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On the Calculation of the Poiseuille Number in the Annular Region for Non-isothermal Gas Flow

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Abstract. A slow longitudinal non-isothermal gas flow in the annular region caused by small pressure and temperature drops across a long micro-channel is considered in the paper. A method for calculating the values of the Poiseuille number in the transitional gas flow regime is proposed. The method is based on the solution of the model linearised Bhatnagar-Gross-Krook (BGK) kinetic equation using Chebyshev polynomials. The calculated values are compared with similar results obtained using analytical solutions of the Navier–Stokes equations with no-slip and slip boundary conditions. The effect of the accommodation coefficient of the tangential momentum of the gas molecules and the gas rarefaction parameter on the change in the Poiseuille number is analysed for small ratios of the temperature and pressure gradients of the gas in the channel.

Keywords: Chebyshev polynomials of the first kind, collocation method, nonisothermal gas flow in a channel, Poiseuille number, kinetic equation, models of boundary conditions.

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The miniaturization of technological processes has recently become actively implemented in the chemical industry [1]. Modelling of flows in micro-channels has become an intensively developing area of research [1–4]. Micro-channel devices are widely used in various fields of science and technology such as micro-reactors, micro-heat exchangers, micro-mixers, etc. [1, 2]. The obvious advantages are that use of micro-channels makes it possible to significantly intensify physiochemical processes. One of the characteristic similarity parameters that is used to describe gas mass transfer processes in micro-channels is the Poiseuille number [5]. It is calculated using the Boltzmann kinetic equation [6] or model kinetic equations [7]. In the presented paper, the value of the Poiseuille number is calculated using the linearized BGK model kinetic equation for the gas flow in the annular region under the influence of pressure and temperature gradients [8]. Formulation of the problem is close to that given in [7] and [9]. However, unlike [7] and [9] the influence of cross-effects caused by the action of pressure and temperature gradients is studied. Comparison with the results from [7] and results for the sliding flow regime using the Navier-Stokes equation was carried out . The values of the Poiseuille number were found using Chebyshev polynomials [10]. They depend on the rarefaction parameter, the accommodation coefficient of the tangential momentum of gas molecules, ratios of cylinder radii and temperature and pressure gradients. The ratio of temperature gradients and gas pressure in the channel is assumed to be small.

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1. Formulation of basic equations

Let us consider a rarefied gas flow in a long channel under the action of given pressure and temperature gradients with incomplete accommodation of the tangential momentum of gas molecules on the channel walls. The cross section of the channel represents the annular region with radii R'_1 and R'_2 ($R'_1 < R'_2$). The channel connects two reservoirs at pressures p'_1 and p'_2, temperatures T'_1 and T'_2, with $p'_1 > p'_2$ and $T'_1 < T'_2$. The tangential momentum accommodation coefficients of gas molecules on the inner and outer cylinders are the same $\alpha_1 = \alpha_2 = \alpha$. Physical quantities are presented in non-dimensional form as in [9], except for the length. The hydraulic diameter $D'_h = 2(R'_2 - R'_1)$ [11] is chosen as the length scale. In what follows the prime symbol for dimensionless quantities is omitted.

The Poiseuille number P_0 is defined according to [6] as the product of the Darcy coefficient of friction f_d and the Reynolds number Re

$$\mathrm{P}_0 = f_d \mathrm{Re} = -\frac{2G_p p'_0 D'_h {\beta'}^{1/2}}{\mu' \bar{u}_z},$$

where μ' is the dynamic viscosity of the gas, $\beta' = m'/(2k'_B T'_0)$, m' is the mass of gas molecules, k'_B is the Boltzmann constant, p'_0 and T'_0 are pressure and temperature of the gas taken as the origin; G_p is the dimensionless pressure gradient, $\bar{u}_z$ is the average value of the dimensionless component of the gas mass velocity u_z.

Assuming that the absolute values of the dimensionless gradients G_p and G_T are small, linearised distribution function is written as

$$f(\mathbf{r}, \mathbf{C}) = f_0(C)\left(1 + G_T\left(C^2 - \frac{5}{2}\right)z + G_p z + h(\rho, \mathbf{C})\right), \tag{1}$$

$$h(\rho, \mathbf{C}) = G_p h_1(\rho, \mathbf{C}) + G_T h_2(\rho, \mathbf{C}).$$

Here $f_0(C) = \pi^{-3/2}\exp\left(-C^2\right)$ is the dimensionless absolute Maxwellian, $h_1(\rho, \mathbf{C})$ and $h_2(\rho, \mathbf{C})$ are perturbations of the distribution function due to the presence of pressure and temperature gradients. In the configuration space and velocity space cylindrical coordinates $\mathbf{r} = (\rho, r_\varphi, r_z)$ are used and $\mathbf{C} = (C_\perp, C_\psi, C_z)$.

The average macroscopic velocity of the gas $\bar{u}_z$ is expressed in terms of the perturbation functions $h_1(\rho, \mathbf{C})$ and $h_2(\rho, \mathbf{C})$ as

$$\bar{u}_z = -G_p \bar{U}_{1,z} + G_T \bar{U}_{2,z}, \tag{2}$$

$$\bar{U}_z = -\frac{2}{R_2^2 - R_1^2}\int_{R_1}^{R_2}(U_{1,z}(\rho) - G U_{2,z}(\rho))\rho \mathrm{d}\rho, \tag{3}$$

$$U_{i,z}(\rho) = \pi^{-3/2}\int \exp\left(-C^2\right) C_z h_i(\rho, \mathbf{C}) \mathrm{d}^3\mathbf{C}, \tag{4}$$

$$G = \frac{G_T}{G_p}.$$

Substituting (2) into (4) and taking into account that in the case of using the solid sphere model the sparsity parameter δ satisfies the ratio $\delta = 2(R'_2 - R'_1)p'{\beta'}^{1/2}{\mu'}^{-1}$ [12], we obtain the following expression for the Poiseuille number

$$\mathrm{P}_0 = \frac{2\delta}{\bar{U}_z}. \tag{5}$$

Let us introduce functions $Z_1 = Z_1(\rho, \zeta, C_\perp)$ and $Z_2 = Z_2(\rho, \zeta, C_\perp)$ as

$$Z_1(\rho, \zeta, C_\perp) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \exp(-C_z^2) C_z h(\rho, \mathbf{C}) dC_z,$$

$$Z_2(\rho, \zeta, C_\perp) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \exp(-C_z^2) C_z^3 h(\rho, \mathbf{C}) dC_z, \ \zeta = \cos\psi.$$

Then using Z_1, the component $U_{1,z}$ is written as follows

$$U_{1,z} = \frac{2}{\pi} \int_0^{+\infty} C_\perp \exp(-C_\perp^2) \int_{-1}^{1} \frac{1}{\sqrt{1-\zeta^2}} Z_1 d\zeta dC_\perp. \tag{6}$$

Function Z_1 can be found from the solution of the linearised BGK model of the kinetic Boltzmann equation [9]

$$\left(\frac{\partial Z_1}{\partial \rho} \zeta + \frac{\partial Z_1}{\partial \zeta} \frac{(1-\zeta^2)}{\rho} \right) C_\perp + \delta Z_1(\rho, \zeta, C_\perp) + \frac{1}{2} = \delta U_{1,z}(\rho), \ \zeta = \cos\psi, \tag{7}$$

using Maxwell's mirror-diffuse boundary condition

$$Z_1(R_i, \zeta, C_\perp) = (1-\alpha) Z_1(R_i, -\zeta, C_\perp), \ (-1)^i \zeta < 0, \ i = 1, 2. \tag{8}$$

To find $\overline{U}_{2,z}$, the Onsager ratio $\overline{U}_{2,z} = \overline{Q}_{1,z}$ is used [13], where $2G_p\overline{Q}_{1,z}$ is dimensionless heat flow due to the presence of the pressure gradient

$$\bar{Q}_{1,z} = \frac{2}{R_2^2 - R_1^2} \int_{R_1}^{R_2} q_{1,z}(\rho) \rho d\rho. \tag{9}$$

Here, $q_{1,z}$ is the dimensionless z-component of the heat flux vector

$$q_{1,z} = \frac{2}{\pi} \int_0^{+\infty} C_\perp \exp(-C_\perp^2) \int_{-1}^{1} \frac{1}{\sqrt{1-\zeta^2}} (C_\perp^2 Z_1 + Z_2) d\zeta dC_\perp - \frac{5}{2} U_{1,z}. \tag{10}$$

After the solution of the boundary value problem (7), (8) function Z_2 can be found from the equation [15]

$$\left(\frac{\partial Z_2}{\partial \rho} \zeta + \frac{\partial Z_2}{\partial \zeta} \frac{(1-\zeta^2)}{\rho} \right) C_\perp + \delta Z_2(\rho, \zeta, C_\perp) + \frac{3}{4} = \frac{3\delta}{2} U_{1,z}(\rho), \tag{11}$$

with the boundary condition

$$Z_2(R_i, \zeta, C_\perp) = (1-\alpha) Z_2(R_i, -\zeta, C_\perp), \ (-1)^i \zeta < 0, \ i = 1, 2. \tag{12}$$

2. Solution of the boundary value problem

Unknown functions $Z_1(\rho, \zeta, C_\perp)$ and $Z_2(\rho, \zeta, C_\perp)$ are represented as series in Chebyshev polynomials of the first kind $\{T_{k_i}\}$ $(i = \overline{1,3})$. Limiting the resulting decompositions to terms with numbers $k_i \leqslant n_i$ $(i = \overline{1,3})$, we have

$$Z_j(\rho, \zeta, C_\perp) = \mathbf{T_1}(x_1) \otimes \mathbf{T_2}(x_2) \otimes \mathbf{T_3}(x_3) \mathbf{A_j}, \ j = 1, 2, \tag{13}$$

where $x_1 = (2\rho - R_2 - R_1)/(R_2 - R_1)$, $x_2 = \zeta$, $x_3 = (C_\perp - 1)/(C_\perp + 1)$, $\mathbf{T_i}$ is the matrix of dimension $1 \times n_i'$ $(n_i' = n_i + 1, \ i = 1 \ldots 3)$

$$\mathbf{T_i}(x_i) = (T_0(x_i)\, T_1(x_i) \dots T_{n_i-1}(x_i)\, T_{n_i}(x_i))\,,$$

and $\mathbf{A_j}$ $(j = 1, 2)$ are unknown matrices of dimension $n'_1 n'_2 n'_3 \times 1$

$$\mathbf{A_j} = \left(a^{(j)}_{000}\, a^{(j)}_{001} \dots a^{(j)}_{n_1 n_2 n_3 - 1}\, a^{(j)}_{n_1 n_2 n_3}\right)^T,\ j = 1, 2.$$

Expression $\mathbf{T_1}(x_1) \otimes \mathbf{T_2}(x_2)$ means the Kronecker product of two matrices.

Zeros of polynomial $T_{n'_i}$ on the interval $[-1, 1]$

$$x_{i,k_i} = \cos\left(\frac{\pi(2n_i - 2k_i + 1)}{2(n_i + 1)}\right), \quad k_i = \overline{0, n_i},\ i = \overline{1, 3} \tag{14}$$

are selected as collocation nodes in (7) and (11) for x_i. The values of Chebyshev polynomials and their derivatives at points (14) are found according to the definition $T_{j_i}(x_i) = \cos(j_i \arccos x_i)$, where $x_i \in [-1, 1]$ [10].

To calculate integrals (6) and (10), the Clenshaw–Curtis method [14] and the recurrence relations [10] are used

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_i(x) = 2xT_{i-1}(x) - T_{i-2}(x), \quad i \geqslant 2.$$

Substituting (13) and (14) into (7) and (11), we obtain two systems of $n'_1 n'_2 n'_3$ equations in which equations at points $x_{1,0}$, x_{2,k_2} $(k_2 = n'_2/2 \dots n_2)$ are replaced with the equations arising from boundary conditions (8) and (12) for $x_2 > 0$

$$\mathbf{T_1}(-1) \otimes (\mathbf{T_2}(x_{2,k_2}) - (1-\alpha)\mathbf{T_2}(x_{2,n_2-k_2})) \otimes \mathbf{T_3}(x_{3,k_3})\mathbf{A_j} = 0, \quad k_3 = \overline{0, n_3},\ j = 1, 2.$$

Similarly, at the points x_{1,n_1} x_{2,k_2} $(k_2 = \overline{0, n'_2/2 - 1})$ equations corresponding to (8) and (12) for $x_2 < 0$ are

$$\mathbf{T_1}(1) \otimes (\mathbf{T_2}(x_{2,k_2}) - (1-\alpha)\mathbf{T_2}(x_{2,n_2-k_2})) \otimes \mathbf{T_3}(x_{3,k_3})\mathbf{A_j} = 0, \quad k_3 = \overline{0, n_3},\ j = 1, 2.$$

Here and below, it is assumed that n_2 is an odd number.

In order to reduce the computational error for $\bar{U}_{1,z}$ and $\bar{U}_{2,z}$ coefficients in (13) are expressed in terms of values of functions Z_1 and Z_2 at points (14). As this takes place, we have the following equalities at points (14)

$$\frac{2}{n'_i} \sum_{k_i=0}^{n_i}{}' T_{k_i}(x_{l_i}) T_{k_i}(x_{j_i}) = \delta_{l_i, j_i}, \quad l_i, j_i = \overline{0, n_i}, \quad i = \overline{1, 3},$$

where δ_{l_i, j_i} is the Kronecker symbol, and notation $\sum\limits_{k_i=0}^{n_i}{}'$ means the partial sum in which the first term is multiplied by 1/2.

Denoting matrices that contain values of functions Z_1 and Z_2 at points (14) as $\mathbf{Z_1}$ and $\mathbf{Z_2}$, we obtain

$$\mathbf{A_j} = \frac{8}{n'_1 n'_2 n'_3} \mathbf{J}^{T'} \otimes \mathbf{H}^{T'} \otimes \mathbf{L}^{T'} \mathbf{Z_j}, \quad j = 1, 2, \tag{15}$$

where $\mathbf{J}$, $\mathbf{H}$ and $\mathbf{G}$ are square matrices of size $n'_i \times n'_i$ with elements $J_{k_1+1, j_1+1} = T_{j_1}(x_{1,k_1})$, $H_{k_2+1, j_2+1} = T_{j_2}(x_{2,k_2})$, $L_{k_3+1, j_3+1} = T_{j_3}(x_{3,k_3})$, $j_i, k_i = \overline{0, n_i}$, $i = 1 \dots 3$. The symbol T means the transposition of matrices $\mathbf{J}$, $\mathbf{H}$ and $\mathbf{L}$. The prime symbol means that the first rows of matrices $\mathbf{J}^T$, $\mathbf{H}^T$ and $\mathbf{L}^T$ are multiplied by 1/2

Using (15), we obtain system of equations with respect to unknown matrices $\mathbf{Z_1}$ and $\mathbf{Z_2}$. They are found by the LU method. Next, using the obtained elements of matrices $\mathbf{Z_1}$ and $\mathbf{Z_2}$, $U_{1,z}(\rho)$ and $q_{1,z}(\rho)$ are restored

$$U_{1,z}(\rho) = \frac{8}{n_1' n_2' n_3'} \mathbf{T_1}\left(\frac{2\rho - R_2 - R_1}{R_2 - R_1}\right)\mathbf{J}^{T'} \otimes \mathbf{B_1}\mathbf{Z_1}, \tag{16}$$

$$q_{1,z}(\rho) = \frac{8}{n_1' n_2' n_3'} \mathbf{T_1}\left(\frac{2\rho - R_2 - R_1}{R_2 - R_1}\right)\mathbf{J}^{T'} \otimes \sum_{j=1,2} \mathbf{B_{3-j}}\mathbf{Z_j}, \tag{17}$$

where $\mathbf{B_j}$ is the block matrix of size $1 \times n_2' n_3'$ that consists of n_2'-identical blocks $\mathbf{K_j}\mathbf{L}^{T'}$ of dimension $1 \times n_3'$,

$$\mathbf{K_j} = 2\int_{-1}^{1} \frac{(1+x_3)^{i_j}}{(1-x_3)^{2+i_j}} \mathbf{T_3}(x_3) \exp\left(-\frac{(1+x_3)^2}{(1-x_3)^2}\right) dx_3, \quad j = 1,2, \quad i_1 = 1, \quad i_2 = 3.$$

Substituting (16) and (17) into (3) and considering that $u_{2,z} = q_{1,z}$, the values of the Poiseuille number can be calculated from (5). The variable parameters in this case are α, δ, G and $r = R_1'/R_2'$. Radii R_1 and R_2 are expressed in terms of r as

$$R_1 = \frac{r}{2(1-r)}, \quad R_2 = \frac{1}{2(1-r)}.$$

3. Analysis of the obtained results

Relationship between the Poiseuille number P_0 and δ for $r = R_1'/R_2' = 0.1, 0.5, 0.9$ and $G = 0$ are shown in Fig. 1 (a)(α =1) and in Fig. 1 (b) ($\alpha = 0.85$) at $n_1 = n_2$ =15, n_3 =11. Interpolation of values of the Poiseuille number P_0 (5) is performed on the basis of cubic splines with values of the sparsity parameter δ from 0 to 100. Curves 1–3 correspond to $r = 0.1, 0.5, 0.9$. The dots mark the values of P_0 from [7]. It is clear that results obtained in this paper based on the Chebyshev polynomials are in good agreement with [7]. The difference between results does not exceed 2 %. It should be noted that there is a rapid convergence of curves 2 and 3 for $r = 0.5$ and $r = 0.9$ with decreasing values of the tangential momentum accommodation coefficient of the gas molecules α. Next, the simulation results for $r = 0.1$ and $r = 0.9$ are presented.

To analyse the results obtained in the sliding and hydrodynamic modes, solution of the Navier–Stokes equation with boundary conditions of sliding and sticking were found. In the hydrodynamic limit ($\delta \to \infty$), we obtain from (7)

$$U_z(\rho) = \frac{\delta}{4}\left(R_1^2 \ln\left(\frac{R_2}{\rho}\right) + R_2^2 \ln\left(\frac{\rho}{R_1}\right) - \rho^2\right)\left(\ln\left(\frac{R_2}{R_1}\right)\right)^{-1}. \tag{18}$$

It corresponds to the solution of the Navier–Stokes equation [16]

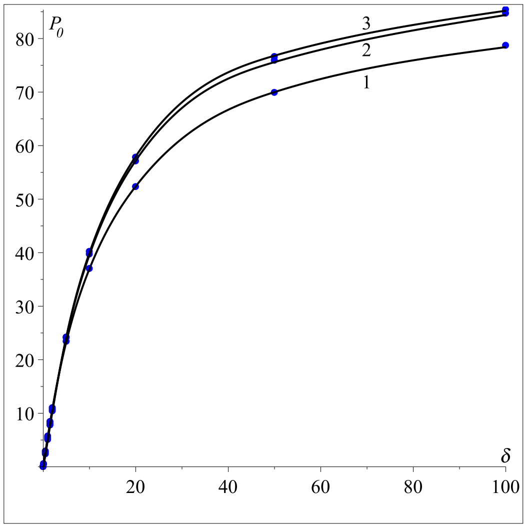
$$\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho \frac{\mathrm{d}U_z(\rho)}{\mathrm{d}\rho}\right) = -\delta, \tag{19}$$

with boundary conditions of adhesion on the inner and outer surfaces of the cylinders

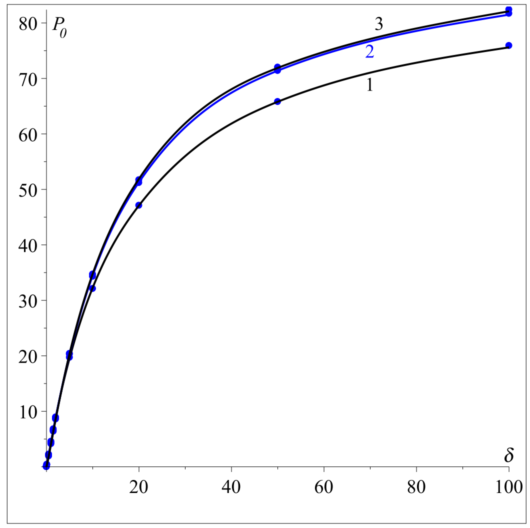
$$U_z(R_i) = 0, \quad i = 1,2.$$

Substituting (18) into (3) and (5) and considering that $\bar{U}_{2,z} = 0$, we obtain the following expression for the Poiseuille number

$$\mathrm{P}_0 = \frac{64(1-r)^2 \ln r}{(\ln r - 1)r^2 + \ln r + 1}. \tag{20}$$



(a)



(b)

Fig. 1. The relationship between P_0 and δ at $G = 0$ for $\alpha = 1$ (a) and $\alpha = 0.85$ (b): (1–3) corresponds to $r = 0.1$, 0.5 and 0.9

For $r \to 0$ we obtain from (20) $\mathrm{P}_0 = 64$ that corresponds to the laminar flow in the cylinder [11]. In the case of $r \to 1$, the value of P_0 tends to the value of 96 that is characteristic of the flow between two parallel planes [6].

In the sliding flow mode, boundary conditions for Navier–Stokes equation (19) are written in the form [12]

$$U_{1,z}(R_i) = (-1)^{i+1} \frac{\sigma_p}{\delta} \frac{\mathrm{d}U_z}{\mathrm{d}\rho}(R_i), \;\; i = 1, 2, \tag{21}$$

$$U_{2,z}(R_i) = \frac{\sigma_T}{2\delta}, \;\; i = 1, 2. \tag{22}$$

Here σ_p and σ_T are the coefficients of isothermal and thermal slip, respectively. For the BGK equations considered in the paper the relationship between σ_p, σ_T and α can be represented as [12, 17]

$$\begin{gathered} \sigma_p(\alpha) = \frac{2-\alpha}{\alpha} \left(\sigma_p(1) - 0.1211(1-\alpha)\right), \quad \sigma_p(1) = 1.016, \\ \sigma_T(\alpha) = 0.75 + 0.3993\alpha. \end{gathered} \tag{23}$$

Solving boundary value problems (19), (21) and (22), we find

$$\begin{aligned} U_{1,z}(\rho) = \Bigg[&\left(\rho^2 \ln r - R_1^2 \ln\left(\frac{\rho}{R_2}\right) - R_2^2 \ln\left(\frac{R_1}{\rho}\right) \right) R_1 R_2 \delta^2 + (R_1 + R_2)\, \rho^2 \sigma_p \delta + 2\sigma_p^2 \left(R_2^2 - R_1^2\right) + \\ &+ \left(R_1^3 + R_2^3 - R_1^2 R_2 \ln\left(\frac{R_2^2}{\rho^2}\right) - R_1 R_2^2 \ln\left(\frac{R_1^2}{\rho^2}\right) \right) \sigma_p \delta \Bigg] \Bigg[4(\sigma_p (R_1 + R_2) - R_1 \delta R_2 \ln r) \Bigg]^{-1}, \end{aligned} \tag{24}$$

$$U_{2,z}(\rho) = \frac{\sigma_T}{2\delta}. \tag{25}$$

One can see from (25) that component $\overline{U}_{2,z}$ does not depend on $r = R_1/R_2$.

Substituting (24) and (25) into (3), we find from (5) that

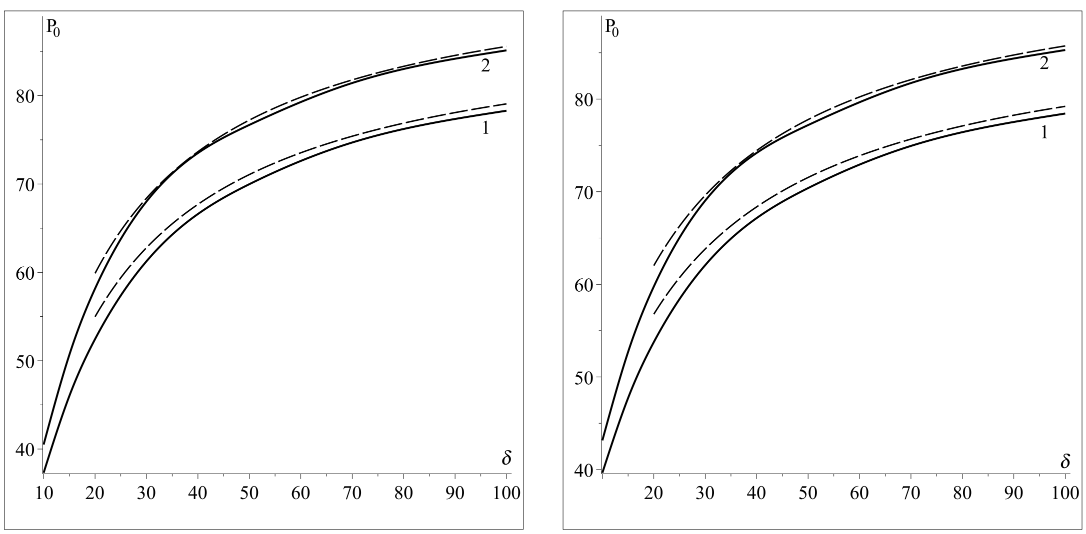
$$\mathrm{P}_0 = \frac{4\delta^2\beta_1}{\beta_2}, \tag{26}$$

where $\beta_1 = 32(\delta r \ln r + (2r^2 - 2)\sigma_p)(1 - r)^2$,

$$\beta_2 = 2(r^3(\ln r - 1) + r(\ln r + 1))\delta^3 + 4(r^4 + 4r^3(\ln r - 1) - 4r^2 \ln r + 4(\ln r + 1)r - 1)\sigma_p\delta^2 +$$

$$+32(r^4 - 2r^3 + 2r - 1)\delta\sigma_p^2 - \beta_1\sigma_T.$$

The results of calculation of the Poiseuille number P_0 at $\alpha = 1$ for $G = G_T/G_p = 0.1$ (a) and $G = 0.9$ (b) using the BGK model (curves 1 and 2 for $r = 0.1$ and 0.9, respectively) in comparison with results obtained on the basis of the Navier–Stokes equation with a sliding boundary condition (dashed lines) are shown in Fig. 2. To reconstruct $\overline{U}_{2,z}$ component using the BGK model the expression $2\delta/3$ [12] was used for the sparsity parameter, since this model of the kinetic Boltzmann equation leads to the value of the Prandtl number equal to 1. The difference in the results is less than 5 % at $\delta = 20$ and about 1 % at $\delta = 40$. It can be seen from Fig. 2 that with increasing values of δ and r the simulation results are slowly approaching the hydrodynamic limit. At $r = 0.1$, the value of the Poiseuille number calculated by formula (20) is $\mathrm{P}_0 = 89.4$, that is, there is a shift towards a flat flow.



(a) (b)

Fig. 2. The relationship between P_0 and δ for $\alpha = 1$, $G = 0.1$ (a) and $G = 0.9$ (b): (1 and 2) correspond to $r = 0.1$ and 0.9

Fig. 3 shows the results of interpolation of $\mathrm{P}_0(\delta)$ for $\alpha = 0.85$ with the same parameters as in Fig. 2. It can be seen from Fig. 3 that value of the Poiseuille number increases with an increase in the ratio of temperature and pressure gradients $G = G_T/G_p$. Note that correction in the second-order approximation for $\sigma_T(\alpha)$ in the form $\sigma_T(\alpha) = 0.75(1 + 0.5714\alpha - 0.0422\alpha^2)$ [18] does not significantly contribute to the values of the Prandtl number. Comparison of this coefficient with (23) shows that deviation for $\alpha = 1$ and 0.85 does not exceed 0.2%.

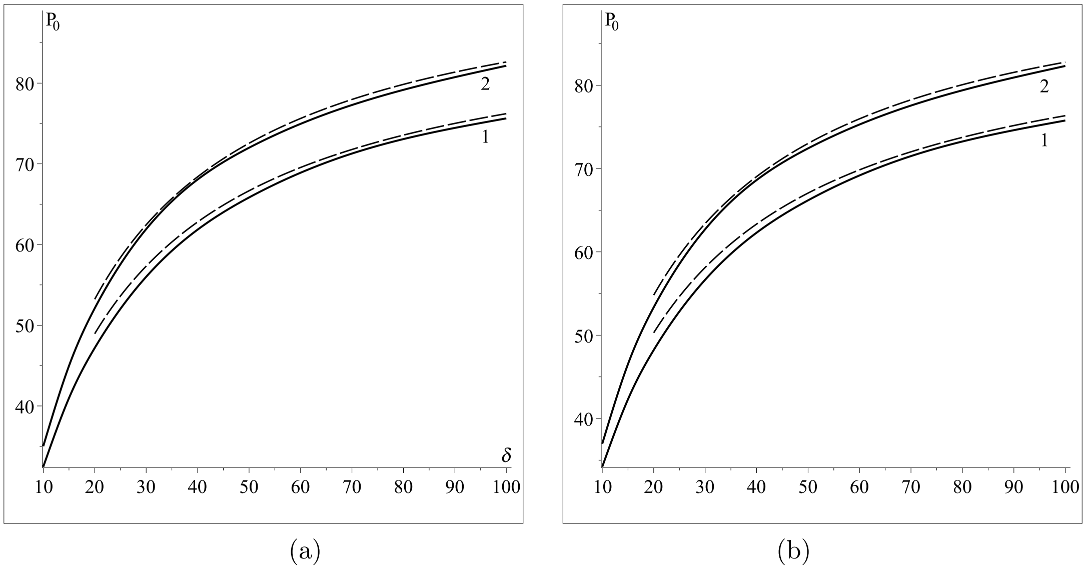


Fig. 3. The relationship between P_0 and δ for $\alpha = 0.85$, $G = 0.1$ (a) and $G = 0.9$ (b): (1 and 2) correspond to $r = 0.1$ and 0.9

Conclusion

The change in the value of the Poiseuille number in the channel formed by two concentric cylinders was studied with the use of Chebyshev polynomials of the first kind. It depends on rarefaction parameter, the tangential momentum accommodation coefficient of the gas molecules, the ratio of the radii of the cylinders and the gradient of temperature and pressure. It was shown that the value of the Poiseuille number increases with an increase in the ratio of dimensionless temperature and pressure gradients for any ratio of cylinder radii, the degree of gas rarefaction and the coefficient of accommodation of the tangential pulse by the channel walls. When the ratio of temperature and pressure gradients is close to zero the presented results correspond to the results characteristic of the isothermal flow of rarefied gas in a channel with the same configuration.

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Вычисление числа Пуазейля в кольцевой области при неизотермическом течении газа

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Аннотация. В статье рассматривается медленный продольный неизотермический поток газа в кольцевой области, обусловленный малыми перепадами давления и температуры на концах длинного микроканала. Предложен метод расчета значений числа Пуазейля в промежуточном режиме течения газа, основанный на решении модельного линеаризованного кинетического уравнения Бхатнагара-Гросса-Крука (БГК, BGK) с использованием полиномов Чебышева первого рода. Вычисленные значения сравниваются с аналогичными результатами, полученными с использованием аналитических решений уравнений Навье–Стокса с граничными условиями прилипания и скольжения. Анализируется влияние коэффициента аккомодации тангенциального импульса молекул и параметра разрежения газа на изменение числа Пуазейля при малых отношениях градиентов температуры и давления газа в канале.

Ключевые слова: полиномы Чебышева первого рода, метод коллокаций, неизотермическое течение газа в канале, число Пуазейля, кинетическое уравнение, модели граничных условий.