Malfunction analysis and safety of mathematical models of technical systems

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Abstract. In this article we discuss methods of computing the guaranteed values of the states of a technical system in order to estimate the safety of a technical system. For most models describing the problems of science and technology, it is necessary to analyze the effect of disturbances on the state of an object or system. It is necessary to evaluate the boundaries of the zones of dangerous states and the boundary values of the system parameters. The danger of a system functioning is a threat, possibility, probability of damage, system catastrophe, that is, potential damage to a technical system in certain conditions and situations. An analysis of the boundaries of the safety areas of technical systems helps to obtain quantitative estimates of the possibility of dangerous situations. Used methods are based on the approximation of the shift operator along the trajectory, and taking into account the influence of constantly acting perturbations on the solutions.

1. Introduction

The article describes the result of applying the methods, which evaluate for these models the regions of solutions under the influence of finite, constantly acting disturbances, that is, check the guaranteed safety conditions (under the influence of all factors) and the reliability of technical systems. The appearance of a malfunction in the model of the system [1-3] corresponds to the intersection of the trajectory of the system with some predetermined surface at any a priori unknown time of the system's operation, at any point inside this surface. In many problems, a technical system (TS) is an object that is artificially designed and includes elements (components that have different properties, function differently in interactions), interconnected (transmission lines of units or flows of something) and interacting with each other and with external influences, in the end, to perform a sequence of actions that change or maintain the state of the TS and realize the goal of the TS (function, purpose, role). The mathematical model of the TS is recorded using mathematical symbols and dependencies and describes the functioning of the TS in the environment, determines the output parameters and characteristics, and evaluates the efficiency and quality of the vehicle, etc. To describe and compile the design of the TS, a set of types of mathematical models is applied, depending on the level of the hierarchy of the system, degrees of decomposition of the system, value, and the stage of studying the TS.

2. Mathematical model

In this approach, the mathematical model of technical system is considered as a dynamic system, the motion of which is given by a system of ordinary differential equations

$$y' = f(t, y) \tag{1}$$

where $y(t), f(t, y)$ are the $n-$ dimensional vector functions. The initial conditions of equation (1) belong to some limited region Z^0 of the state space of the system Y. The right-hand side $f(t, z)$ of this system satisfies the conditions of the existence and uniqueness theorem for a solution in a certain region of space (t, z) .A malfunction is considered to be a change in the functional state in the control system that leads to unacceptable deviations of the system (1) from the control goal, that is, from the programmed movement causes of malfunctions are not considered.

According to [1-3], system (1) is transformed to a composite system of equations that takes into account the structure of the work and control of the TS

$$\begin{aligned} &y' = D(y) + A(y)\xi, \\ &\xi' = \Phi(\delta), \\ &\delta = By + \varphi(\sigma), \\ &\sigma = Cy. \end{aligned} \tag{2}$$

System (2) can be transformed again to (1) by using of elimination of some unknown variables . Here, $y - n -$ dimensional phase state vector of the system, $D(y), A(y) -$ continuous matrix functions of dimension $n \times n$, $n \times 3$, $\sigma, \delta -$ vectors of dimension 3. Next, $\xi -$ is a control vector of dimension 3, $B, C -$ are matrices of dimension 3 with constant coefficients, satisfying the conditions

$$\begin{aligned} &b_{ij} \in \left[\underline{b}_{ij}, \bar{b}_{ij}\right], \ \underline{b}_{ij} \le \bar{b}_{ij}, i = 1, 2, 3. \\ &c_{ij} \in \left[\underline{c}_{ij}, \bar{c}_{ij}\right], \ \underline{c}_{ij} \le \bar{c}_{ij}, \ j = 1, 2, 3, \end{aligned} \tag{3}$$

where $\underline{b}_{ij}, \bar{b}_{ij}, \underline{c}_{ij}, \bar{c}_{ij} -$ are some positive constants. To take into account possible restrictions on the operation of the TS in the model, vector functions $\Phi(\delta), \varphi(\delta)$ are introduced that are defined and continuous for all values δ_k, σ_k, $k = 1, 2, 3$. These functions will belong to the class of permissible characteristics, under the following conditions:

$$\begin{aligned} &1.\ \Phi_k(\delta_k) = 0, \text{ if } \delta_k = 0; \varphi_k(\delta_k) = 0, if\, \sigma_k = 0, \\ &2.\ \delta_k \Phi_k(\delta_k) > 0, \text{ if } \delta_k \ne 0;\ \sigma_k \varphi_k(\delta_k) > 0, \text{ if } \sigma_k \ne 0 \end{aligned} \tag{4}$$

The integrals within the limits $[0, +\infty], [-\infty, 0]$ of these functions diverge, on the basis of this it is possible to prove the convergence (operation of the TS) of the model solutions for $t \to \infty$. To eliminate the complexity of the algorithm, we consider all components of the phase vector y to be accessible to measurement at any time. The last three equations in (2) describe the control system of a dynamic object. The aim of the control is to preserve the trajectory $y(t)$ in a neighborhood close to a certain program trajectory y_s.

Let malfunctions appear in the process of functioning in a controlled system. The control signal ξ generated by the control system of the object, is not correct, that is, the control of the object no longer provides proximity to the location of the trajectory $y(t)$ to the trajectory $y_s(t)$. For these malfunctions, specific values of the matrix coefficients B, C and types of functions φ, Φ can be associated. Suppose that it is possible to determine set $l-$ faults, each of which has its own matrices B_i, C_i and functions φ_i, Φ_i, $i = 1, 2, ..., l$ and solutions $y_i(t)$ of systems $y_i' = f_i(t, y_i)$ with initial conditions $y_i(t_0) \in Y_i^0$. The right-hand sides of these systems are calculated by substituting matrix elements B_i, C_i and functions φ_i, Φ_i, $i = 1, 2, ..., l$ in (2) and further transforming the resulting expression. Further research is reduced to determining the fault number due to a given set of faults, that is, establishing the number i of the function f_i, $i = 1,..,l$, which is substituted for the function f on the right-hand side of (1), at some point in time t_0.Here t_0 is the moment of occurrence of a malfunction in the facility control system. It is useful to note that information about the control system in (1) is created by sensors, devices that use and transform the trajectory information of the object.

Sensors that process trajectory information of various physical nature, for example, trajectory information of different dimensions, are considered different. In [2], the classification was developed in relation to malfunctions in the aircraft control system; it is easy to transfer to other components of the aircraft or expand to other vehicles. A necessary condition for such an extension is the presence of a description of elements that may turn out to be faulty in the mathematical model of the movement of objects. In order to control the intersection of the trajectories of the mathematical models of the TS on given surfaces, it is proposed to use methods for estimating the sets of systems of differential equations with perturbing influences. An analysis of the boundaries of these sets allows us to study the features of the functioning of the TS in various conditions, to design their required characteristics and to reduce the risk of emergency situations. Evaluation methods are based on writing the symbolic form of the solution to the ODE system, depending on the initial data and system parameters specified in the form of symbols [6-12].

From the definition of a shift operator along the trajectories of a differential equation, the problem of finding an arbitrary solution to this equation is equivalent to the problem of finding a shift operator.

It is usually impossible to write down the shift operator using one formula in explicit form (i.e., to find all solutions of the equation). In addition, formulas describing the explicit form of the operator (in cases where this is possible) turn out to be cumbersome in most cases and do not provide the required information about the solutions themselves. Because of this, it is better to interpret the ODE system as the law of motion of the image point in phase space. The construction (recording) of symbolic formulas for approximate solutions as vector functions where vector is a vector of initial values considered as symbolic values is based on the transformation and storage of formulas for approximate solutions (formulas of numerical methods).

The formula of each component of the symbol vector is constructed at each point t^k as a function depending on the symbol initial data $y_1^0, ..., y_n^0$, and substitutions are not used. When this method is completed, after constructing the symbolic decision formula, the range of values of this formula is calculated. The method is based on the execution from step to step of storage and processing of symbolic information. There is a movement along the trajectory of the solutions. Information about the formula of solution is stored permanently based on the processing of memory addresses using stream processing functions.

3. An example of controlling the exit of a trajectory to a given surface

When landing aircraft, it is required to determine the visibility range on the runway, depending on the height of the decision. It is required at the time of the decision to move the system to some area of the state space. This means calculating the boundaries of the region of possible states of the runway in which the plane enters for any admissible number of extreme disturbances, i.e., check $D \subset W$.

Let two phase variables determine the position of the aircraft on the plane, the third coordinate is equal to the angle of the direction of the velocity vector. The plane moves in a horizontal plane. The control (scalar quantity) is equal to the instantaneous radius of the rotation of the velocity vector. Now let define the forces acting on the aircraft: the resistance to movement D, traction T, lifting force L, gravity mg. Vector quantities are located in the plane of symmetry and are orthogonal to the lifting force. This provides a coordinated U-turn. The values of the thrust angle of attack ε, the angle of heel (the angle between the vertical plane and the plane of symmetry) μ are defined. We believe that the plan moves at a constant speed. If all these conditions are met, it is possible to describe the problem of observing the movement of an aircraft in the horizontal plane using a system of ordinary differential equations

$$\begin{aligned}
&\frac{dy_1}{dt} = V\cos\varphi,\\
&\frac{dy_2}{dt} = V\sin\varphi,\\
&\frac{d\varphi}{dt} = \frac{ku}{V},\\
&V = const > 0,\ k = const > 0,\ |u| \le 1
\end{aligned} \tag{5}$$

For numerical estimation, the problem is formulated as the problem of constructing a tube of reachability sets, as well as finding trajectories passing near the center of each time section. Solving this problem analytically even in this formulation is difficult. In fig.1 the numerical boundaries of inclusions of reachability set are shown for the geometric coordinates of system (5) with parameters

$$V = 6\frac{m}{\sec}, k = 1\frac{m}{\sec^2}$$

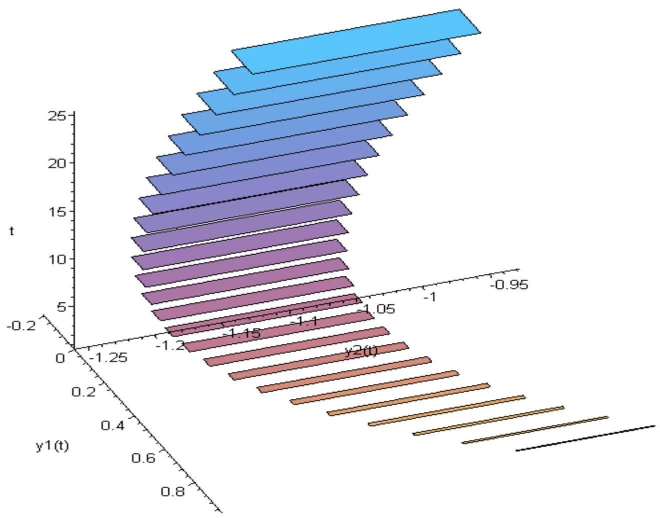

Fig. 1.Boundaries of inclusions of reachability sets

3. Conclusions

The diagnostic problem is realized by tracking the trajectory of the object, starting from the point of exit of the trajectory to the control surface. In this work, it was demonstrated that it is possible to solve the problem of differential diagnostics of mathematical models of technical systems by implementing external control over trajectories. Fig. 1.Boundaries of inclusions of reachability sets

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