + C_4\left(\lambda_4^3 - A\lambda_4\right) = 0,\\
&C_1\lambda_1\exp(\lambda_1 z) + C_2\lambda_2\exp(\lambda_2 z) + C_3\lambda_3\exp(\lambda_3 z) + C_4\lambda_4\exp(\lambda_4 z) = -\frac{I_0\rho_1}{Wh_1},\\
&C_1\lambda_1\left(\lambda_1^3 - A\lambda_1\right) + C_2\lambda_2\left(\lambda_2^3 - A\lambda_2\right) + C_3\lambda_3\left(\lambda_3^3 - A\lambda_3\right) + C_4\lambda_4\left(\lambda_4^3 - A\lambda_4\right) = 0.
\end{aligned} \tag{16}$$

Solving this system, one can find the constants C_1, C_2, C_3, C_4. Such a solution can be implemented analytically by any of the direct methods or numerically using built-in computational procedures of mathematical software systems.

So, from (10) and (16) one can find all λ_i and all C_i. This allows, using (11) and (12), to determine the dependences $U_{c12}(z)$ and $U_{c23}(z)$, and on their basis it is possible to calculate the distributions $I_1(z), I_2(z), I_3(z)$.

To determine the dependence $I_1(z)$, we use (2.1)–(2.3) and (7.1), (7.2), (7.3), pairwise connecting I_1 and U_1, I_2 and U_2, and also I_3 and U_3.

For the current I_1 on the basis of (2.1) we write $d^2U_1(z)/dz^2 = (\rho_1/Wh_1)/[dI_1(z)/dz]$. By replacing the $U_1(z)$ in this relation with the right side of (7.1), we obtain an expression relating $U_{c12}(z)$ and the first derivative $I_1(z)$:

$$\frac{\rho_1}{Wh_1}\frac{dI_1(z)}{dz} = -\frac{\rho_1}{h_1}\frac{U_{c12}(z)}{\rho_{c12}} \quad \text{or} \quad \frac{dI_1(z)}{dz} = -\frac{W}{\rho_{c12}}U_{c12}(z).$$

Integrating the last relation and taking into account $I_1(0) = I_0$, we determine the current variation in the first layer $I_1(z)$:

$$I_1(z) = I_0 - \frac{W}{\rho_{c12}}\int_0^z U_{c12}(z)dz = I_0 - \frac{W}{\rho_{c12}}\sum_{i=1}^{4}\frac{C_i}{\lambda_i}\big[\exp(\lambda_i z) - 1\big]. \tag{17}$$

For current I_3 on the basis of (2.3) we write $d^2U_3(z)/dz^2 = (\rho_3/Wh_3)/[dI_3(z)/dz]$. Based on equation (7.3), which expresses the second derivative of $U_3(z)$ in terms of $U_{c23}(z)$, we can write

$$\frac{\rho_3}{Wh_3}\frac{dI_3(z)}{dz} = \frac{\rho_3}{h_3}\frac{U_{c23}(z)}{\rho_{c23}} \quad \text{or} \quad \frac{dI_3(z)}{dz} = \frac{W}{\rho_{c23}}U_{c23}(z).$$

Integrating the last relation, taking into account $I_3(0) = 0$, we determine the current variation in the third layer $I_3(z)$:

$$I_3(z) = \frac{W}{\rho_{c23}}\int_0^z U_{c23}(z)dz = \frac{1}{B}\frac{W}{\rho_{c23}}\sum_{i=1}^{4}C_i\Big[\exp(\lambda_i z) - 1\Big]\left(\lambda_i - \frac{A}{\lambda_i}\right). \tag{18}$$

Using (2.2) for the current I_2 we obtain $d^2U_2(z)/dz^2 = (\rho_2/Wh_2)/[dI_2(z)/dz]$. On the other hand, according to (7.2) $d^2U_2(z)/dz^2 = (\rho_2/h_2)/[(U_{c12}/\rho_{c12}) - (U_{c23}/\rho_{c23})]$. Then

$$\frac{\rho_2}{Wh_2}\frac{dI_2(z)}{dz} = \frac{\rho_2}{h_2}\left[\frac{U_{c12}(z)}{\rho_{c12}} - \frac{U_{c23}(z)}{\rho_{c23}}\right] \quad \text{or} \quad \frac{dI_2(z)}{dz} = W\left[\frac{U_{c12}(z)}{\rho_{c12}} - \frac{U_{c23}(z)}{\rho_{c23}}\right].$$

Integrating the last relation, one can find $I_2(z)$ and, taking into account (17) and (18), obtain:

$$I_2(z) = \frac{W}{\rho_{c12}}\int_0^z U_{c12}(z)dz - \frac{W}{\rho_{c23}}\int_0^z U_{c23}(z)dz = I_0 - I_1(z) - I_3(z). \tag{19}$$

Relation (19) shows that in any section of the three-layer structure the equality $I_0 = I_1+I_2+I_3$ and the dependence of the current $I_2(z)$ for the middle layer can be found if the distributions of $I_1(z)$ and $I_3(z)$ are known.

Integration (2.1) allows us to find the distribution $U_1(z)$ in the upper layer of the structure:

$$U_1(z) - U_1(0) = \frac{\rho_1}{Wh_1}\int_0^z I_1(z)dz. \tag{20}$$

By substituting in (20) the dependence of $I_1(z)$ from (17) and taking into account $U_1(0) = 0$, we obtain

$$U_1(z) = \frac{\rho_1}{Wh_1}\int_0^z I_1(z)dx - U_1(0) = \frac{\rho_1 I_0}{Wh_1}z - \frac{\rho_1}{\rho_{c12}h_1}\sum_{i=1}^{4}\frac{C_i}{\lambda_i}\left[\frac{\exp(\lambda_i z)}{\lambda_i} - z\right] - \sum_{i=1}^{4}\frac{C_i}{\lambda_i^2}. \tag{21}$$

The total voltage drop over the entire length of the three-layer structure is determined from (21) as $U_0 = U_1(L)$. Accordingly, the total resistance of the structure is equal to

$$R = \left[U_1(L) - U_1(0)\right]/I_0 = U_1(L)/I_0. \tag{22}$$

4. Simulation results

The distributions of voltages and currents along the three-layer structure obtained as a result of the calculations are shown in Fig. 2. Taking into account the fact that the value of I_0, as well as the width of the structure W, does not affect the nature of the distribution of currents and voltages (this can be seen from the calculated relations (17)–(19), (21)), graphs are given in a normalized form: for voltages $\tilde{U}_{c12} = U_{c12}/U_0$, $\tilde{U}_{c23} = U_{c23}/U_0$, $\tilde{U}_1 = U_1/U_0$, $\tilde{U}_2 = U_2/U_0$, $\tilde{U}_3 = U_3/U_0$ and currents $\tilde{I}_1 = I_1/I_0$, $\tilde{I}_2 = I_2/I_0$, $\tilde{I}_3 = I_3/I_0$ relative to the reduced coordinate $\tilde{z} = z/L$.

Dependences $U_{c12}(z)$ and $U_{c23}(z)$ are calculated on the basis of (11) and (12). The distributions $U_1(z), U_2(z), U_3(z)$ are obtained using (21) and using (4.1) and (4.2): $U_2(z) = U_1(z) + U_{c12}(z)$; $U_3(z) = U_2(z) + U_{c23}(z)$. Dependences $I_1(z), I_2(z)$ and $I_3(z)$ are constructed in accordance with (17), (18) and (19).

Calculations were made for the following parameters: $L = 0.01$ m; $W = 0.01$ m; $\rho_1 = 2\cdot 10^{-6}$ Ohm·m; $\rho_2 = 1\cdot 10^{-6}$ Ohm·m; $\rho_3 = 4\cdot 10^{-6}$ Ohm·m. Specific contact resistivities ρ_{c12} and ρ_{c23} were chosen from the condition: $\rho_{c12} = \min(\rho_1,\rho_2)\times 1$ m; $\rho_{c23} = \min(\rho_2,\rho_3)\times 1$ m, so that for the specified layer parameters $\rho_{c12} = \rho_{c23} = 1\cdot 10^{-6}$ Ohm·m^2 . The layer thicknesses were set equal: $h_1 = h_2 = h_3/2 = L \times 10^{-3} = 10^{-5}$ m (Fig. 2, a) and $h_1 = h_2 = h_3/2 = L \times 10^{-4} = 10^{-6}$ m (Fig. 2, b). Such thicknesses are typical in works on thin-film microelectronics [1–10].

The intensity of redistribution of the total current I_0 between the layers of the structure can be judged from the gradients of $I_1(z), I_2(z)$ and $I_3(z)$. As can be seen from Fig. 2, this process is most active in areas near the left and right boundaries. As a result, for $z = 0$ and $z = L$ (in Fig. 2 $\tilde{z} = 0$ and $\tilde{z} = 1$) voltages U_{c12} and U_{c23} have maximum absolute values, which is

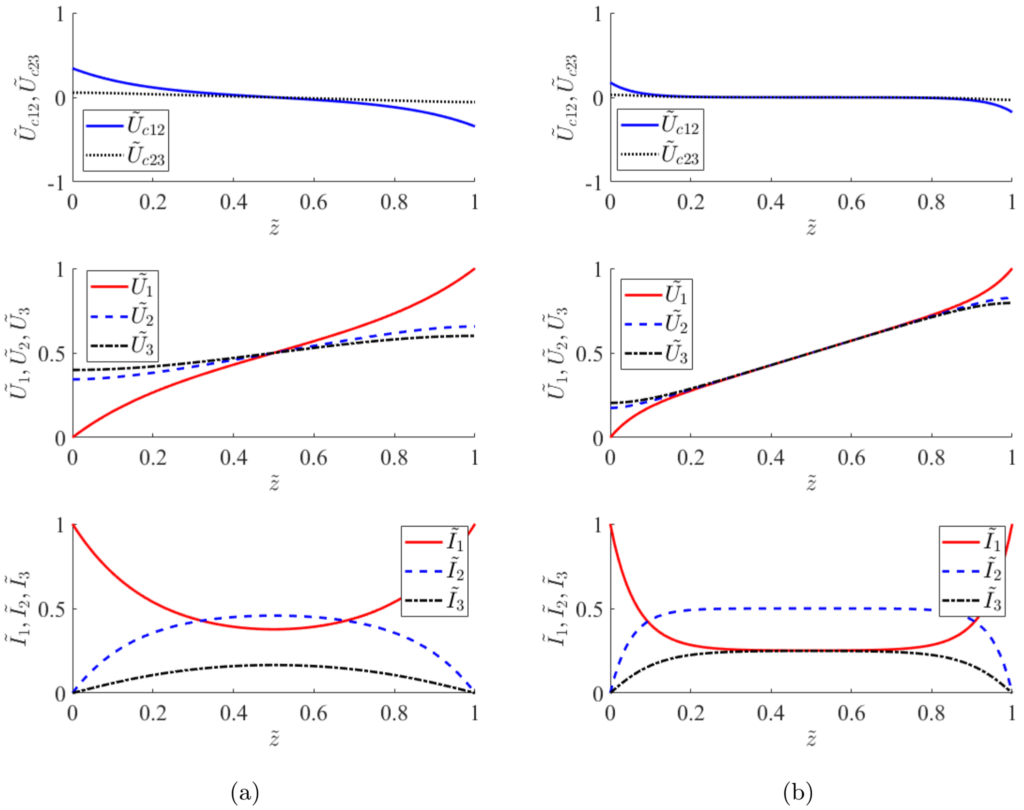

Fig. 2. Voltage and current distributions along a three-layer structure: a) $h_1 = h_2 = h_3/2 = L \times 10^{-3} = 10\,\mu$m; b) $h_1 = h_2 = h_3/2 = L \times 10^{-4} = 1\,\mu$m

consistent with (5.1) and (5.2), from which it follows that $U_{c12} \sim dI_{c12}$ and $U_{c23} \sim dI_{c23}$. The slope of the curves $U_1(z), U_2(z)$ and $U_3(z)$ also changes along the coordinate z, and near the left and right boundaries of the structure, the gradient $U_1(z)$ is maximum, and the gradients $U_2(z)$ and $U_3(z)$ are minimal as U_{c12} and U_{c23} increase .

Dependencies in Fig. 2, a, corresponding to the ratio $h_i/L \sim 10^{-3}$, show that the redistribution of the current and the variation in voltages are observed over the entire length of the structure. In the middle part of the structure, the current I_1 has a minimum value, while the currents I_2 and I_3, on the contrary, reach maximum values due to the branching of a part of the total current I_0 into the lower layers. In this case, in any section, $\tilde{z}$ the relation is fulfilled $\tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3 = 1$, which is similar to the condition $I_1 + I_2 + I_3 = I_0$.

Current and voltage distributions presented in Fig. 2, b are obtained for thinner conductive layers ($h_i/L \sim 10^{-4}$), while keeping other initial calculated parameters unchanged. It can bee seen that the length of the segments in which the redistribution of currents I_1, I_2, and I_3 mainly occurs does not exceed half the length of the structure. It is also worth noting that a similar result can be obtained not only by decreasing the layer thickness, but also by increasing the length L.

The difference in the character of dependences in Fig. 2, a and Fig. 2, b can be attributed to the fact that the variation in currents I_1, I_2 and I_3, according to (17), (19) and (18), is determined by the roots λ_i of equation (10), depending on the specific contact resistances ρ_{c12}, ρ_{c23} and parameters that, for a given structure width W characterize the longitudinal conductivity of the layers – $\rho_1/h_1, \rho_2/h_2, \rho_3/h_3$. Therefore, an increase or decrease in $\rho_1/h_1, \rho_2/h_2, \rho_3/h_3$ leads to a change in the parameters λ_i and, accordingly, to a reduction or increase in the length of the regions in which the redistribution of currents I_1, I_2 mainly occurs and I_3.

Due to the fact that in Fig. 2, b, the regions of growth and decay of currents in the layers make up a relatively small part of the total length L , in the middle part of the structure, the dependences $I_1(z), I_2(z)$ and $I_3(z)$ have flat sections, within which $dI_1(z)/dz \approx dI_2(z)/dz \approx dI_3(z)/dz \approx 0$ and, respectively, $I_{c12}(z) \approx 0, I_{c23}(z) \approx 0, U_{c12}(z) \approx 0, U_{c23}(z) \approx 0, U_1(z) \approx U_2(z) \approx U_3(z)$. For the given design parameters, these sections are located in the range approximately from $\tilde{z} \approx 0.4$ to $\tilde{z} \approx 0.6$. Obviously, with an increase in the length of the structure, the extent of these flat sections will increase.

It should be noted that the contribution of each of the currents I_1, I_2 and I_3 in the total current I_0 at (that is, at $z = L/2$) is inversely proportional to the ratio $\rho_1/h_1, \rho_2/h_2$ and ρ_3/h_3 respectively for each of the layers. From the dependencies in Fig. 2, b, for example, it can be seen that the currents in the first and third layers are equal, since $\rho_1/h_1 = \rho_3/h_3$.

An analysis of the influence of geometric factors on the nature of the distribution of currents and voltages shows that for the considered three-layer structure, a nonlinear dependence of its total resistance R on the length L can be observed. Fig. 3 shows the dependences $R(L)$ calculated in the range of L from 10^{-4} m to 10^{-2} m for three options corresponding to the layer thicknesses: $h_1 = h_2 = h_3/2 = 10^{-6}$ m; $h_1 = h_2 = h_3/2 = 3 \cdot 10^{-6}$ m; $h_1 = h_2 = h_3/2 = 10^{-5}$ m. The values of the other parameters of the structure were set the same as in the previous calculations.

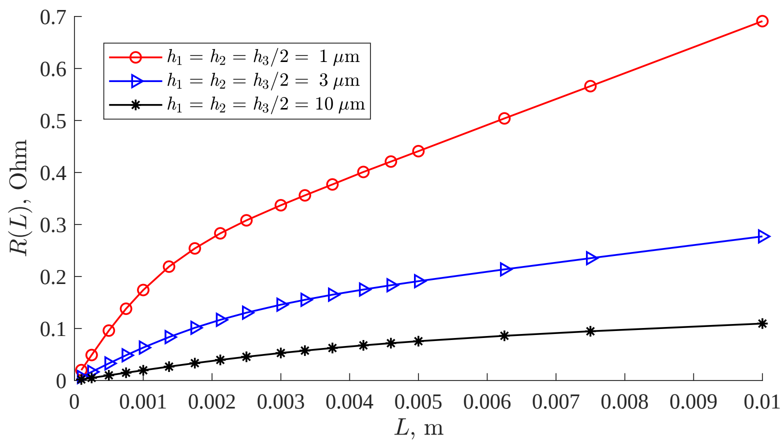

Fig. 3. Length dependence of the three-layer structure resistance $R(L)$ for different layers thicknesses

On the entire length of the upper curve, the condition of smallness of the layer thickness $h_i/L \leqslant 0.02$ is satisfied, and on the second curve located below it, – $h_i/L \leqslant 0.02$. For the lower dependence in the range of L from 0.001 m to 0.01 m, this condition corresponds to $h_i/L \leqslant 0.02$. At $L < 0.001$m, the ratio h_3/L can reach 0.2, so this section can be considered as an extrapolation of the dependence based on the proposed model.

Plots in Fig. 3 show that for the given design parameters, the dependence $R(L)$ in its initial section is non-linear, approximately up to $L \sim (2 \ldots 3) \cdot 10^3 \times h_1$. The non-linear nature of the curves at small values of the structure length is due to the fact that in this range of L variation, the redistribution of the current between the layers occurs over its entire length.

It can be shown that for a structure length not exceeding approximately $L \sim 1/\lambda_i$ (see (10), (17)), the current flowing through the structure is mainly concentrated in the upper layer, while the fraction of the current in the two lower layers is very small. As L increases , the part of the current branched into the second and third layers of the structure increases, which leads to a decrease in the rate of increase in the resistance of the structure $dR(L)/dL$ with an increase in its length.

With an increase in the length of the structure, approximately from $L \sim (2 \ldots 3) \cdot 10^3 \times h_1$, the resistance R begins to increase linearly. At large values of L, the most significant influence on the nature of the $R(L)$ dependence is exerted by the middle part of the structure, within which the distributions of $I_1(z), I_2(z)$, and $I_3(z)$ have flat areas. In this case, an increment in the length L leads to a corresponding increase in the length of these flat sections, which determines the linear nature of the dependence $R(L)$.

Conclusion

The analysis of the current flow mechanism in a three-layer conductive structure made it possible to obtain a model that describes the regularities in the distribution of electric current and voltage in the structure. The analysis of the obtained relations describing the three-layer structure, as well as the calculations performed on their basis, allow us to draw the following conclusions.

1. The length of the sections of current redistribution between the layers of the structure within the framework of the proposed model is determined by the specific contact resistances ρ_{c12}, ρ_{c23} at the interfaces of the conductive layers and the ratios of the volume resistivity of the layers to their thicknesses — $\rho_1/h_1, \rho_2/h_2, \rho_3/h_3$.

2. For "short" three-layer structures, in which the redistribution of current between the layers occurs over their entire length L, the dependence of the total resistance R on L is non-linear.

3. For "long" three-layer structures, in which the regions of growth and decay of currents in the layers make up a relatively small part of the total length L, in the middle part of the structure, the dependences $I_1(z), I_2(z)$ and $I_3(z)$ have low slope graphs. For such structures, a linear dependence of the resistance R on the length L is observed .

The approach used in this work can be applied to the construction of similar models of multilayer structures, for example, for other boundary conditions that determine their connection to an external circuit.

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Анализ распределения электрического тока в трехслойной проводящей структуре

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Аннотация. В работе представлена аналитическая модель, позволяющая исследовать характер распределения электрического тока в трехслойной проводящей структуре. Предложенная модель учитывает характеристики трех проводящих слоев и переходных сопротивлений между ними. Также получены выражения для распределения тока и изменения электрического потенциала вдоль структуры, а также её общего сопротивления. Кроме того, представлены количественные оценки, показывающие особенности перераспределения электрического тока между слоями при изменении параметров слоев.

Ключевые слова: трехслойная проводящая структура, распределение тока, сопротивление, удельное контактное сопротивление, TLM-метод.