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## Scaling Behavior of Complex Low-dimensional Spin Systems

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**Abstract.** We studied the non-equilibrium properties of spin-valve structures like a main elements of spintronics. We observed the aging effects in non-equilibrium critical behavior like a two-time dependence of measurement quantities: waiting time and observation time. The other essential property of ageing systems is dynamic scaling which we detected in our systems.

**Keywords:** dynamic scaling, spin-valve structures, thin films, aging effects, phase transitions and critical phenomena.

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In the early 1990s, there was a significant rise in the physics of magnetic phenomena due to the discovery of the giant magnetoresistance effect in the Fe/Cr structure [1]. This is how spintronics appeared — a new area of research. The basic elements of spintronics include spin valve nanostructures and multilayer.

Spin valves consist of magnetic metals (Fe, Co, Ni) separated by a nonmagnetic interlayer. The magnetization direction of one of the ferromagnetic films is fixed due to the strong bond with the antiferromagnetic metal layer (IrMn, FeMn, FeO) unlike multilayer structures. The structure of the spin valve is mainly represented as NiFe/NM/FM/FeMn, where one of the ferromagnetic films is the NiFe alloy — permalloy. The advantage of spin valves is their high sensitivity to an external magnetic field. For comparison, saturation fields at room temperature

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in multilayer structures are 100–2000 Oe, while in spin valves 5–50 Oe. This makes them the most advantageous in practical applications [2].

One of the fundamental properties of spin valves is magnetic anisotropy. In its absence, these elements do not have the characteristic features of changing magnetic and magnetoresistive properties. Magnetic anisotropy is one of the dominant factors influencing the behavior and properties of magnetic materials [3]. Symmetry breaking at the boundaries is always present during crystal growth due to the influence of various external and internal factors. This why the properties of a ferromagnetic system dependence on the direction of the magnetization vector. It is possible to create nanostructures from several layers using various materials. The magnetic moments of these layers will be directed along or across the plane of the film surface. It is convenient to describe the properties of such structures using the anisotropic Heisenberg model, which belongs to statistical spin models with slow dynamics.

Critical slowing down, aging effects, memory effects [4] are one of the most phenomena which we can see in statistical systems with slow dynamics. That is why these systems have theoretical and experimental interest.

The critical slowing down is the increasing of the autocorrelation time  $\tau_{\text{cor}}$  near the critical temperature  $T_c$ . In the spin system this critical slowing down characterized by the universal dynamical critical index  $z$

$$\tau_{\text{cor}} \sim |T - T_c|^{-\nu z} \quad (1)$$

with the correlation length  $\xi$  critical exponent  $\nu$ .

## 1. Model and Methods

We simulated the systems with size  $N_s = L \times L \times N$ . Here  $N$  is number of layers and  $L \times L$  is number of atom in one layer. We updated spin configurations with help of Metropolis algorithm. In this paper, we investigated time dependencies next quantities. For ferromagnetic films we defined the magnetization

$$m = \left\langle \frac{1}{N_s} \left[ \left( \sum_i^{N_s} S_i^x \right)^2 + \left( \sum_i^{N_s} S_i^y \right)^2 + \left( \sum_i^{N_s} S_i^z \right)^2 \right]^{1/2} \right\rangle, \quad (2)$$

where angle brackets denote the statistical averaging.

In this paper, we calculated time dependencies of two-time autocorrelation function in order to inspected ageing phenomena

$$C(t, t_w) = \left\langle \frac{1}{N_s} \sum_i \vec{S}_i(t) \vec{S}_i(t_w) \right\rangle - m(t) \cdot m(t_w), \quad (3)$$

where  $t$  is the observation time is time from the sample preparation;  $t_w$  is the waiting time is the time characterizing the time got by from the moment of sample preparation to the measurement of its characteristics.

In this paper we calculated the correlation time. But Monte Carlo simulation have some troubles in obtaining the relaxation time. One of them is slow dynamics of our systems. The second problem is the method of determining the relaxation time. This method requires accurate evaluation of a long-time tail of the autocorrelation function. In order to fixed this problems, in

paper [5] had suggested a dimensionless dynamic correlation function

$$R(t, t_w) = \frac{C(t, t_w)}{\sqrt{\left\langle \left( \frac{1}{N_s} \sum_i \vec{S}_i(t) \vec{S}_i(t_w) \right)^2 \right\rangle}} \sim e^{-\delta t / \tau_{cor}}. \quad (4)$$

Equation (4) is the ratio of an odd to an even moment of the autocorrelation function.

## 2. Dynamic scaling in thin films

We researched the relaxation of process in thin Heisenberg films. In this paper, we studied how initial state influences these processes. In order to simulate relaxation processes in our systems we used Heisenberg hamiltonian with easy axis anisotropy

$$H = -J \sum_{\langle i, j \rangle} \left[ (1 - \Delta)(S_i^x S_j^x + S_i^y S_j^y) + S_i^z S_j^z \right] - h \sum_i S_i^z. \quad (5)$$

We examined the thickness dependence of the Curie temperature  $T_C$  for thin films of  $Ni(111)/W(110)$  [6] to determine the effective anisotropy constant  $\Delta(N)$  [7, 8].

We simulated our systems at critical temperature  $T_c = 1.15$  for  $N = 3$  monolayer (ML),  $T_c = 1.31$  for  $N = 5$  ML,  $T_c = 1.39$  for  $N = 7$  ML [7–9] from height-temperature initial state  $m_0 = 0.0001$ . Simulation was carry out for different values of the waiting time  $t_w = 200, 100, 70, 50, 20, 0$  Monte-Carlo steps per spin (MCS/s).

Autocorrelation function  $C(t, t_w)$ , which we calculated, depends on two time  $t$  and  $t_w$ . For different values of waiting time  $t_w$  the data are characterized the different curves. This fall on different curves means that time-translation invariance is broken. There are the effects of aging.

Therefore, the aging dynamics of systems may depend on the entire background history of our sample. In the aging regime for  $t - t_w \sim t_w \gg 1$ , the two-time dependence of the autocorrelation function is characterized by a scaling form:

$$C(t, t_w) \sim t_w^{-c} F_c(t/t_w), \quad (6)$$

where the index  $c$  is expressed in terms of the critical indices  $c = 2\beta/z\nu$  at the quenching temperature  $T_s = T_c$ . When we plotted the same data for  $C(t, t_w)$  depending on  $t/t_w$ , they fall onto a one curve. We observed this effect only the time  $t_w$  is large enough. In Fig. 1 the dynamic scaling for autocorrelation function presented for  $N = 3$  ML and  $N = 7$  ML. The collapse in Fig. 1 means that  $C(t, t_w) = f(L(t)/L(t_w))$ . Presence of dynamic scaling is the one of the properties of ageing systems.

We calculated the dimensionless dynamic correlation function  $R(t, t_w)$  [10] to estimate the correlation time of our systems. For sufficiently long times,  $R(t, t_w)$  decreases exponentially:

$$R(t, t_w) \sim \exp(-t/\tau_{cor}). \quad (7)$$

The values of the correlation time are presented in table:

$N$	$t_w = 50$	$t_w = 70$	$t_w = 100$	$t_w = 200$
3	2255(47)	2704(5)	2835(6)	3039(8)
5	1858(3)	2326(3)	2398(4)	2626(4)
7	1498(2)	1987(3)	2074(3)	2237(4)

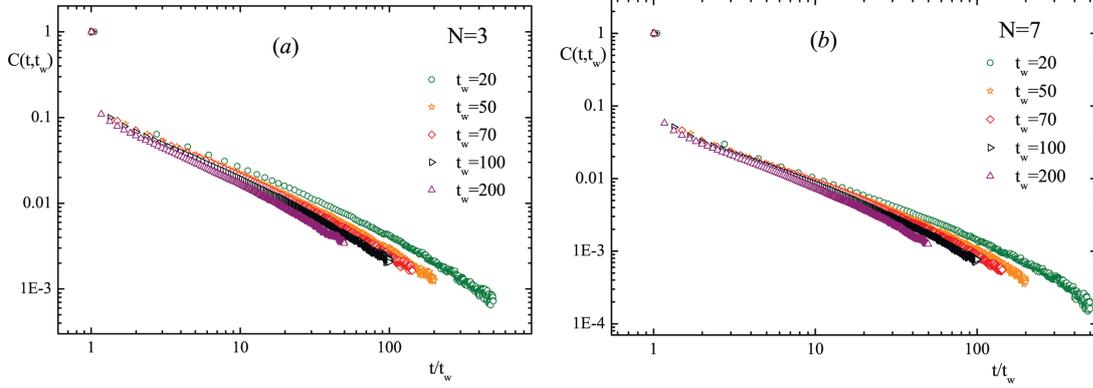


Fig. 1. Dynamical scaling of the two-time autocorrelation function  $C(t, t_w)$  for several values of the waiting time  $t_w = 200; 100; 70; 50; 20$  MCS/s with  $N = 3$  ML (a) and  $N = 7$  ML (b) for initial state  $m_0 \ll 1$

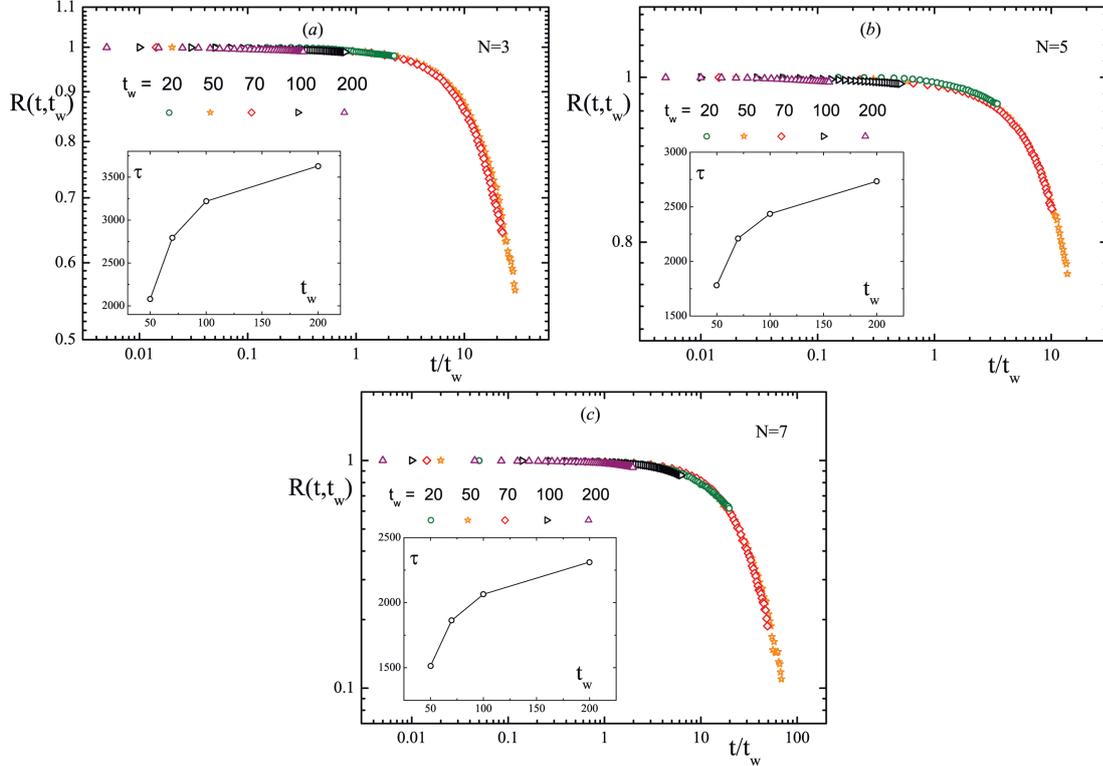


Fig. 2. Dynamical scaling of the two-time dimensionless dynamic correlation function  $R(t, t_w)$  for several values of the waiting time  $t_w = 200; 100; 70; 50; 20$  MCS/s with  $N = 3$  ML (a),  $N = 5$  ML (b) and  $N = 7$  ML (c) for initial state  $m_0 \ll 1$

The dynamic scaling for  $R(t, t_w)$  is presented on Fig. 2 for thin film with different thickness.  $R(t, t_w)$  is dimensionless correlation function. That is why their dynamical scaling form is given by [5]

$$R(t, t_w) \sim F_R(t/t_w) \quad (8)$$

without a factor involving the power of  $t_w$ , as for  $C(t, t_w)$  in Eq. (6) or magnetization  $m(t, t_w)$  in Eq. (9).

### 3. Dynamic relaxation of the complicated spin valve

Thin films from magnetic metals are the main elements of magnetic spin valve structures. This spin valve structures are used in devices with giant magnetoresistance (GMR) effects [1,11].

In order to simulate dynamic processes in spin valve structures we used Heisenberg hamiltonian with easy axis anisotropy (5). We considered system with size  $L \times L \times N$ . Our structures presented on Fig. 3. There are  $N_1 = 3$  ML,  $N_2 = 3$  ML,  $N_3 = 3$  ML. The neighboring spins interacted with exchange integral  $J_1$ . This exchange integral is assumed to be  $J_1/kT = 1$ . Other exchange integrals between the neighboring spins and the interlayer interaction are  $J_{01} = 0.4J_1$ ,  $J_{02} = 0.75J_1$ ,  $J_2 = 0.01J_1$ ,  $J_3 = -3.0J_1$ ,  $J_4 = J_5 = -2.0J_1$  [2].

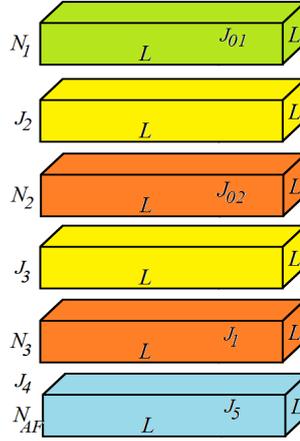


Fig. 3. Model of complicated spin valve.  $L$ ,  $N_1$ ,  $N_2$ ,  $N_3$ ,  $N_{AF}$  – linear sizes;  $J_{01} > 0$ ,  $J_1 > 0$ ,  $J_2 > 0$ ,  $J_3 < 0$ ,  $J_4 < 0$ ,  $J_5 < 0$  – exchange interaction constants

In order to investigate the dynamic properties of spin valve structure we calculated time dependencies of magnetization  $m_1^z(t)$ . The simulation was carry out for waiting times  $t_w = 10, 20, 30, 40, 60, 80, 100$  MCS/s. The following form of scaling dependence of magnetization is predicted from the theory of non-equilibrium processes:

$$m(t, t_w) \sim t_w^{-b} F_m(t/t_w), \quad (9)$$

where parameter  $b = \beta/\nu z$  at temperature  $T_s = T_c$ . The time dependence of magnetization is construct in double logarithmic scale. It is necessary for finding the exponents, which characterize power-law character behavior of magnetization. In Fig. 4, there is a collapse of data for various  $t_w$  on a universal curve corresponding to the scaling form from the formula (9). We determined the value of the parameter  $b = 0.0146(1)$  for system in critical point  $T = T_c$  and  $b = 0.0071(1)$  for system at  $T = T_c/2$ . We used next procedure to determine value of the parameter  $b$ : we chose the time interval according to the minimum of mean-square approximation error for the magnetization for each waiting time  $t_w$ ; then, we picked up the parameter  $b$  at the selected intervals so that the data collapsed on universal curve.

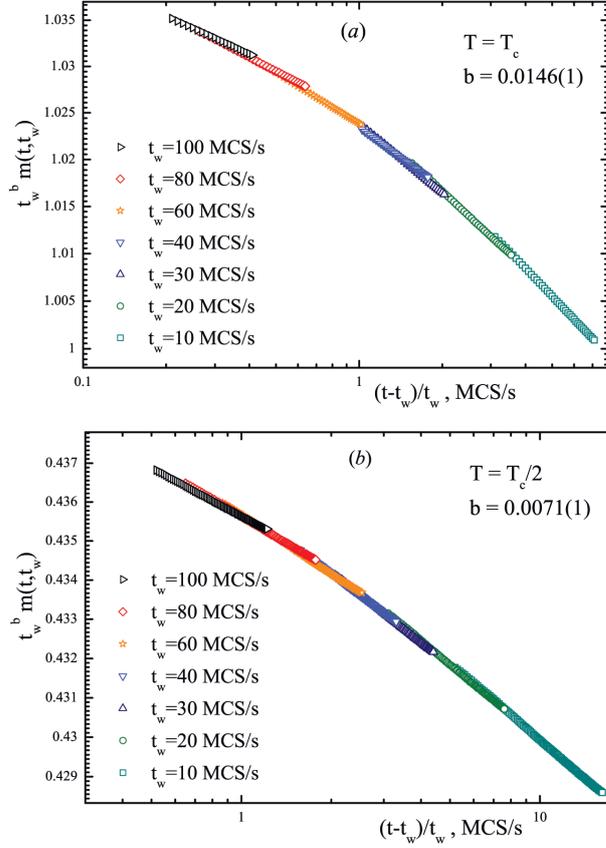


Fig. 4. Scaling dependence of magnetization  $t_w^b \cdot m(t, t_w)$  from ratio  $(t - t_w)/t_w$  at temperature  $T = T_c$  (a) and  $T = T_c/2$  (b)

## Conclusion

In this paper, we studied the non-equilibrium behavior of a complicated spin-valve structure. The dependence of the reduced magnetization for free ferromagnetic film  $m_1^z/m_0^z(t, t_w)$  on the observation time was shown. It characterized the behavior of magnetization relaxation. In the systems we study, the effects of critical slowdown of relaxation processes are observed. Our slow-dynamic systems exhibit the effects of aging.

We considered the structure with thicknesses  $N_1 = N_2 = N_3 = 3$  ML. In this structure, the study of the manifestation of aging effects becomes clearer with an increase in the external magnetic field  $h$ . For  $h = 2.0 J_1$ , the system relaxes over a longer period of time with an increase in the waiting time  $t_w$ . This confirms that aging effects are observed in this dependence in the complicated spin-valve structure. In this paper, the presence of aging effects is confirmed by the data collapse for dynamic scaling dependencies for magnetization  $m(t, t_w)$ , autocorrelation function  $C(t, t_w)$  and dimensionless autocorrelation function  $R(t, t_w)$ .

An important feature of non-equilibrium behavior in systems with slow dynamics is the occurrence of aging effects, which are observed in the present study. For different values of  $t_w$  the data clearly fall on different curves which means that time-translation invariance is broken. When we plotted the same data for  $C(t, t_w)$  depending on  $t/t_w$ , they collapse onto a single

curve. This effect we observed only the time  $t_w$  is large enough. The observed effects of aging in magnetic structures such as thin films are in good agreement with the experimental results presented in [12].

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## Скейлинговое поведение сложных низкоразмерных спиновых систем

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**Аннотация.** Изучены неравновесные свойства спин-вентильных структур как основных элементов спинтроники. Эффекты старения были обнаружены в неравновесном критическом поведении в виде двувременной зависимости измеряемых величин: времени ожидания и времени наблюдения. Другим важным свойством стареющих систем является динамическое масштабирование, которое было обнаружено в рассматриваемых системах.

**Ключевые слова:** динамический скейлинг, спин-вентильные структуры, тонкие пленки, эффекты старения, фазовые переходы и критические явления.