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A Fixed Point Approach to Study a Class of Probabilistic Functional Equations Arising in the Psychological Theory of Learning

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Abstract. Many biological and learning theory models have been investigated using probabilistic functional equations. This article focuses on a specific kind of predator–prey relation in which a predator has two prey options, each with a probability of x and $1-x$, respectively. Our aim is to investigate the animal's responses in such situations by proposing a general probabilistic functional equation. The noteworthy fixed-point results are used to investigate the existence, uniqueness, and stability of solutions to the proposed functional equation. An example is also given to illustrate the importance of our results in this area of research.

Keywords: probabilistic functional equations, stability, fixed points.

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1. Introduction and preliminaries

Various mathematical learning experiments have recently shown that the behavior of a simple learning experiment follows a stochastic model. Thus, it is not a novel idea (for detail, see [1,2]). Following 1950, however, two critical features were apparent, most notably in the Bush, Estes, and Mosteller research. First, one of the most critical characteristics of the proposed models is the inclusive nature of the learning process. Second, such models may be evaluated in such a manner that their statistical properties are revealed.

In 1976, Istrăţescu [3] examined the participation of predatory animals that feed on two different kinds of prey using the following functional equation

$$\mathscr{R}(x) = x\mathscr{R}(r + (1-r)x) + (1-x)\mathscr{R}((1-s)x), \tag{1}$$

for all $x \in \mathscr{J} = [0,1]$ and $0 < r \leqslant s < 1$, where $\mathscr{R} : \mathscr{J} \to \mathbb{R}$ is an unknown function.

The states x and $(1-x)$ to $r+(1-r)x$ and $(1-s)x$, respectively, were converted into Markov transitions to explain such behavior by $\mathbb{P}(r+(1-r)x) = x$ and $\mathbb{P}((1-s)x) = 1-x$. Sintunavarat and Turab [4] discussed the properties of the above model (1) under the experimenter-subject controlled events.

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In a two-choice scenario, in [2,5], the authors utilized such operators to monitor the movement of a paradise fish under the reinforcement-extinction and the habit formation circumstances (for detail, see Tab. 1).

Table 1. Operators for reinforcement-extinction and habit formation model

Operators for reinforcement-extinction model			
Fish's Responses	**Outcomes (Left side)**	**Outcomes (Right side)**	**Events**
Reinforcement	rx	$rx+1-r$	E_1^{RE}
Non-reinforcement	$sx+1-s$	sx	E_2^{RE}
Operators for habit formation model			
Fish's Responses	**Outcomes (Left side)**	**Outcomes (Right side)**	**Events**
Reinforcement	rx	$rx+1-r$	E_1^{HF}
Non-reinforcement	sx	$sx+1-s$	E_2^{HF}

Berinde and Khan [6] extended the preceding concept by introducing the subsequent functional equation

$$\mathscr{R}(x) = x\mathscr{R}(\mathscr{V}_1(x)) + (1-x)\mathscr{R}(\mathscr{V}_2(x)), \tag{2}$$

for all $x \in \mathscr{J}$, where $\mathscr{V}_1, \mathscr{V}_2 : \mathscr{J} \to \mathscr{J}$ are given mappings and satisfied the following boundary conditions

$$\begin{cases} \mathscr{V}_1(1) = 1, \quad and \\ \mathscr{V}_2(0) = 0. \end{cases} \tag{3}$$

Recently, Turab and Sintunavarat [7] utilized the above ideas and suggested the functional equation stated below

$$\mathscr{R}(x) = x\mathscr{R}(\varpi_1 x + (1-\varpi_1)\Theta_1) + (1-x)\mathscr{R}(\varpi_2 x + (1-\varpi_2)\Theta_2) \quad \forall x \in \mathscr{J}, \tag{4}$$

where $\mathscr{R} : \mathscr{J} \to \mathbb{R}$ is an unknown, $0 < \varpi_1 \leqslant \varpi_2 < 1$ and $\Theta_1, \Theta_2 \in \mathscr{J}$. The aforementioned functional equation was used to investigate a particular kind of psychological resistance in dogs who were kept in a confined enclosure.

Several additional research on the behaviors of humans and animals in probability-learning situations have yielded a variety of diverse conclusions (see [8–13]).

As a result of the previous research, we propose the following general probabilistic functional equation

$$\begin{aligned} \mathscr{R}(x) = &\left(\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_1(x)) + \left(\frac{w-j}{k-j}\right)\left(1-\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_2(x)) + \\ &+ \left(1-\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_3(x)) + \left(1-\frac{w-j}{k-j}\right)\left(1-\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_4(x)), \end{aligned} \tag{5}$$

for all $x \in [j,k]$, where $0 \leqslant w \leqslant 1$, $\mathscr{R} : [j,k] \to \mathbb{R}$ is an unknown and $\tau, \mathscr{V}_1, \mathscr{V}_2, \mathscr{V}_3, \mathscr{V}_4 : [j,k] \to [j,k]$ are given mappings.

The Banach fixed point theorem will be used to establish the existence and uniqueness results of the above equation (5). Finally, we examine the stability of the suggested stochastic equation's solution.

The following stated result will be needed in later sections.

Theorem 1.1 ([14]). *Let $(\mathscr{J}, d)$ be a complete metric space and $\mathscr{R} : \mathscr{J} \to \mathscr{J}$ be a mapping defined by*

$$d(\mathscr{R}s, \mathscr{R}t) \leqslant \Lambda d(s, t) \tag{6}$$

for some $\Lambda < 1$ and for all $s, t \in \mathscr{J}$. Then $\mathscr{R}$ has precisely one fixed point. Furthermore, the Picard iteration $\{s_n\}$ in $\mathscr{J}$ which is defined by $s_n = \mathscr{R}s_{n-1}$ for all $n \in \mathbb{N}$, where $s_0 \in \mathscr{J}$, converges to the unique fixed point of $\mathscr{R}$.

2. Main results

Let $\mathscr{J} = [j, k]$ with $j < k$, where $j, k \in \mathbb{R}$. We indicate the class $\mathscr{R} : \mathscr{J} \to \mathbb{R}$ consisting of all continuous real-valued functions by $\mathscr{T}$ such that $\mathscr{R}(j) = 0$ and

$$\sup_{s \neq t} \frac{|\mathscr{R}(s) - \mathscr{R}(t)|}{|s - t|} < \infty.$$

We can see that $(\mathscr{T}, \|\cdot\|)$ is a normed space (for the detail, see [5, 15]), where $\|\cdot\|$ is given by

$$\|\mathscr{R}\| = \sup_{s \neq t} \frac{|\mathscr{R}(s) - \mathscr{R}(t)|}{|s - t|} \tag{7}$$

for all $\mathscr{R} \in \mathscr{T}$.

Next, we rewrite (5) as

$$\begin{aligned} \mathscr{R}(x) \quad = \quad & \left(\frac{w - j}{k - j}\right) \left(\frac{\tau(x) - j}{k - j}\right) \mathscr{R}(\mathscr{V}_1(x)) + \left(\frac{w - j}{k - j}\right) \left(\frac{k - \tau(x)}{k - j}\right) \mathscr{R}(\mathscr{V}_2(x)) + \\ & + \left(\frac{k - w}{k - j}\right) \left(\frac{\tau(x) - j}{k - j}\right) \mathscr{R}(\mathscr{V}_3(x)) + \left(\frac{k - w}{k - j}\right) \left(\frac{k - \tau(x)}{k - j}\right) \mathscr{R}(\mathscr{V}_4(x)), \end{aligned} \tag{8}$$

where $\mathscr{R} : \mathscr{J} \to \mathbb{R}$ is an unknown function such that $\mathscr{R}(j) = 0$. Also, $\mathscr{V}_1, \mathscr{V}_2, \mathscr{V}_3, \mathscr{V}_4 : \mathscr{J} \to \mathscr{J}$ are contraction mappings with contractive coefficients b_1, b_2, b_3 and b_4 respectively. Also, the following condition holds

$$\mathscr{R}(\mathscr{V}_2(j)) = j = \mathscr{R}(\mathscr{V}_4(j)). \tag{9}$$

Furthermore, $\tau : \mathscr{J} \to \mathscr{J}$ is a non-expansive mapping with $\tau(j) = j$ and $|\tau(x)| \leqslant b_5$, for all $x \in \mathscr{J}$ with $b_5 \geqslant 0$.

Before proving the main results, we mention the following conditions here.

$(\mathcal{A}_1)$: For the mappings $\mathscr{V}_1, \mathscr{V}_2 : \mathscr{J} \to \mathscr{J}$, we have

$$|\mathscr{V}_1(u) - \mathscr{V}_2(v)| \leqslant b_6 |u - v|, \tag{10}$$

for all $u, v \in \mathscr{J}$ with $u \neq v$, where $b_6 \in [0, 1)$.

$(\mathcal{A}_2)$: For the mappings $\mathscr{V}_3, \mathscr{V}_4 : \mathscr{J} \to \mathscr{J}$, we have

$$|\mathscr{V}_3(u) - \mathscr{V}_4(v)| \leqslant b_7 |u - v|, \tag{11}$$

for all $u, v \in \mathscr{J}$ with $u \neq v$, where $b_7 \in [0, 1)$.

$(\mathcal{A}_3)$: For the mappings $\mathscr{V}_1, \mathscr{V}_2, \mathscr{V}_3, \mathscr{V}_4 : \mathscr{J} \to \mathscr{J}$, there exist points $u^\star, v^\star \in [j, k]$ such that

$$\mathscr{V}_1(u^\star) = \mathscr{V}_2(u^\star) \quad \text{and} \quad \mathscr{V}_3(v^\star) = \mathscr{V}_4(v^\star). \tag{12}$$

We begin with the succeeding outcome.

Theorem 2.1. *Consider the probabilistic functional equation (8) with (9). Assume that the conditions $(\mathcal{A}_1)$ and $(\mathcal{A}_2)$ hold with $\Lambda_1 < 1$, where*

$$\begin{aligned}\Lambda_1 \ &:= \ \Bigg|\left(\frac{w-j}{k-j}\right)\left[b_1\left(1+\frac{b_5-j}{k-j}\right)+b_2\left(\frac{k-b_5}{k-j}\right)+b_6\right]+\\ &+\left(\frac{k-w}{k-j}\right)\left[b_3\left(1+\frac{b_5-j}{k-j}\right)+b_4\left(\frac{k-b_5}{k-j}\right)+b_7\right]\Bigg|, \qquad (13)\end{aligned}$$

and there is a nonempty subset $\mathscr{E}$ of $\mathscr{S} := \{\mathscr{R} \in \mathscr{T} | \mathscr{R}(k) \leqslant k\}$ such that $(\mathscr{E}, \|\cdot\|)$ is a Banach space, where $\|\cdot\|$ is given in (7), and the mapping $\mathscr{K}$ from $\mathscr{E}$ defined for each $\mathscr{R} \in \mathscr{T}$ by

$$\begin{aligned}(\mathscr{K}\mathscr{R})(x) \ &= \ \left(\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_1(x))+\left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_2(x))+\\ &+\left(\frac{k-w}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_3(x))+\left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_4(x)), \quad (14)\end{aligned}$$

for all $x \in \mathscr{J}$ is a self mapping. Then $\mathscr{K}$ is a Banach contraction mapping with the metric d induced by $\|\cdot\|$.

Proof. Let $\mathscr{R}_1, \mathscr{R}_2 \in \mathscr{E}$. For each $u, v \in \mathscr{J}$ with $u \neq v$, we obtain

$$\begin{aligned}
&|\Theta_{u\neq v}| := \frac{|\mathscr{K}(\mathscr{R}_1-\mathscr{R}_2)(u)-\mathscr{K}(\mathscr{R}_1-\mathscr{R}_2)(v)|}{|u-v|} =\\
&= \Bigg|\frac{1}{u-v}\Bigg[\left(\frac{w-j}{k-j}\right)\left(\frac{\tau(u)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_1(u))+\left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(u)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_2(u))\\
&+\left(\frac{k-w}{k-j}\right)\left(\frac{\tau(u)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_3(u))+\left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(u)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_4(u))\\
&-\left(\frac{w-j}{k-j}\right)\left(\frac{\tau(v)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_1(v))-\left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(v)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_2(v))\\
&-\left(\frac{k-w}{k-j}\right)\left(\frac{\tau(v)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_3(v))-\left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(v)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_4(v))\Bigg]\Bigg| =\\
&= \Bigg|\frac{1}{u-v}\left[\left(\frac{w-j}{k-j}\right)\left(\frac{\tau(u)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_1(u))-\left(\frac{w-j}{k-j}\right)\left(\frac{\tau(u)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_1(v))\right]\\
&+\frac{1}{u-v}\left[\left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(u)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_2(u))-\left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(u)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_2(v))\right]\\
&+\frac{1}{u-v}\left[\left(\frac{k-w}{k-j}\right)\left(\frac{\tau(u)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_3(u))-\left(\frac{k-w}{k-j}\right)\left(\frac{\tau(u)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_3(v))\right]\\
&+\frac{1}{u-v}\left[\left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(u)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_4(u))-\left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(u)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_4(v))\right]\\
&+\frac{1}{u-v}\left[\left(\frac{w-j}{k-j}\right)\left(\frac{\tau(u)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_1(v))-\left(\frac{w-j}{k-j}\right)\left(\frac{\tau(v)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_1(v))\right]\\
&+\frac{1}{u-v}\left[\left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(u)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_2(v))-\left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(v)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_2(v))\right]\\
&+\frac{1}{u-v}\left[\left(\frac{k-w}{k-j}\right)\left(\frac{\tau(u)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_3(v))-\left(\frac{k-w}{k-j}\right)\left(\frac{\tau(v)-j}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_3(v))\right]\\
&+\frac{1}{u-v}\left[\left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(u)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_4(v))-\left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(v)}{k-j}\right)(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_4(v))\right]\Bigg|
\end{aligned}$$

As $\mathscr{V}_1 - \mathscr{V}_4$ are contraction mappings with the contractive coefficients $b_1 - b_4$, respectively. Thus, by using the definition of the norm (7), we have

$$|\Theta_{u \neq v}| \leqslant \Lambda_1 \|\mathscr{R}_1 - \mathscr{R}_2\|,$$

where Λ_1 is defined in (13). This gives that

$$d(\mathscr{K}\mathscr{R}_1, \mathscr{K}\mathscr{R}_2) = \|\mathscr{K}\mathscr{R}_1 - \mathscr{K}\mathscr{R}_2\| \leqslant \Lambda_1 \|\mathscr{R}_1 - \mathscr{R}_2\| = \Lambda_1 d(\mathscr{R}_1, \mathscr{R}_2).$$

As $0 < \Lambda_1 < 1$, we conclude that $\mathscr{K}$ is a Banach contraction mapping with metric d induced by $\|\cdot\|$. □

Theorem 2.2. *Consider the probabilistic functional equation (8) associated with (9). Assume that the conditions $(\mathcal{A}_1)$ and $(\mathcal{A}_2)$ hold with $\Lambda_1 < 1$, where Λ_1 is defined in (13). Also, there exist a nonempty subset $\mathscr{E}$ of $\mathscr{S} := \{\mathscr{R} \in \mathscr{T} | \mathscr{R}(k) \leqslant k\}$ such that $(\mathscr{E}, \|\cdot\|)$ is a Banach space, where $\|\cdot\|$ is given in (7), and the mapping $\mathscr{K}$ from $\mathscr{E}$ defined for each $\mathscr{R} \in \mathscr{T}$ by (10) is a self mapping. Then, the functional equation (8) with (9) has a unique solution in $\mathscr{E}$. Furthermore, the iteration $\mathscr{R}_n$ in $\mathscr{E}$ can be defined by*

$$\begin{aligned}(\mathscr{R}_n)(x) = \left(\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_1(x)) + \left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_2(x)) + \\ + \left(\frac{k-w}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_3(x)) + \left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_4(x)), \quad (15)\end{aligned}$$

for all $n \in \mathbb{N}$, where $\mathscr{R}_0 \in \mathscr{E}$, converges to the unique solution of (8).

Proof. We get the conclusion of this theorem by combining Theorem 2.1 with the Banach fixed point theorem. □

Remark 2.3. *Our proposed probabilistic equation (5) is a generalization of the functional equations discussed in [3, 5–7].*

Here, we shall look at different conditions. If $\mathscr{V}_1, \mathscr{V}_2, \mathscr{V}_3, \mathscr{V}_4 : \mathscr{J} \to \mathscr{J}$ are contraction mappings with contractive coefficients $b_1 \leqslant b_2 \leqslant b_3 \leqslant b_4$, respectively, then by Theorems 2.1 and 2.2, the outcomes are as follows.

Corollary 2.4. *Consider the probabilistic functional equation (8) associated with (9). Assume that the conditions $(\mathcal{A}_1)$ and $(\mathcal{A}_2)$ hold with $\tilde{\Lambda}_1 < 1$, where*

$$\tilde{\Lambda}_1 := \left| 2b_4 + \frac{1}{k-j}\left[(w-j)b_6 + (k-w)b_7\right] \right|, \quad (16)$$

and there is a nonempty subset $\mathscr{E}$ of $\mathscr{S} := \{\mathscr{R} \in \mathscr{T} | \mathscr{R}(k) \leqslant k\}$ such that $(\mathscr{E}, \|\cdot\|)$ is a Banach space, where $\|\cdot\|$ is given in (7), and the mapping $\mathscr{K}$ from $\mathscr{E}$ defined for each $\mathscr{R} \in \mathscr{T}$ by

$$\begin{aligned}(\mathscr{K}\mathscr{R})(x) = \left(\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_1(x)) + \left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_2(x)) + \\ + \left(\frac{k-w}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_3(x)) + \left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_4(x)), \quad (17)\end{aligned}$$

for all $x \in \mathscr{J}$ is a self mapping. Then $\mathscr{K}$ is a Banach contraction mapping with the metric d induced by $\|\cdot\|$.

Corollary 2.5. *Consider the probabilistic functional equation (8) associated with (9). Assume that the conditions* $(\mathcal{A}_1)$ *and* $(\mathcal{A}_2)$ *hold with* $\tilde{\Lambda}_1 < 1$*, where* $\tilde{\Lambda}_1$ *is given in (16), and there is a nonempty subset* $\mathscr{E}$ *of* $\mathscr{S} := \{\mathscr{R} \in \mathscr{T} | \mathscr{R}(k) \leqslant k\}$ *such that* $(\mathscr{E}, \|\cdot\|)$ *is a Banach space, where* $\|\cdot\|$ *is given in (7), and the mapping* $\mathscr{K}$ *from* $\mathscr{E}$ *defined for each* $\mathscr{R} \in \mathscr{T}$ *by (17) is a self mapping. Then, the functional equation (8) with (9) has a unique solution in* $\mathscr{E}$*. Furthermore, the iteration* $\mathscr{R}_n$ *in* $\mathscr{E}$ *is defined as*

$$(\mathscr{R}_n)(x) = \left(\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_1(x)) + \left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_2(x)) + \\ + \left(\frac{k-w}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_3(x)) + \left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_4(x)), \quad (18)$$

for all $n \in \mathbb{N}$*, where* $\mathscr{R}_0 \in \mathscr{E}$*, converges to the unique solution of (8).*

Theorem 2.6. *Consider the probabilistic functional equation (8) with (9). Assume that the condition* $(\mathcal{A}_3)$ *holds with* $\Lambda_2 < 1$*, where*

$$\Lambda_2 := \left|\left(\frac{w-j}{k-j}\right)\left(b_1\left(1+\frac{b_5-j}{k-j}\right) + b_2\left(1+\frac{k-b_5}{k-j}\right)\right) + \right. \\ \left. + \left(\frac{k-w}{k-j}\right)\left(b_3\left(1+\frac{b_5-j}{k-j}\right) + b_4\left(1+\frac{k-b_5}{k-j}\right)\right)\right|. \quad (19)$$

Suppose that there is a nonempty subset $\mathscr{E}$ *of* $\mathscr{S} := \{\mathscr{R} \in \mathscr{T} | \mathscr{R}(k) \leqslant k\}$ *such that* $(\mathscr{E}, \|\cdot\|)$ *is a Banach space, where* $\|\cdot\|$ *is given in (7), and the mapping* $\mathscr{K}$ *from* $\mathscr{E}$ *defined for each* $\mathscr{R} \in \mathscr{T}$ *by*

$$(\mathscr{K}\mathscr{R})(x) = \left(\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_1(x)) + \left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_2(x)) + \\ + \left(\frac{k-w}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_3(x)) + \left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_4(x)), \quad (20)$$

for all $x \in \mathscr{J}$ *is a self mapping. Then* $\mathscr{K}$ *is a Banach contraction mapping with the metric* d *induced by* $\|\cdot\|$.

Proof. The line of proof of this theorem is the same as Theorem 2.1. Here, we highlight those parts which are different from the previous theorem.

Let $\mathscr{R}_1, \mathscr{R}_2 \in \mathscr{E}$. For each $u, v \in \mathscr{J}$ with $u \neq v$, we obtain

$$|\Theta_{u\neq v}| \leqslant \\ \leqslant \frac{1}{|u-v|}\left(\frac{w-j}{k-j}\right)\left(\frac{\tau(u)-j}{k-j}\right) \times \left[\frac{|(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_1(u)) - (\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_1(v))|}{|\mathscr{V}_1(u)-\mathscr{V}_1(v)|} \times |\mathscr{V}_1(u)-\mathscr{V}_1(v)|\right] \\ + \frac{1}{|u-v|}\left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(u)}{k-j}\right) \times \left[\frac{|(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_2(u)) - (\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_2(v))|}{|\mathscr{V}_2(u)-\mathscr{V}_2(v)|} \times |\mathscr{V}_2(u)-\mathscr{V}_2(v)|\right] \\ + \frac{1}{|u-v|}\left(\frac{k-w}{k-j}\right)\left(\frac{\tau(u)-j}{k-j}\right) \times \left[\frac{|(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_3(u)) - (\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_3(v))|}{|\mathscr{V}_3(u)-\mathscr{V}_3(v)|} \times |\mathscr{V}_3(u)-\mathscr{V}_3(v)|\right] \\ + \frac{1}{|u-v|}\left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(u)}{k-j}\right) \times \left[\frac{|(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_4(u)) - (\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_4(v))|}{|\mathscr{V}_4(u)-\mathscr{V}_4(v)|} \times |\mathscr{V}_4(u)-\mathscr{V}_4(v)|\right] \\ + \left(\frac{1}{k-j}\right)\left(\frac{w-j}{k-j}\right) \times \left[\frac{|(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_1(v)) - (\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_1(u^\star))|}{|\mathscr{V}_1(v)-\mathscr{V}_1(u^\star)|} \times |\mathscr{V}_1(v)-\mathscr{V}_1(u^\star)|\right]$$

$$+\left(\frac{1}{k-j}\right)\left(\frac{w-j}{k-j}\right)\times\left[\frac{|(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_2(u^\star))-(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_2(v))|}{|\mathscr{V}_2(u^\star)-\mathscr{V}_2(v)|}\times|\mathscr{V}_2(u^\star)-\mathscr{V}_2(v)|\right]$$
$$+\left(\frac{1}{k-j}\right)\left(\frac{w-j}{k-j}\right)\times\left[\frac{|(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_3(v))-(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_3(v^\star))|}{|\mathscr{V}_3(v)-\mathscr{V}_3(v^\star)|}\times|\mathscr{V}_3(v)-\mathscr{V}_3(v^\star)|\right]$$
$$+\left(\frac{1}{k-j}\right)\left(\frac{w-j}{k-j}\right)\times\left[\frac{|(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_4(v^\star))-(\mathscr{R}_1-\mathscr{R}_2)(\mathscr{V}_4(v))|}{|\mathscr{V}_4(v^\star)-\mathscr{V}_4(v)|}\times|\mathscr{V}_4(v^\star)-\mathscr{V}_4(v)|\right]. \quad (21)$$

Here, we discuss the following cases.

Case 1: If $v = u^\star = v^\star$, then by (21) we have

$$|\Theta_{u\neq v}| \leqslant \Lambda_2 \|\mathscr{R}_1-\mathscr{R}_2\|.$$

Case 2: If $v \neq u^\star$, $v = v^\star$, then by (21) we have

$$|\Theta_{u\neq v}| \leqslant \Lambda_2 \|\mathscr{R}_1-\mathscr{R}_2\|.$$

Case 3: If $v = u^\star$, $v \neq v^\star$, then by (21) we have

$$|\Theta_{u\neq v}| \leqslant \Lambda_2 \|\mathscr{R}_1-\mathscr{R}_2\|.$$

Case 4: If $v \neq u^\star \neq v^\star$, then by (21) we have

$$|\Theta_{u\neq v}| \leqslant \Lambda_2 \|\mathscr{R}_1-\mathscr{R}_2\|,$$

where Λ_2 is defined in (19). This gives that

$$d(\mathscr{K}\mathscr{R}_1,\mathscr{K}\mathscr{R}_2)=\|\mathscr{K}\mathscr{R}_1-\mathscr{K}\mathscr{R}_2\|\leqslant\Lambda_2\|\mathscr{R}_1-\mathscr{R}_2\|=\Lambda_2 d(\mathscr{R}_1,\mathscr{R}_2).$$

As a result of $0<\Lambda_2<1$, we can conclude that $\mathscr{K}$ is a Banach contraction mapping with metric d induced by $\|\cdot\|$. □

Theorem 2.7. *Consider the probabilistic functional equation (8) associated with (9). Assume that the condition $(\mathcal{A}_3)$ holds with $\Lambda_2<1$, where Λ_2 is defined in (19). Also, there is a nonempty subset $\mathscr{E}$ of $\mathscr{S}:=\{\mathscr{R}\in\mathscr{T}|\mathscr{R}(k)\leqslant k\}$ such that $(\mathscr{E},\|\cdot\|)$ is a Banach space, where $\|\cdot\|$ is given in (7), and the mapping $\mathscr{K}$ from $\mathscr{E}$ defined for each $\mathscr{R}\in\mathscr{T}$ by (20) is a self mapping. Then, the functional equation (8) with (9) has a unique solution in $\mathscr{E}$. Furthermore, the iteration $\mathscr{R}_n$ in $\mathscr{E}$ can be defined by*

$$(\mathscr{R}_n)(x)=\left(\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_1(x))+\left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_2(x))+$$
$$+\left(\frac{k-w}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_3(x))+\left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_4(x)), \quad (22)$$

for all $n\in\mathbb{N}$, where $\mathscr{R}_0\in\mathscr{E}$, converges to the unique solution of (8).

Proof. By coupling the Banach fixed point theorem with Theorem 2.6, we obtain the conclusion of this theorem. □

If $\mathscr{V}_1,\mathscr{V}_2,\mathscr{V}_3,\mathscr{V}_4:\mathscr{J}\to\mathscr{J}$ are contraction mappings with contractive coefficients $b_1\leqslant b_2\leqslant b_3\leqslant b_4$, respectively, then by Theorems 2.6 and 2.7, the outcomes are as follows.

Corollary 2.8. *Consider the probabilistic functional equation (8) associated with (9). Assume that the condition* $(\mathcal{A}_3)$ *holds with* $\tilde{\Lambda}_2 := 3b_4 < 1$. *Also, there is a nonempty subset* $\mathscr{E}$ *of* $\mathscr{S} := \{\mathscr{R} \in \mathscr{T} | \mathscr{R}(k) \leqslant k\}$ *such that* $(\mathscr{E}, \|\cdot\|)$ *is a Banach space, where* $\|\cdot\|$ *is given in (7), and the mapping* $\mathscr{K}$ *from* $\mathscr{E}$ *defined for each* $\mathscr{R} \in \mathscr{T}$ *by*

$$(\mathscr{K}\mathscr{R})(x) = \left(\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_1(x)) + \left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_2(x)) + \\ + \left(\frac{k-w}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_3(x)) + \left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_4(x)), \tag{23}$$

for all $x \in \mathscr{J}$ *is a self mapping. Then* $\mathscr{K}$ *is a Banach contraction mapping with the metric* d *induced by* $\|\cdot\|$.

Corollary 2.9. *Consider the probabilistic functional equation (8) associated with (9). Assume that the condition* $(\mathcal{A}_3)$ *holds with* $\tilde{\Lambda}_2 := 3b_4 < 1$. *Also, there is a nonempty subset* $\mathscr{E}$ *of* $\mathscr{S} := \{\mathscr{R} \in \mathscr{T} | \mathscr{R}(k) \leqslant k\}$ *such that* $(\mathscr{E}, \|\cdot\|)$ *is a Banach space, where* $\|\cdot\|$ *is given in (7), and the mapping* $\mathscr{K}$ *from* $\mathscr{E}$ *defined for each* $\mathscr{R} \in \mathscr{T}$ *by (23) is a self mapping. Then, the functional equation (8) with (9) has a unique solution in* $\mathscr{E}$. *Furthermore, the iteration* $\mathscr{R}_n$ *in* $\mathscr{E}$ *is defined as*

$$(\mathscr{R}_n)(x) = \left(\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_1(x)) + \left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_2(x)) + \\ + \left(\frac{k-w}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_3(x)) + \left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}_{n-1}(\mathscr{V}_4(x)), \tag{24}$$

for all $n \in \mathbb{N}$, *where* $\mathscr{R}_0 \in \mathscr{E}$, *converges to the unique solution of (7).*

Remark 2.10. *The authors of [3, 4, 6] utilized the boundary conditions to prove their major findings. However, compared to them, our results are independent of such conditions.*

We now offer the following example to enhance our findings.

Example. Consider the probabilistic functional equation given below

$$\mathscr{R}(x) = wx\mathscr{R}\left(\frac{x}{16} + \frac{1}{11}\right) + w(1-x)\mathscr{R}\left(\frac{x}{6}\right) + (1-w)x\mathscr{R}\left(\frac{x}{19} + \frac{1}{33}\right) + (1-w)(1-x)\mathscr{R}\left(\frac{x}{12}\right) \tag{25}$$

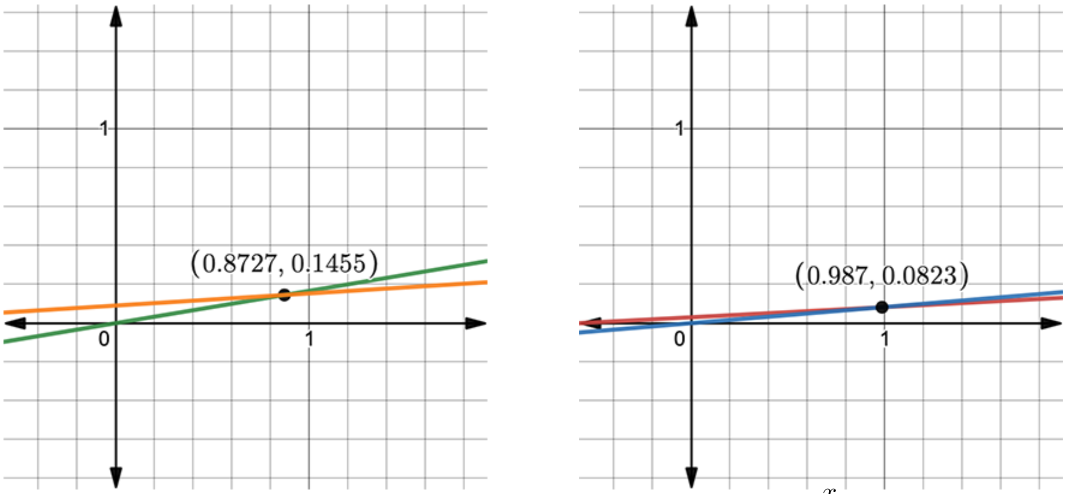
for all $x \in \mathscr{J} = [0,1]$ and $\mathscr{R} \in \mathscr{T}$. If we set the mappings $\tau, \mathscr{V}_1, \mathscr{V}_2, \mathscr{V}_3, \mathscr{V}_4 : \mathscr{J} \to \mathscr{J}$ by

$$\tau(x) = x, \quad \mathscr{V}_1(x) = \frac{x}{16} + \frac{1}{11}, \quad \mathscr{V}_2(x) = \frac{x}{6}, \quad \mathscr{V}_3(x) = \frac{x}{19} + \frac{1}{33}, \quad \mathscr{V}_4(x) = \frac{x}{12},$$

for all $x \in \mathscr{J}$. So, our equation (25) is decreased to the equation (8). It is easy to see that $\mathscr{V}_1, \mathscr{V}_2, \mathscr{V}_3, \mathscr{V}_4$ satisfy our boundary conditions (9). Also,

$$|\mathscr{V}_1 u - \mathscr{V}_1 v| \leqslant \frac{1}{16}|u-v|, \ |\mathscr{V}_2 u - \mathscr{V}_2 v| \leqslant \frac{1}{6}|u-v|, \ |\mathscr{V}_3 u - \mathscr{V}_3 v| \leqslant \frac{1}{19}|u-v|, \ |\mathscr{V}_4 u - \mathscr{V}_4 v| \leqslant \frac{1}{12}|u-v|$$

for all $u, v \in \mathscr{J}$. This implies that $\mathscr{V}_1 - \mathscr{V}_4$ are contraction mappings with coefficients $b_1 = \frac{1}{16}$, $b_2 = \frac{1}{6}$, $b_3 = \frac{1}{19}$ and $b_4 = \frac{1}{12}$ respectively. Also, there exist points $u^\star, v^\star \in [0,1]$ such that $\mathscr{V}_1(u^\star) = \mathscr{V}_2(u^\star)$ and $\mathscr{V}_3(v^\star) = \mathscr{V}_4(v^\star)$ (see Fig. 1).



(a) $\mathscr{L}_1(x) = \dfrac{x}{16} + \dfrac{1}{11}$ (orange) and $\mathscr{L}_2(x) = \dfrac{x}{6}$ (green)

(b) $\mathscr{L}_3(x) = \dfrac{x}{19} + \frac{1}{33}$ (red) and $\mathscr{L}_4(x) = \dfrac{x}{12}$ (blue)

Fig. 1. Graphs of $\mathscr{L}_1(x)$, $\mathscr{L}_2(x)$, $\mathscr{L}_3(x)$, *and* $\mathscr{L}_4(x)$

Moreover, $\Lambda_2 = \dfrac{47w+86}{456} < 1$, for all $w \in [0,1]$, and there is a nonempty set $\mathscr{E}$ of $\mathscr{S} := \{\mathscr{R} \in \mathscr{T} | \mathscr{R}(1) \leqslant 1\}$ such that $(\mathscr{E}, \|\cdot\|)$ is a Banach space, and the mapping $\mathscr{K}$ from $\mathscr{E}$ defined in (25) for all $x \in \mathscr{J}$ is a self mapping, thus it fulfill all the requirements of Theorem 2.6, and therefore, we get the results related to the existence of the given equation (25)' solution.

If we define $\mathscr{R}_0 = x$ as a starting approximation, then by Theorem 2.7, the iteration stated below converges to a unique solution of (25):

$$
\begin{aligned}
\mathscr{R}_1(x) &= \frac{1}{10032}\left[-737wx^2 - 308x^2 + 1444wx + 1140x\right],\\
\mathscr{R}_2(x) &= \frac{wx}{28250112}\left[-8107wx^2 - 3388x^2 + 230560wx + 190784x + 352512w + 284672\right] +\\
&\quad + \frac{w(1-x)}{361152}\left[-737wx^2 - 308x^2 + 8664wx + 6840x\right] +\\
&\quad + \frac{(1-w)x}{358533648}\left[72963wx^2 - 30492x^2 + 2632146wx + 2109228x + 1539665w + 1224512\right] +\\
&\quad + \frac{(1-w)(1-x)}{1444608}\left[-737wx^2 - 308x^2 + 17328wx + 13680x,\right]\\
&\cdots\\
\mathscr{R}_n(x) &= wx\mathscr{R}_{n-1}\left(\mathscr{V}_1(x)\right) + w(1-x)\mathscr{R}_{n-1}\left(\mathscr{V}_2(x)\right) + (1-w)x\mathscr{R}_{n-1}\left(\mathscr{V}_3(x)\right) +\\
&\quad + (1-w)(1-x)\mathscr{R}_{n-1}\left(\mathscr{V}_4(x)\right)
\end{aligned}
$$

for all $n \in \mathbb{N}$.

3. Stability analysis of the proposed probabilistic functional equation

Now, we shall discuss the stability of the suggested functional equation (7) (for the details of stability, we refer [16], [17].

Theorem 3.1. *Under the hypothesis of Theorem 2.1, the equation* $\mathscr{K}\mathscr{R} = \mathscr{R}$*, where* $\mathscr{K} : \mathscr{E} \to \mathscr{E}$ *is defined as*

$$\begin{aligned}(\mathscr{K}\mathscr{R})(x) \;=\; & \left(\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_1(x)) + \left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_2(x)) \;+ \\ & + \left(\frac{k-w}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_3(x)) + \left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_4(x)), \quad (26)\end{aligned}$$

for all $\mathscr{R} \in \mathscr{E}$ *and* $x \in \mathscr{J}$*, has Hyers-Ulam-Rassias stability; that is, for a fixed function* $\varphi : \mathscr{E} \to [0, \infty)$*, we have that for every* $\mathscr{R} \in \mathscr{E}$ *with* $d(\mathscr{K}\mathscr{R}, \mathscr{R}) \leqslant \varphi(\mathscr{R})$*, there exists a unique* $\tilde{\mathscr{R}} \in \mathscr{E}$ *such that* $\mathscr{K}\tilde{\mathscr{R}} = \tilde{\mathscr{R}}$ *and* $d(\mathscr{R}, \tilde{\mathscr{R}}) \leqslant \varsigma\varphi(\mathscr{R})$ *for some* $\varsigma > 0$.

Proof. Let $\mathscr{R} \in \mathscr{E}$ such that $d(\mathscr{K}\mathscr{R}, \mathscr{R}) \leqslant \varphi(\mathscr{R})$. From Theorem 2.1, there exists a unique $\tilde{\mathscr{R}} \in \mathscr{E}$ such that $\mathscr{K}\tilde{\mathscr{R}} = \tilde{\mathscr{R}}$. Then we have

$$d(\mathscr{R}, \tilde{\mathscr{R}}) \leqslant d(\mathscr{R}, \mathscr{K}\mathscr{R}) + d(\mathscr{K}\mathscr{R}, \tilde{\mathscr{R}}) \leqslant \varphi(\mathscr{R}) + d(\mathscr{K}\mathscr{R}, \mathscr{K}\tilde{\mathscr{R}}) \leqslant \varphi(\mathscr{R}) + \Lambda_1 d(\mathscr{R}, \tilde{\mathscr{R}}),$$

where Λ_1 is defined in (13), and so

$$d(\mathscr{R}, \tilde{\mathscr{R}}) \leqslant \varsigma\varphi(\mathscr{R}),$$

where $\varsigma := \dfrac{1}{1-\Lambda_1}$. This completes the proof. □

From the above analysis, we obtain the following result related to the Hyers-Ulam stability.

Corollary 3.2. *Under the hypothesis of Theorem 2.1, the equation* $\mathscr{K}\mathscr{R} = \mathscr{R}$*, where* $\mathscr{K} : \mathscr{E} \to \mathscr{E}$ *is defined as*

$$\begin{aligned}(\mathscr{K}\mathscr{R})(x) \;=\; & \left(\frac{w-j}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_1(x)) + \left(\frac{w-j}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_2(x)) + \\ & + \left(\frac{k-w}{k-j}\right)\left(\frac{\tau(x)-j}{k-j}\right)\mathscr{R}(\mathscr{V}_3(x)) + \left(\frac{k-w}{k-j}\right)\left(\frac{k-\tau(x)}{k-j}\right)\mathscr{R}(\mathscr{V}_4(x)), \quad (27)\end{aligned}$$

for all $\mathscr{R} \in \mathscr{E}$ *and* $x \in \mathscr{J}$*, has Hyers-Ulam stability; that is, for a fixed* $\nu > 0$*, we have that for every* $\mathscr{R} \in \mathscr{E}$ *with* $d(\mathscr{K}\mathscr{R}, \mathscr{R}) \leqslant \nu$*, there exists a unique* $\tilde{\mathscr{R}} \in \mathscr{E}$ *such that* $\mathscr{K}\tilde{\mathscr{R}} = \tilde{\mathscr{R}}$ *and* $d(\mathscr{R}, \tilde{\mathscr{R}}) \leqslant \varsigma\nu$*, for some* $\varsigma > 0$.

Conclusion

The predator-prey paradigm, especially in a two-choice situation, is one of the most exciting frameworks in mathematical biology. A predator has two possible prey choices in these models, and the solution exists when the predator is fixed to one of them. We extended the research by introducing a generic stochastic functional equation in this paper that may cover a wide range of learning theory models in the existing literature. The existence, uniqueness, and stability of the proposed stochastic equation's solution were investigated using a fixed-point method. Our techniques do not rely on the boundary conditions discussed in [6, 9], which implies that the proposed results cover more problems than the results described in the literature. Our method is unique, and it may be used to solve a wide variety of mathematical models in the fields of mathematical psychology and learning theory.

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Подход с фиксированной точкой для изучения класса вероятностных функциональных уравнений, возникающих в психологической теории обучения

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Аннотация. *Многие биологические модели и модели теории обучения были исследованы с использованием вероятностных функциональных уравнений. В этой статье основное внимание уделяется особому типу отношений хищник–жертва, в котором у хищника есть два варианта добычи, каждый с вероятностью* x *и* $1-x$*, соответственно. Наша цель состоит в том, чтобы исследовать реакцию животного в таких ситуациях, предложив общее вероятностное функциональное уравнение. Заслуживающие внимания результаты с фиксированной точкой используются для исследования существования, единственности и устойчивости решений предложенного функционального уравнения. Приведен также пример, иллюстрирующий важность наших результатов в этой области исследований.*

Ключевые слова: *вероятностные функциональные уравнения, устойчивость, неподвижные точки.*