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## Supercapacitor as a Buffer Electrical Source

## for Induction Motor

Ruslan F. Saifulin*<br>Karaganda Technical University Karaganda, Kazakhstan

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#### Abstract

Mathematical substitutions in the equations and the used methods have shown the adequacy and correctness of their use. Analysis of the data, obtained from simulation modeling of the developed model, showed small discrepancies with the standard block from the Simulink library. The use of the obtained mathematical equations, as well as the model assembled in Matlab Simulink, will make it possible to qualitatively evaluate the operation of an induction drive in statics and dynamics. This model will be used in future research, including the creation of a buffer power source based on a supercapacitor for an induction electric drive.


Keywords: electric drive, modeling, Matlab Simulink, supercapacitor, math model, simulation.

# Суперконденсатор <br> как буферный источник электроэнергии <br> <br> Для асинхронного двигателя 

 <br> <br> Для асинхронного двигателя}

Р. Ф. Сайфулин<br>Карагандинский технический университет<br>Казахстан, Караганда


#### Abstract

Аннотация. Математические замены в уравнениях и используемые методы показали адекватность и правильность их применения. Анализ данных, полученных при имитационном моделировании


[^0]разработанной модели, показал небольшие расхождения со стандартным блоком из библиотеки Simulink. Полученные математические уравнения, а также модель, собранная в Matlab Simulink, позволят качественно оценить работу асинхронного привода в статике и динамике. Эта модель будет использована в дальнейших исследованиях, в том числе при создании буферного источника питания на основе суперконденсатора для асинхронного электропривода.

Ключевые слова: электропривод, моделирование, Matlab Simulink, суперконденсатор, математическая модель, симуляция

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## Introduction

Mathematical substitutions in the equations, as well as the used methods have shown the adequacy and correctness of their use. Analysis of the data, obtained from simulation modeling of the developed model, showed small discrepancies with the standart block from the Simulink library.

The use of the obtained mathematical equations, as well as the model assembled in Matlab Simulink, will make it possible to qualitatively evaluate the operation of an asynchronous drive in statics and dynamics.

This model will be used in the future research, including the creation of a buffer power source based on a supercapacitor for an asynchronous electric drive.

The electromechanical properties of an asynchronous electric motor are most easily and conveniently studied using mathematical modeling, which determines the relevance of the topic accepted for consideration.

At present, the development of mathematical methods for the study of electrical machines is associated with the widespread use of computers, which makes it possible to implement the most complete models of transient processes with a minimum number of assumptions. The essence of the methods consists in the development system of a model and their implementation on a computer in the form of software systems for carrying out computational experiments in any possible conditions for the functioning of electrical machines. Possessing the simplicity of varying the structure and parameters of the design scheme, the mathematical model, with an appropriate level of adequacy, makes it possible to obtain, in the course of computational experiments, the necessary information for the development and design of electrical machines, their control and protection systems. However, as you know, the complexity of the phenomena, occurring in electric machines of alternating current during transient processes, makes their mathematical description and study without a number of simplifying assumptions practically impossible. The desire to take into account the main factors that determine the properties of the machine, and neglect the secondary factors leads to the consideration of an idealized electrical machine. Such a machine is usually characterized by the absence of saturation, hysteresis and eddy currents in the magnetic circuit, the absence of current displacement in the winding conductors, complete symmetry of the stator windings and a number of other assumptions. [1] These are the assumptions that are made when simulating the operation of an asynchronous electric motor in Matlab Simulink when choosing default blocks from the Simscape/Machines library. For more accurate results of electromechanical processes, flowing in an asynchronous electric motor, it is proposed to use a model of an electric motor in a rotating coordinate system. At the same time, there are
various ways of calculating this model. [2,3,4,5,6,7] However, these methods use vector multiplication, Euler transformations, and the substitutions that are made when calculating differential equations lead to a large number of terms in the equation. Therefore, this article proposes the calculation of the asynchronous motor model in a rotating coordinate system during the transition from a three-phase system to a two-phase one.

## Mathematical calculation of the induction drive model

Initially, the induction drive is a three-phase electric machine with an implicit-pole rotor, it is proposed to simplify this model to a two-phase one. To simplify the mathematical description of induction drive, the space vector method turned out to be suitable. The method allows linking the rotor flux linkage equations into a single system with vector state variables. The essence of the method is that the instantaneous values of symmetric three-phase state variables (voltage, currents, flux linkage) can be mathematically transformed so that they are represented by one space vector. We represent a system of equations with vector state variables for the case with an arbitrary orientation of the coordinate system.

$$
\begin{align*}
& \vec{U}_{S}=r_{S} \overrightarrow{i_{S}}+\frac{d \overrightarrow{\Psi_{S}}}{d t}+j \alpha_{k} \overrightarrow{\Psi_{S}}  \tag{1.1}\\
& \vec{U}_{R}=r_{R} \overrightarrow{i_{R}}+\frac{d \overrightarrow{\Psi_{R}}}{d t}+j\left(\alpha_{k}-p \vartheta_{m}\right) \overrightarrow{\Psi_{R}}  \tag{1.2}\\
& \overrightarrow{\Psi_{S}}=x_{S} \overrightarrow{i_{S}}+x_{m} \overrightarrow{i_{R}}  \tag{1.3}\\
& \overrightarrow{\Psi_{R}}=x_{R} \overrightarrow{i_{R}}+x_{m} \vec{i}_{S}  \tag{1.4}\\
& m=k \operatorname{Mod}\left(\overrightarrow{\Psi_{i}} * \overrightarrow{i_{k}}\right)  \tag{1.5}\\
& \overrightarrow{T_{m}} \frac{d \vartheta_{m}}{d t}=m m_{c} \tag{1.6}
\end{align*}
$$

Here $\vec{U}_{S}, \vec{U}_{R}, \overrightarrow{i_{S}}, \overrightarrow{i_{R}}, \overrightarrow{Y_{S}}$ and $\overrightarrow{\Psi_{R}}$ are two-element vectors of voltages, currents and flux linkages, presented in an arbitrarily oriented orthogonal (two-phase) coordinate system in the form of components along the coordinate axes. The variable $\omega_{k}$ is used to set an arbitrary frequency of rotation of the coordinate system. The auxiliary matrix constant $j$ serves to "flip" the components of vector variables and simplifies the form of writing the system of equations. $r_{S}, r_{R}-$ Resistance of stator and rotor, m - torque.

Expanding the content of space vectors, the flux linkage equations 1.3, 1.4 are substituted into the stator and rotor voltage formulas, while equating 1.2 to zero, since the electric motor is short-circuited.

$$
\begin{align*}
& \vec{U}_{S}=r_{S} \overrightarrow{i_{S}}+\frac{d\left(x_{S} \overrightarrow{i_{S}}+x_{m} \overrightarrow{i_{R}}\right)}{d t}+j \alpha_{k}\left(x_{S} \overrightarrow{i_{S}}+x_{m} \overrightarrow{i_{R}}\right)  \tag{2}\\
& \vec{U}_{R}=r_{R} \overrightarrow{i_{R}}+\frac{d\left(x_{R} \overrightarrow{i_{R}}+x_{m} \overrightarrow{i_{S}}\right)}{d t}+j\left(\alpha_{k}-p \vartheta_{m}\right)\left(x_{R} \overrightarrow{\vec{i}_{R}}+x_{m} \overrightarrow{\vec{i}_{S}}\right)=0 \tag{3}
\end{align*}
$$

The change in the rotor current vector is expressed from equation 3

$$
\begin{equation*}
\frac{d \overrightarrow{i_{R}}}{d t}=-\frac{r_{R} \overrightarrow{i_{R}}}{x_{R}} \frac{x_{m}}{x_{R}} \frac{d \overrightarrow{i_{S}}}{d t}-j \alpha_{k} \overrightarrow{i_{R}}-j \alpha_{k} \frac{x_{m}}{x_{R}} \overrightarrow{i_{S}}+j p \vartheta_{m} \frac{\overrightarrow{\Psi_{R}}}{x_{R}} \tag{4}
\end{equation*}
$$

The rotor current vector is expressed from equation 1.4 and then substituted into equation 4

$$
\begin{align*}
& \overrightarrow{i_{R}}=\frac{\overrightarrow{\Psi_{R}}-x_{m} \overrightarrow{i_{S}}}{x_{R}}  \tag{5}\\
& \frac{d \overrightarrow{i_{R}}}{d t}=-\frac{r_{R} x_{m} \overrightarrow{\Psi_{R}}}{x_{R}^{2}}+\frac{r_{R} x_{m}^{2} \overrightarrow{i_{S}}}{x_{R}^{2}}-\frac{x_{m}^{2}}{x_{R}} \frac{\overrightarrow{d_{S}}}{d t}-j \alpha_{k} \frac{x_{m} \overrightarrow{\Psi_{R}}}{x_{R}}+j p \vartheta_{m} \frac{x_{m} \overrightarrow{\Psi_{R}}}{x_{R}} \tag{6}
\end{align*}
$$

Equations 5 and 6 are inserted into equation 2 to calculate the stator voltage vector

$$
\begin{equation*}
\vec{U}_{S}=r_{S} \overrightarrow{i_{S}}+\frac{x_{S} d \overrightarrow{i_{S}}}{d t}-\frac{r_{R} x_{m} \overrightarrow{\Psi_{R}}}{x_{R}^{2}}+\frac{r_{R} x_{m}^{2} \vec{i}_{S}}{x_{R}^{2}}-\frac{x_{m}^{2}}{x_{R}} \frac{\overrightarrow{i_{S}}}{d t}+j p \vartheta_{m} \frac{x_{m} \overrightarrow{Y_{R}}}{x_{R}}+j \alpha_{k} x_{S} \overrightarrow{i_{S}-} \tag{7}
\end{equation*}
$$

The following replacements are introduced

$$
\begin{align*}
& r=r_{S}+k_{R}^{2} r_{R}  \tag{8.1}\\
& k_{R}=\frac{x_{m}}{x_{R}}  \tag{8.2}\\
& x_{S}^{\prime}=x_{S^{-}} \frac{x_{m}^{2}}{x_{R}}  \tag{8.3}\\
& T_{R}=\frac{X_{R}}{r_{R}} \tag{8.4}
\end{align*}
$$

Let's describe the vectors of stator and rotor voltages with introduced substitutions

$$
\begin{align*}
& \vec{U}_{S}=r_{S} \vec{i}_{S}+\frac{x_{S}^{\prime} d \overrightarrow{i_{S}}}{d t}+j \alpha_{k} x_{S}^{\prime} \overrightarrow{i_{S}}-\frac{k_{R} \overrightarrow{\Psi_{R}}}{T_{R}}+j p \vartheta_{m} k_{R} \overrightarrow{\Psi_{R}}  \tag{9}\\
& \vec{U}_{R}=-k_{R} r_{R} \overrightarrow{i_{S}}+\frac{\overrightarrow{\Psi_{R}}}{T_{R}}+\frac{d \overrightarrow{\Psi_{R}}}{d t}+j\left(\alpha_{k}-p \vartheta_{m} \overrightarrow{\Psi_{R}}=0\right. \tag{10}
\end{align*}
$$

In the case of coordinates orientation along the stator flux linkage, the moment can be expressed from equation 1.5 as

$$
\begin{equation*}
m=\overrightarrow{\Psi_{S \alpha} i_{S \beta}}-\overrightarrow{\Psi_{S \beta} i_{S \alpha}} \overrightarrow{ } \tag{11}
\end{equation*}
$$

In this case, the stator flux linkage vector from equation 1.3 is transformed by replacing the rotor current vector from equation 5

$$
\begin{equation*}
\overrightarrow{\Psi_{S}}=x_{S} \overrightarrow{i_{S}}+x_{m}\left(\frac{\overrightarrow{\Psi_{R}}-x_{m} \overrightarrow{i_{S}}}{x_{R}}\right) \tag{12}
\end{equation*}
$$

And this equation 12 of the stator flux linkage vector can now be written along the axes, taking into account the replacements 8.1-8.4

$$
\begin{align*}
& \overrightarrow{\Psi_{S \alpha}}=x_{S}^{\prime} \overrightarrow{i_{S \alpha}}+k_{R} \overrightarrow{\Psi_{R \alpha}}  \tag{13}\\
& \overrightarrow{\Psi_{S \beta}}=x_{S}^{\prime} \overrightarrow{i_{S \beta}}+k_{R} \overrightarrow{\Psi_{R \beta}} \tag{14}
\end{align*}
$$

Accordingly, in the equation of moment 11 , it is possible to replace the vectors of the stator flux linkage along the axes

$$
\begin{equation*}
m=k_{R}\left(\overrightarrow{\Psi_{R \alpha}} \cdot \overrightarrow{i_{S \beta}}-\overrightarrow{\Psi_{R \beta} i_{S \alpha}}\right) \tag{15}
\end{equation*}
$$

Space vectors of stator voltage, stator current, rotor flux linkage can be described by these equations

$$
\begin{align*}
& \vec{U}_{S}=U_{S x}+j U_{S y}  \tag{16.1}\\
& \vec{i}_{S}=i_{S x}+j i_{S y}  \tag{16.2}\\
& \vec{\Psi}_{R}=\Psi_{R x}+j \Psi_{R y} \tag{16.3}
\end{align*}
$$

As a result of changes, substitutions and transformations, the system of equations 1.1-1.6, which mathematically describes the work of the induction drive in a rotating coordinate system, will take the following new form

$$
\begin{align*}
& \vec{U}_{S x}=r \overrightarrow{i_{S x}}+\frac{x_{s x}^{\prime} d \overrightarrow{i_{S x}}}{d t}-\alpha_{k}^{\prime} x_{S}^{\prime} \overrightarrow{i_{S y}}-\frac{k_{R}}{T_{R}} \overrightarrow{\Psi_{R x}}-p \vartheta_{m} k_{R} \overrightarrow{\Psi_{R y}}  \tag{17.1}\\
& \vec{U}_{S y}=r \overrightarrow{i_{S y}}+\frac{x_{S x}^{\prime}}{d t} \overrightarrow{d \overrightarrow{i_{S y}}}-\alpha_{k}^{\prime} x_{S S}^{\prime} \overrightarrow{i_{S x}}-\frac{k_{R}}{T_{R}} \overrightarrow{\Psi_{R y}}+p \vartheta_{m} k_{R} \overrightarrow{\Psi_{R x}}  \tag{17.2}\\
& \vec{U}_{R x}=0=\frac{1}{T_{R}} \overrightarrow{\Psi_{R x}}+\frac{d \overrightarrow{\Psi_{R x}}}{d t}-k_{R} r_{R} \overrightarrow{i_{S x}}-\left(\alpha_{k}-p \vartheta_{m}\right) \overrightarrow{\Psi_{R y}}  \tag{17.3}\\
& \vec{U}_{R y}=0=\frac{1}{T_{R}} \overrightarrow{\Psi_{R y}}+\frac{d \overrightarrow{\Psi_{R y}}}{d t}-k_{R} r_{R} \overrightarrow{i_{S y}}+\left(\alpha_{k}-p \vartheta_{m}\right) \overrightarrow{\Psi_{R x}}  \tag{17.4}\\
& m=k_{R}\left(\overrightarrow{\Psi_{R x}} \overrightarrow{i_{S y}}-\overrightarrow{\Psi_{R y}} \overrightarrow{i_{S x}}\right)  \tag{17.5}\\
& \overrightarrow{T_{m}} \frac{d \vartheta_{m}}{d t}=m-m_{c} \tag{17.6}
\end{align*}
$$

Accepting $\mathrm{T}_{\mathrm{S}}^{\prime}=\frac{\mathrm{x}_{\mathrm{S}}}{\mathrm{r}}$ the system of equations 17.1-17.6 transforms into operator form

$$
\begin{align*}
& \vec{U}_{S x}=r\left(1+T_{S}^{\prime} p\right) i_{S x}-\alpha_{k} x_{S}^{\prime} i_{S y}-\frac{k_{R}}{T_{R}} \Psi_{R x}-p \vartheta_{m} k_{R} \Psi_{R y}  \tag{18.1}\\
& \vec{U}_{S y}=r\left(1+T_{S}^{\prime} p\right) i_{S y}+\alpha_{k} x_{S}^{\prime} i_{S x}-\frac{k_{R}}{T_{R}} \Psi_{R y}+p \vartheta_{m} k_{R} \Psi_{R x}  \tag{18.2}\\
& \vec{U}_{R x}=0=\frac{1}{T_{R}}\left(1+T_{R} p\right) \Psi_{R x}-k_{R} r_{R} i_{S x}-\left(\alpha_{k}-p \vartheta_{m}\right) \Psi_{R y}  \tag{18.3}\\
& \vec{U}_{R y}=0=\frac{1}{T_{R}}\left(1+T_{R} p\right) \Psi_{R y}-k_{R} r_{R} i_{S y}+\left(\alpha_{k}-p \vartheta_{m}\right) \Psi_{R x}  \tag{18.4}\\
& m=k_{R}\left(\Psi_{R x} i_{S y}-\Psi_{R y} i_{S x}\right)  \tag{18.5}\\
& \overrightarrow{T_{m}} \vartheta_{m}=m-m_{c} \tag{18.6}
\end{align*}
$$

From equations 18.1, 18.2 the stator current is expressed

$$
\begin{equation*}
i_{S x}=\left(U_{S x}+\alpha_{k} x_{S}^{\prime} i_{S y}+\frac{k_{R}}{T_{R}} \Psi_{R x}-p \vartheta_{m} k_{R} \Psi_{R y}\right) \frac{1}{r\left(1+T_{S}^{\prime} p\right)} \tag{19}
\end{equation*}
$$

$$
\begin{equation*}
i_{S y}=\left(U_{S y}-\alpha_{k} x_{S}^{\prime} i_{i_{X}}+\frac{k_{R}}{T_{R}} \Psi_{R y}-p \vartheta_{m} k_{R} \Psi_{R x}\right) \frac{1}{r\left(1+T_{S}^{\prime} p\right)} \tag{20}
\end{equation*}
$$

And from equations 18.3, 18.4 the rotor flux linkage is expressed

$$
\begin{align*}
& \Psi_{R x}=\left(k_{R} r_{R} i_{S x}+\left(\alpha_{k}-p \vartheta_{m}\right) \Psi_{R y} \frac{T_{R}}{1+T_{R} p}\right.  \tag{21}\\
& \Psi_{R y}=\left(k_{R} r_{R} i_{S y}-\left(\alpha_{k}-p \vartheta_{m}\right) \Psi_{R x} \frac{T_{R}}{1+T_{R} p}\right. \tag{22}
\end{align*}
$$

The speed from equation 18.6 will be

$$
\begin{equation*}
\vartheta_{m}=\frac{1}{\overrightarrow{T_{m} p}}\left(m-m_{c}\right) \tag{23}
\end{equation*}
$$

Thus, from the obtained final equations 19-23, a mathematical model of induction drive in a rotating coordinate system was created. At the same time, this model was transformed from a threephase into a two-phase one - Fig. 1.


Fig. 1. Model assembled in Simulink

## Imitation modeling

In this paper, a model of an AIR 160S 4 induction electric motor is considered with the following parameters presented in Table 1.

Table 1. AIR 160S 4 parameters

| Motor | Power, <br> kW | Speed, <br> rpm | Voltage, <br> V | Efficiency, <br> $\%$ | Power <br> factor | $\frac{I_{s}}{I_{n}}$ | $\frac{M_{s}}{M_{n}}$ | $\frac{M_{\max }}{M_{n}}$ | Moment of <br> inertia, $\mathrm{kgm}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AIR 160S 4 | 15 | 1450 | 400 | 89.5 | 0.86 | 7,7 | 2.2 | 2.6 | 0.075 |

To assess the adequacy and correctness of the assembled model, it was proposed to compare it with the induction motor model from the standard Simulink/Simscape library - Fig.2.

The rotor speed and the torque developed by the motors are shown in the Fig. 3 for the model in a rotating coordinate system - blue, and for the standard model - orange.


Fig. 2. Standard model of an induction motor in Simulink

(a)
(b)
(c)

Fig. 3. Output characteristics of electric drives: $\mathrm{a}-$ speed, $\mathrm{b}-$ torque, $\mathrm{c}-$ stator current

From the obtained characteristics, it can be seen that the output values coincide, and the processes take place at the same time. However, the system from the standard library is more oscillatory, so the model in the rotating coordinate system has a better transient process. It is planned to compare these models with a real electric drive to obtain the most accurate results in further researches.

## Conclusion

Mathematical substitutions in the equations and the used methods have shown the adequacy and correctness of their use. Analysis of the data, obtained from simulation modeling of the developed model, showed small discrepancies with the standard block from the Simulink library. The use of the obtained mathematical equations, as well as the model assembled in Matlab Simulink, will make it possible to qualitatively evaluate the operation of an induction drive in statics and dynamics. This model will be used in future research, including the creation of a buffer power source based on a supercapacitor for an induction electric drive.

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    * Corresponding author E-mail address: azoorjke@gmail.com

