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Fantappiè G-transform of Analytic Functionals

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Abstract. To each analytic functional on the space $O'(\mathbb{C}^n)$, a function $f(z)$ holomorphic in a neighborhood of the origin and an entire function of exponential type $F(z)$ are associated so that the coefficients c_α of the power series expansion of $f(z)$ are given by values of $F(\alpha)$. We study the problem of finding a connection between the domain where the function $f(z)$ extends to and the growth of the function $F(z)$.

Keywords: analytic functionals, G-transform, Fantappiè transformation, entire functions, continuation of holomorphic functions.

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Introduction

Let the function

$$f(z) = \sum_{n=0}^{\infty} c_n z^n \tag{1}$$

be holomorphic in a neighborhood of the origin. In many branches of mathematics, the question arises of existence of the *coefficient function*, i.e. such a function $F(z)$ that

$$F(n) = c_n, \quad n = 0, 1, 2, \dots \tag{2}$$

and the relationship between the properties of functions $f(z)$ and $F(z)$.

There always exists an entire coefficient function $F(z)$ of exponential type. Indeed, if γ is a contour around the point $z = 0$ in the positive direction, then by the well-known formula

$$c_n = \frac{1}{2\pi i} \int_\gamma f(z) z^{-n-1}\, dz.$$

After the change $z = e^{-\zeta}$ the contour γ turns to a contour Γ connecting two points P and Q such that $\Im(Q - P) = 2\pi$ and we get the integral representation

$$c_n = \frac{1}{2\pi} \int_\Gamma f(e^\zeta) e^{n\zeta}\, d\zeta.$$

As is well known, the function

$$F(z) = \frac{1}{2\pi} \int_\Gamma f(e^\zeta) e^{z\zeta}\, d\zeta$$

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is an entire function of exponential type (see, for example, [1, Sec. 1] for which the equality (2) holds.

Recall that an entire function $F(z)$ is said to be of exponential type if there is an a such that for a sufficiently large z we have

$$|F(z)| < e^{a|z|}. \tag{3}$$

The lower bound H of such numbers a is called the type of the function $F(z)$.

A change of the contour γ leads to another function $F^*(z)$ with similar properties. If Γ and Γ^* are two such contours and Γ^* connects points P^* and Q^* for which $\Im(Q^* - P^*) = 2\pi$, then

$$F^*(z) - F(z) = B(z)\sin\pi z, \quad B(z) = \frac{e^{\pi}}{\pi}\int_P^{P^*} f(e^{\zeta})e^{z\zeta}\,d\zeta.$$

As noted by L. Bieberbach ([1, Sec. 1]), the question arises of finding an entire function of exponential type with the minimal growth, and the problem of finding a connection between the domain where the function $f(z)$ extends to and the growth of the function $F(z)$.

To formulate the result we introduce the indicator function $h(\varphi)$ of the entire function $F(z)$ of exponential type which characterizes the growth along the ray $\arg z = \varphi$:

$$h(\varphi) = \limsup_{r\to\infty}\frac{\ln|F(re^{i\varphi})|}{r}.$$

Let K be the indicator diagram of the function $F(z)$, i.e. a convex compact set for which $h(\varphi)$ is the support function.

Theorem 1. *Let $F(z)$ be an entire function of exponential type with the indicator diagram K. The function $f(z)$, for which $F(n) = c_n$, $n = 1, 2, \dots$ and c_0 is arbitrary, is holomorphic in the connected component of the complement of the set e^{-K} to the whole plane that contains the point $z = 0$. For the convergence radius R of the series* (1) *the following estimate is valid*

$$R \geqslant e^{-h(0)}. \tag{4}$$

The proof of this theorem as well as a number of other results related to the case of entire functions of coefficients $F(z)$ of exponential type can be found in the book [1]. A closer connection between the properties of the function $f(z)$ and the entire function of coefficients $F(z)$ of exponential type can be formulated as conditions on a compact set K.

Theorem 2. *Let K be a closed bounded convex set. In order for the function* (1) *to be holomorphic in the connected component of the complement of the set e^{-K} to the whole plane that contains the point $z = 0$ and not holomorphic in any larger domain of the same kind, it is necessary and sufficient that the width of the set K in the direction of the imaginary axis is less than 2π and there exists an entire function of the coefficients $F(z)$ of exponential type with the indicator diagram K.*

If an entire function of the coefficients $F(z)$ with the specified properties exists, then it is unique,

$$R = e^{-h(0)}, \tag{5}$$

and the function $f(z)$ itself can be analytically continued to an infinite point along one of the radii and in a neighborhood of infinity its series decomposition is

$$f(z) = c_0 - \sum_{n=1}^{\infty} F(-n)z^{-n}.$$

Further application of the theory of analytic functionals and their G-transformation allowed more universal methods to study the question of the continuation of the series (1) depending on the nature of growth of functions $F(z)$ in the one-dimensional case and study the case of several dimensions. The main references are [2–5]

1. Laplace and Avanissan-Gay transforms of analytic functionals

Definition 1. *The elements in the dual space $\mathcal{O}'(\mathbb{C}^n)$ of the space $\mathcal{O}(\mathbb{C}^n)$ of entire functions, equipped with the topology of uniform convergence on compact sets, are called analytic functionals. An analytic functional T is said to be carried by a compact set K if for every neighborhood ω of K there is a constant C_ω such that*

$$|T(\varphi)| \leqslant C_\omega \sup_\omega |\varphi|, \quad \varphi \in \mathcal{O}(\mathbb{C}^n). \tag{6}$$

If K is compact, we denote by $O'(\mathbb{C}^n, K)$ the space of all analytic functionals carried by the set K.

Definition 2. *If $T \in \mathcal{O}'(\mathbb{C}^n)$, we define its Laplace transform by*

$$F_T(\zeta) = T_z(e^{(\zeta,z)}), \quad \zeta \in \mathbb{C}^n, \quad (\zeta, z) = \zeta_1 z_1 + \cdots + \zeta_n z_n.$$

The Laplace transform is an entire analytic function of ζ. From (6) we obtain the estimate

$$|F_T(\zeta)| \leqslant C_\omega \exp(\sup_{z\in\omega} \Re(z,\zeta)).$$

Set

$$H_K(\zeta) = \sup_{z\in K} \Re(z,\zeta).$$

If K is convex we have

$$K = \{z \colon \Re(z,\zeta) \leqslant H_K(\zeta), \zeta \in \mathbb{C}^n\}$$

otherwise

$$K \subset \{z \colon \Re(z,\zeta) \leqslant H_K(\zeta), \zeta \in \mathbb{C}^n\}.$$

The following Ehrenpreis-Martineau theorem characterizes the Laplace transform of analytic functionals (see [6]).

Theorem 3. *If $T \in \mathcal{O}'(\mathbb{C}^n, K)$, then $F_T(\zeta)$ is an entire analytic function and for every $\delta > 0$ there is a constant C_δ such that*

$$|F_T(\zeta)| \leqslant C_\delta \exp(H_K(\zeta) + \delta|\zeta|). \tag{7}$$

Conversely, if K is a convex compact set and $F(\zeta)$ an entire function satisfying (7) for every $\delta > 0$, there exists a unique *analytic functional $T \in \mathcal{O}'(\mathbb{C}^n, K)$ such that the Laplace transform of T is $F(\zeta)$.*

Let $U = \{z \in \mathbb{C}\colon -\pi < \Im z < \pi\}$ and $\Omega = U^n$. If $K \subset \Omega$ is a compact set and K_j its j-projection on $\mathbb{C}$, $(z_1, \dots, z_n) \to z_j$, then we denote by $e^{-K} = \{e^{-z} : z \in K\}$ and we put

$$\Omega(K) = \prod_{j=1}^{n} [\mathbb{C} \setminus \exp(-K_j)],$$

$$\widetilde{\Omega}(K) = \prod_{j=1}^{n} [\widetilde{\mathbb{C}} \setminus \exp(-K_j)],$$

where $\widetilde{\mathbb{C}}$ is a compactification of $\mathbb{C}$. Let $\mathcal{O}_0(\Omega(K))$ denote the space of functions holomorphic in $\Omega(K)$ and continuous in $\widetilde{\Omega}(K)$ vanishing in $\widetilde{\Omega}(K) \setminus \Omega(K)$.

If $T \in \mathcal{O}'(\mathbb{C}^n, K)$, $K \subset \Omega$ we denote by $G(T)$ its Avanissan–Gay transform (G-transform):

$$G(T)(z) := T_\zeta\left(\prod_{j=1}^{n} \frac{1}{1 - z_j \exp \zeta_j}\right).$$

Proposition 1. *Let $T \in \mathcal{O}'(\mathbb{C}^n, K)$, $K \subset \Omega$. Then*

1. *$G(T)(z) \in \mathcal{O}_0(\Omega(K))$.*
2. *In a neighborhood of the origin, we have*

$$G(T)(z) = \sum_{\alpha \in (\mathbb{N} \cup \{0\})^n} F_T(\alpha) z^\alpha$$

while the following expansion is valid at infinity:

$$G(T)(z) = (-1)^n \sum_{\alpha \in \mathbb{N}^n} F_T(-\alpha) z^{-\alpha}.$$

3. *The map $G : \mathcal{O}'(K) \to H_0(\Omega(K))$, given by $T \mapsto G(T)$, is an injection, but in general it is not a surjection. When $K \subset \Omega$ is a direct product, then the G-transform gives an isomorphism between $\mathcal{O}'(K)$ and $H_0(\Omega(K)$.*

As a consequence of the these results on G-transform, we obtain an important uniqueness theorem for entire functions of exponential type.

Proposition 2. *A necessary and sufficient condition for two analytic functionals $T_1, T_2 \in \mathcal{O}'(\Omega)$ to coincide is the identity $F_{T_1}(\alpha) = F_{T_2}(\alpha)$ for every $\alpha \in \mathbb{N}^n$.*

G-transforms of analytic functionals are similar to their Cauchy transforms and proofs of the main results use duality of spaces of functionals and spaces of holomorphic functions on suitable sets. Because of the structure of the Cauchy kernel in several dimensions similar results are possible only in the case of compact sets given as products of plane convex compacts.

In this paper we propose to consider a projective analogue of G-transform associated with the Fantappiè transform of analytic functionals and to use a general form of analytic functionals for convex domains and compacts, which was first established in the works of L. Aizenberg [13] and A. Martineau.

2. Sets and maps in projective space

Suppose, as usual, $\mathbb{C}^n$ is the space of row-vectors of dimension n with elements from the field $\mathbb{C}$. The complex projective space $\mathbb{CP}^n$ is defined as the set of one-dimensional linear subspaces (or what is the same, complex lines passing through 0) in $\mathbb{C}^{n+l}$. We denote by p the map of the set $\mathbb{C}^{n+1} \setminus \{0\}$ into $\mathbb{CP}^n$, which assigns to a point the subspace containing it. Projecting the open sets from $\mathbb{C}^{n+l} \setminus 0$, this map gives a topology in $\mathbb{CP}^n$. As is familiar, p is continuous and $\mathbb{CP}^n$ is compact in this topology.

The complex lines lying in the plane

$$\{z \in \mathbb{C}^{n+1} \colon z_{n+1} = 0\}$$

are called infinite points of $\mathbb{CP}^n$, and the other complex lines are called finite points. The map p maps the plane

$$\{z \in \mathbb{C}^{n+1} \colon z_{n+1} = 1\}$$

homomorphically onto the set of finite points of $\mathbb{CP}^n$. Each point $z = (z_1, \dots, z_n)$ can be identified with the corresponding finite point $p(z_1, \dots, z_n, 1)$, and $\mathbb{C}^n$ is represented as an open, everywhere dense set in the compact $\mathbb{CP}^n$.

As a rule, we shall define continuous functions on open sets in $\mathbb{CP}^n$, defining their values (by a formula) only on everywhere dense subsets.

Let D be an open set in $\mathbb{CP}^n$. The space $\mathcal{O}(D)$ consists of functions which are holomorphic in D, and convergence in $\mathcal{O}(D)$ by definition means uniform convergence on each compact subset of D. We define $\mathcal{O}_0(D)$ as the closed subspace in $\mathcal{O}(D)$, consisting of functions satisfying the following condition: for any $z \in D \setminus \mathbb{C}^n$ one can find a neighborhood U of it in $\mathbb{CP}^n$, in which f is holomorphic and $f(\zeta) = O(|\zeta|^{-n}), \zeta \to \infty$.

By a plane of dimension k in $\mathbb{CP}^n$ is meant the image of a $(k+1)$-dimensional linear subspace of $\mathbb{C}^{n+l}$ under the map p. In particular, the infinite points of $\mathbb{CP}^n$ form a hyperplane in it, and the closure of a complex line from $\mathbb{C}^n$ is a line.

We denote by $M(m, n)$ the set of all matrices of size $m \times n$ over the field $\mathbb{C}$. In particular, $\mathbb{C}^n = M(1, n)$. For any matrix $A \in M(m, n)$ we shall denote by A' the transposed matrix. Then for $z \in \mathbb{C}^n$ and $\zeta \in \mathbb{C}^n$ we have $z\zeta' = z_1\zeta_1 + \cdots + z_n\zeta_n$. To each k-dimensional plane $\alpha \subset \mathbb{CP}^n$ there corresponds in a one-to-one fashion its dual $(n - k - 1)$-dimensional plane

$$\alpha' = \{\zeta \in \mathbb{CP}^n \colon z\zeta' = 0 \ \text{ for all } \ z \in \alpha\}.$$

In particular, points and hyperplanes are the duals of each other.

For an arbitrary $M \subset \mathbb{CP}^n$ the adjoint set of

$$M^* := \{\zeta \in \mathbb{CP}^n \colon z_1\zeta_1 + \cdots + z_{n+1}\zeta_{n+1} \neq 0 \ \text{ for all } \ z \in M\}$$

consists of points dual to hyperplanes not passing through M. One can also consider M^* as the complement of the union of hyperplanes dual to points of M. Obviously if $M_1 \subset M_2$, then $M_1^* \supset M_2^*$. The set $M^{**} = (M^*)^*$ always contains M.

The condition $M^{**} = M$ means that through each point $z \notin M$ there passes a hyperplane not passing through M, i.e., that M is Martineau linearly convex. It is known (cf., e.g., [21]), that if M is open, then M^* is compact, and if M is compact, then M^* is open.

Let D be an open set in $\mathbb{CP}^n$. The space $\mathcal{O}(D)$ consists of functions which are holomorphic in D, and convergence in $\mathcal{O}(D)$ by definition means uniform convergence on each compact subset of D. We define the space $\mathcal{O}(K)$ for K as the inductive limit of the spaces $\mathcal{O}(D)$ over all open $D \supset K$.

We define $\mathcal{O}_0(D)$ as the closed subspace in $\mathcal{O}(D)$, consisting of functions satisfying the following condition: for any $z \in D \setminus \mathbb{C}^n$ one can find a neighborhood U of it in $\mathbb{CP}^n$, in which f is holomorphic and $f(\zeta) = O(|\zeta|), \zeta \to \infty$. We denote by $\mathcal{O}_0^*(D)$ the space of continuous linear functionals on $\mathcal{O}_0(D)$.

Definition 3. *Let E be an open or compact set in $\mathbb{CP}^n$ and assume that E^* is non-empty. We define the Fantappiè transform*

$$\mathcal{F} : \mathcal{O}_0^*(E) \to \mathcal{O}_0(E^*) \quad by \quad \mathcal{F}\mu(z) = \mu\left(\frac{1}{z_1\zeta_1 + \cdots + z_{n+1}\zeta_{n+1}}\right).$$

We call E strongly linearly convex if this correspondence establishes an isomorphism of the spaces $\mathcal{O}_0^*(E)$ and $\mathcal{O}_0(E^*)$.

It is known ([11, 17]), cf. also [12, p. 237]), that all convex compacta and all convex domains are strongly linearly convex. Aizenberg showed in [13] that all Martineau linearly convex domains with sufficiently smooth boundary, their closures, and also domains and compacta which can be approximated by such sets are strongly linearly convex.

The main references are [7, 8, 10–15]

3. Fantappiè G-transform of analytic functionals

Definition 4. *If $T \in \mathcal{O}'(\mathbb{C}^n, K)$, $K \subset \Omega$ we denote by $FG(T)$ its Fantappiè G-transform (FG-transform):*

$$FG(T)(z) := T_\zeta\left(\frac{1}{1 + z_1 e^{\zeta_1} + \cdots + z_n e^{\zeta_n}}\right).$$

Theorem 4. *Let $T \in \mathcal{O}'(\mathbb{C}^n, K)$, $K \subset \Omega$. Then*

1. *$FG(T)(z) \in \mathcal{O}_0((e^K)^*)$.*
2. *In a neighborhood of the origin we have*

$$FG(T)(z) = \sum_{|\alpha| \geqslant 0} (-1)^{|\alpha|} \frac{\alpha!}{|\alpha|!} F_T(\alpha) z^\alpha. \tag{8}$$

3. *The map $FG : \mathcal{O}'(\mathbb{C}^n, K) \to \mathcal{O}_0\left(((e^{-K})^*)\right)$ given by $T \longmapsto FG(T)$ is an injection, but in general it is not a surjection. When e^{-K} is a convex compact set the FG-transform gives an isomorphism between $\mathcal{O}'(\mathbb{C}^n, K)$ and $O_0\left((e^K)^*\right)$.*

Proof.

1. Let $T \in \mathcal{O}'(\mathbb{C}^n, K)$. Since the space $\mathcal{O}(\mathbb{C}^n)$ is dense in $\mathcal{O}(\Omega)$, there is an extension of T to the analytic functional $S \in \mathcal{O}'(\Omega)$ which is carried by the set K.

   Consider a biholomorphic mapping $(\zeta_1, \ldots, \zeta_n) \mapsto (w_1, \ldots, w_n) = \left(e^{\zeta_1}, \ldots, e^{\zeta_n}\right)$ of the set Ω to the set $(\mathbb{C} \setminus (-\infty, 0])^n$. Under this mapping the functional $S \in \mathcal{O}'(\mathbb{C}^n, K)$ corresponds to the functional $S_1 \in \mathcal{O}'((\mathbb{C} \setminus (-\infty, 0])^n)$ carried by the set e^K, and more precisely by the set $(e^K)^{**} \supset e^K$. The restriction of the functional S_1 to the space $\mathcal{O}(((e^K)^{**})$ defines some functional $\mu \in \mathcal{O}'((e^k)^{**})$.

The FG-transform of the functional T takes the form of the Fantappiè transform of the functional μ:

$$G(T)(z) = T_\zeta\left(\frac{1}{1+z_1e^{\zeta_1}+\dots+z_ne^{\zeta_n}}\right) = \mu\left(\frac{1}{1+z_1w_1+\dots+z_nw_n}\right). \tag{9}$$

But according to the known properties of the Fantappiè transform, the function $G(T)(z)$ is holomorphic in $\mathcal{O}_0\left((e^K)^*\right)$.

2. Consider a series expansion of the kernel

$$\frac{1}{1+z_1e^{\zeta_1}+\dots+z_ne^{\zeta_n}} = \sum_{|\alpha|\geqslant 0}(-1)^{|\alpha|}\frac{|\alpha|!}{\alpha!}z^\alpha(e^\zeta)^\alpha, \tag{10}$$

$z^\alpha(e^\zeta)^\alpha = z_1^{\alpha_1}\dots z_n^{\alpha_n}(e^{\zeta_1})^{\alpha_1}\dots(e^{\zeta_n})^{\alpha_n}$.

For z in a sufficiently small neighborhood of the origin and for all ζ in some neighborhood U of the compact K, each term of the series (10) is majorized by a term of the convergent multiple geometric series

$$\sum_{|\alpha|\geqslant 0}(-1)^{|\alpha|}\frac{|\alpha|!}{\alpha!}q^\alpha = \frac{1}{1+q_1+\dots+q_n}.$$

It follows that the series (9) converges uniformly in $\zeta\in U$ and we get

$$G(T)(z) = T_\zeta\left(\frac{1}{1+z_1e^{\zeta_1}+\dots+z_ne^{\zeta_n}}\right) = T_\zeta\left(\sum_{|\alpha|\geqslant 0}(-1)^{|\alpha|}\frac{|\alpha|!}{\alpha!}z^\alpha(e^\zeta)^\alpha\right) =$$
$$= \sum_{|\alpha|\geqslant 0}(-1)^{|\alpha|}\frac{|\alpha|!}{\alpha!}z^\alpha T_\zeta\left(e^{(\zeta,\alpha)}\right) = \sum_{|\alpha|\geqslant 0}(-1)^{|\alpha|}\frac{|\alpha|!}{\alpha!}F_T(\alpha)z^\alpha.$$

3. The map $FG : \mathcal{O}'(\mathbb{C}^n, K) \to \mathcal{O}_0\left(((e^{-K})^*)\right)$ is an injection.

Let $T\in\mathcal{O}'(\mathbb{C}^n,K)$ and $FG(T)(z)=0$. The expansion (8) implies that $F_T(\alpha)=0$ for all α. According to Proposition 2 we have $T=0$.

When e^{-K} is a convex compact set the FG-transform gives an isomorphism between $\mathcal{O}'(\mathbb{C}^n,K)$ and $O_0\left((e^K)^*\right)$. This follows from the following sequence of isomorphisms

$$\mathcal{O}'(\mathbb{C}^n,K)\simeq\mathcal{O}'(K)\simeq\mathcal{O}'(e^K)\simeq O_0\left((e^K)^*\right).$$

The last isomorphism is valid due to the strong linear convexity of the compact set e^K. □

Example. Consider the function

$$f(z) = \frac{1}{1+z_1+\dots+z_n} = \sum_{|\alpha|\geqslant 0}(-1)^{|\alpha|}\frac{|\alpha|!}{\alpha!}z^\alpha.$$

Here $F(\alpha)\equiv 1$ for all $\alpha\in\mathbb{N}^n$. Take $T=\delta_0$. This analytic functional is the only one for which the Laplace transform $F_T(\alpha)\equiv 1$. According to Theorem 4 the function $f(z)$ extends to the domain

$$(e^{-K})^* = \{(1,\dots,1)\}^* = \{z : 1+z_1+\dots+z_n\neq 0\},$$

which coincides with the domain of holomorphicity of the function $f(z)$.

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G-преобразование Фантаппье аналитических функционалов

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Аннотация. С каждым аналитическим функционалом пространства $O'(\mathbb{C}^n)$ ассоциируются голоморфная в окрестности начала координат функция $f(z)$ и целая функция экспоненциального типа $F(z)$ таким образом, что коэффициенты c_α разложения функции $f(z)$ определяются значениями $F(\alpha)$. Изучается задача о нахождении связи между областью, в которую продолжается функция $f(z)$, и ростом функции $F(z)$.

Ключевые слова: аналитические функционалы, G-преобразование, целые функции, продолжение голоморфных функций.