DOI: 10.17516/1997-1397-2022-15-3-267-272
УДК 517.96

The Computational Simulation of Temperature and Thermal Stress Fields in a Carbon Block under External Thermal Influences

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Received 23.11.2021, received in revised form 28.01.2022, accepted 20.03.2022

Abstract. The paper is devoted to modeling the thermal regime and the stress-strain state of a carbon block when it is partially immersed in an electrolyte. The temperature field in the block was determined from the solution of a non-stationary three-dimensional heat conduction equation. The calculation of temperature stresses was carried out on the basis of the solution of the Poisson equation written for the thermoelastic displacement potential. As a result of modeling the thermal regime, the temperature fields in the carbon block are obtained for different time points. The calculation of the stress-strain state determined the magnitude and location of the greatest temperature stresses and allowed us to assess the possibility of carbon block failure.

Keywords: heat conduction equation, Poisson equation, temperature stresses, thermoelastic displacement potential, computational simulation.

Citation: E.N. Vasil'ev, The Computational Simulation of Temperature and Thermal Stress Fields in a Carbon Block under External Thermal Influences, J. Sib. Fed. Univ. Math. Phys., 2022, 15(3), 267–272.
DOI: 10.17516/1997-1397-2022-15-3-267-272.

Introduction

The technological process of aluminum production requires regular replacement of spent carbon blocks (anodes). In the industrial electrolytic cells, at the initial immersion of the cold anode in a hot electrolytic solution having a temperature of about 960^oC, a heat wave propagates from the contact boundary into the volume of the anode.An increase in the local temperature causes thermal expansion of the anode material, the difference in the magnitude of the expansion of different zones of the anode leads to the appearance of thermal stresses. In the zone of the highest temperature gradients, significant thermal stresses arise, which can exceed the ultimate strength of the material and lead to the formation of cracks and further destruction of the anode. The phenomena accompanying the process of installing a cold carbon anode into the melt are called thermal shock [1, 2]. The state of the carbon block during thermal shock depends on the thermophysical (thermal conductivity, heat capacity) and mechanical (thermal expansion coefficient, shear modulus, Poisson's ratio, tensile strength) properties of carbon graphite, as well as the conditions of heat exchange with electrolyte. Computational simulation makes it possible to analyze the state of the carbon block taking into account these factors.

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The aim of the work is to calculate the temperature field and the stress-strain state of the carbon block of the electrolytic cell. To describe the formation of thermal stresses, the mathematical modeling procedure includes two consecutive stages:

1. Determination of the temperature field in the volume of the carbon block based on the solution of a thermal conduction problem.
2. Calculation of thermal stresses based on the solution of a Poisson equation for the obtained temperature field at different time points.

1. Determination of the temperature field of the carbon block

The anode block is a parallelepiped made of carbon graphite (Fig. 1). In the electrolysis cell, the anode block is mounted using a steel bracket. The geometrical dimensions of the anode along the x, y, and z coordinates are $1450 \times 700 \times 600$ mm^3.

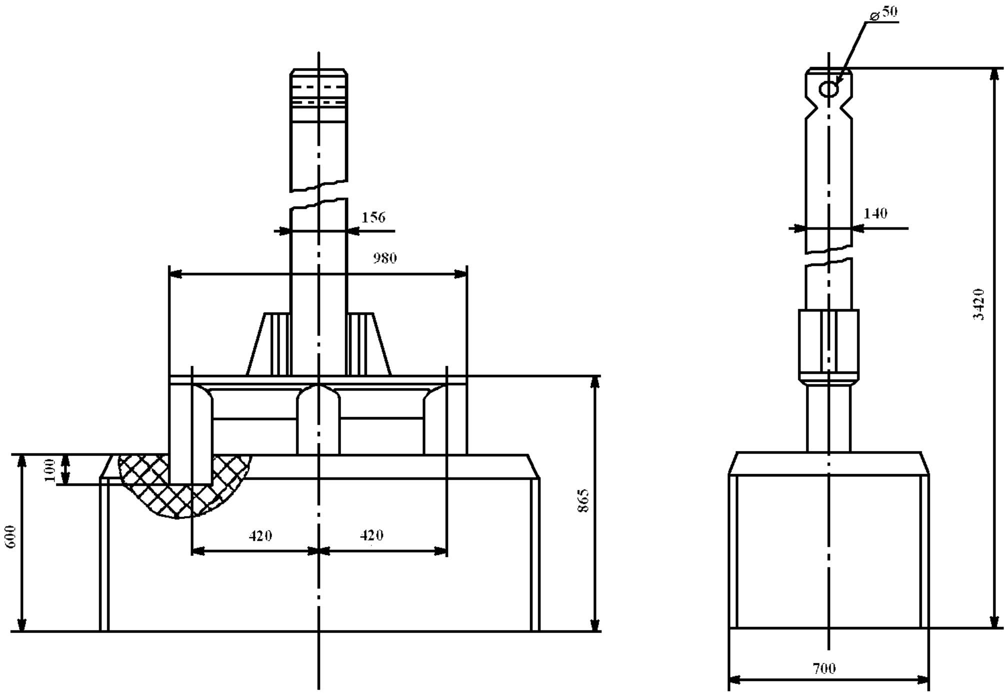

Fig. 1. The anode block of the industrial electrolytic cell

The heat transfer process in a carbon block is described by a non-stationary three-dimensional heat conduction equation

$$c\rho \frac{\partial T}{\partial t} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right), \tag{1}$$

where c, ρ — specific volumetric heat capacity and density of the material; T — temperature; λ — coefficient of thermal conductivity; t — time; x, y, z – spatial coordinates. The solution of equation (1) was carried out by the method of finite differences using the splitting of the problem in spatial coordinates [3, 4].

Calculations are made for the anode block shown in Fig. 1. The depth of the part of the block immersed in the electrolyte is 120 mm. For carbon graphite, the following thermophysical properties were set: $\lambda = 4.4$ W/(m·K), $c = 942$ J/(kg·K), $\rho = 1560$ kg/m^3 [5, 6].The heat exchange coefficient of the surface of the anode block with air $\beta_A = 10$ W/(m^2·K) and the electrolyte solution $\beta_E = 18$ W/(m^2·K). For the calculations, a homogeneous spatial difference grid with the number of nodes $146 \times 71 \times 61$ was used, the time step was 5 s.

The results of calculating the temperature field of the anode for the moment of time $\Delta t = 15$ min are shown in Fig. 2. The temperature field of the lower part of the anode in the middle cross-section of the xz plane is shown in left figure, which shows the temperature field region up to the vertical axis of symmetry. In the right figure, also taking into account the symmetry of the problem, one quarter of the temperature field of the lower surface of the anode (plane xy) is shown. The temperature values on the isolines are given in degrees Celsius. The most intense heating is observed in the zone where the unit is in contact with the electrolyte. In this area, the highest temperature is in the lower corner points and and is 467^{o}C.

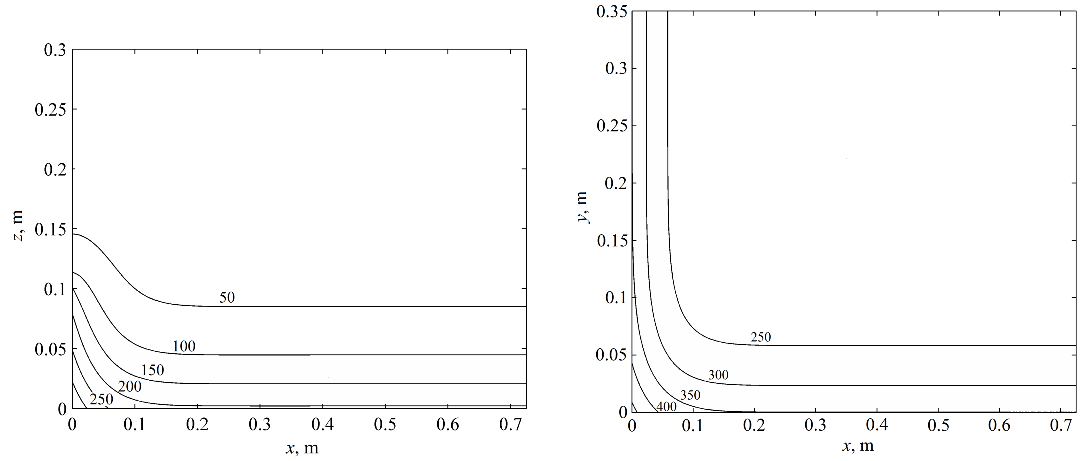

Fig. 2. Distribution of temperature in the middle xz and bottom xy planes of the anode block

The temperature distributions obtained from the solution of equation (1) at different times are the initial data for solving the problem of the stress-strain state of the carbon block.

2. Calculation of the temperature stresses in the carbon block

The temperature stresses are calculated by solving the Poisson equation written for the thermoelastic displacement potential [7]

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{\alpha \Theta (1 + \mu)}{1 - \mu}, \tag{2}$$

where Φ is the thermoelastic potential of displacements; μ, α — Poisson's ratio and coefficient of thermal expansion; $\Theta = (T - T_0)$ is the temperature increment in comparison with the temperature of the natural state of the body T_0. Equation (2) is supplemented by the conditions for the absence of externally applied normal and tangential stresses on the carbon block surface: $\sigma_z = 0,\ \tau_{xz} = 0,\ \tau_{yz} = 0$.

The values of the thermoelastic potential Φ were used to determine the stresses at the corresponding points of the difference grid

$$\sigma_x = 2G\left(\frac{\partial^2\Phi}{\partial x^2} + \bigtriangledown^2\Phi\right), \tag{3}$$

$$\tau_{xy} = 2G\frac{\partial^2\Phi}{\partial x \partial y}(xyz), \tag{4}$$

where σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{xz} are active elastic normal and tangential stresses; G is the shear modulus of the material at a given point and at a given moment in time, (xyz) is a symbol of cyclic permutation of x, y, z. The number of nodes of the difference mesh in the thermal stresses equations (2)–(4) corresponds to the thermal problem.

The distribution of thermal normal stresses of the anode for a time instant of 15 minutes is shown in Fig. 3. The distribution of thermal normal stresses for the middle xz plane is shown in the left figure. The magnitude of the temperature stresses in this plane reaches 3.4 MPa. The highest stresses are observed in the zones of the highest temperature gradients. The maximum values of temperature gradients and stresses occur at the corners of the anode, they can be displayed in a vertical diagonal section passing along the bisector of the angle of the anode base. In right figure shows the distribution of thermal stresses in this diagonal plane. Comparison of the distributions in the middle xz and the diagonal vertical planes shows that in the second case the maximum stress values are more than 1.5 times higher.

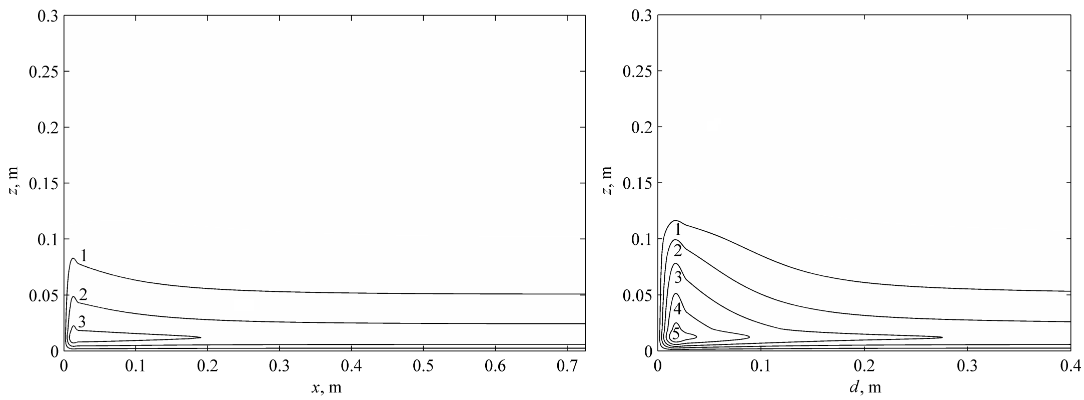

Fig. 3. Distribution of normal temperature stresses in the middle xz and diagonal vertical planes of the anode block

The dynamics of changes in the maximum normal stresses in the anode block is shown in Fig. 4, the calculated maximum values of thermal stresses at different times are marked with circles, the dotted line is obtained by interpolating these values. The greatest increase in thermal stresses occurs at the initial stage of the process, and then the slope of the curve is significantly reduced. Based on the results of calculating the stress-strain state, it is possible to assess the possibility of destruction of the anode by comparing it with the ultimate strength of carbon. The most dangerous from the point of view of cracking and destruction of the anode are tensile stresses arising from the inhomogeneous thermal expansion of the material during heating. The ultimate tensile strength of the anode material is estimated from various data in the range of

5–15 MPa [2, 8, 9].The range of data on the limiting values of thermal stresses is wide for carbon graphite and depends on the manufacturing technology and composition. It follows from the calculation results that when using carbon graphite with a low ultimate strength (less than 8 MPa), there is a probability of destruction of the anode block.

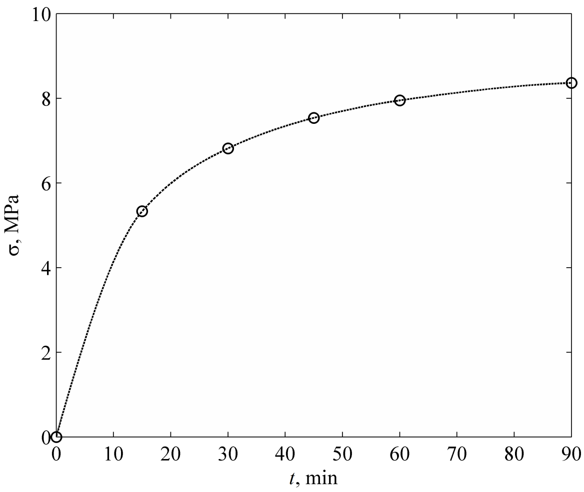

Fig. 4. Dependence of the maximum normal temperature stresses in the anode block on time

Conclusion

Computational modeling of thermal processes occurring when a carbon block is immersed in a hot electrolyte made it possible to determine the magnitude and location of the greatest temperature gradients and stress at different times. Calculations have shown that the maximum values of temperature stresses in the corners of the anode block exceed the lower limit of the tensile strength of graphite carbon, which indicates the possible destruction of the anode block.

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Вычислительное моделирование полей температур и термических напряжений в угольном блоке при внешних тепловых воздействиях

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Аннотация. Работа посвящена моделированию теплового режима и напряженно-деформированного состояния угольного блока при его частичном погружении в электролит. Температурное поле в блоке определялось из решения нестационарного трехмерного уравнения теплопроводности. Расчет температурных напряжений проводился на основе решения уравнения Пуассона, записанного для термоупругого потенциала перемещений. В результате моделирования теплового режима получены температурные поля в угольном блоке для разных моментов времени. Расчет напряженно-деформированного состояния определил величину и расположение наибольших температурных напряжений и позволил оценить возможность разрушения угольного блока.

Ключевые слова: уравнение теплопроводности, уравнение Пуассона, термические напряжения, термоупругий потенциал перемещений, вычислительное моделирование.