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Determining the number of redundant elements of the distribution network in compliance with the specified amount of information entropy

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Abstract. A mathematical economic model for searching an optimal solution is proposed. In addition, a measure of information uncertainty is proposed to solve the problem the construction of a distribution network structure with the redundancy. In the model, there is an objective function for searching minimum costs and the equations for determining an information entropy of operable and non-operable states of the distribution network. The equations are based on the approaches of Ralph Hartley and Claude Shannon for entropy determination. An important place in the mathematical model is the process of constructing of a constraint function to search an optimal solution. The right part of constraints is entropy, its value reflects requirements to provide the required reliability level of power supply of the consumer. The left part is used to determine the connection entropy between an energy source and the consumer. An example of determining the number of redundant elements is offered.

1. Introduction

When designing electrical distribution networks, one of the tasks is the choice of its optimal structure by the criteria of minimum costs and a reliability level [1]. In its turn, the reliability level is taken into account not as a quantitative form, but as a qualitative one. Therefore, it is difficult to find solutions and it doesn't allow using optimization methods actively [2]. To make the formalization of an optimization problem more reasonable, it is better to use both criteria as a quantitative form. If one takes this problem as the reliability analysis of distribution networks and the application of an information theory [3], then a promising direction can be considered as a connection between tools for determining a structural reliability and models of the accounting of the information uncertainty of a structure state [4]. The information uncertainty (its measure) is one of the criteria of reliability evaluation of technical systems [5].

The mathematical model for constructing the optimal structure of the electrical distribution network with the redundancy is offered. When solving the problem, one can be focused to fulfill the criteria of the minimum costs and to ensure a specified reliability level expressed through the measure of information uncertainty (entropy).



2. Information uncertainty in the problem of state estimation of a network structure

The distribution network is a complex technical object, which is impacted by a variety of random effects, indicating the presence of uncertainty in its further behavior. The result of undesirable effects is an event, which can change an electric equipment operation mode (a network element) and then transfer it from an operable state to non-operable one [6]. Accumulation, processing and systematization of information about the states of network elements enlarges the knowledge and allows to develop more effective solutions for the removal of uncertainty in the design of networks.

When using the structural reliability analysis of a network, the informative value can be characterized by information entropy [7]. It allows to express the variability of the structure from impact of random factors both as a quantitative form and as a qualitative one [8]. The constructing of a network structure is connected with its probable state and, therefore, doesn't exclude the calculation of the amount of information entropy, which can to be compared with the value of the boundary entropy. The boundary entropy differentiates the entropy (allowed with reliability conditions) and entropy, the value of which shows inappropriate requirements to ensure a given reliability level.

To express the state of the structure and its reliability using a measure of uncertainty, the authors have published a number of works about the possibility to apply the entropic approach to the analysis of the structure [9-11]. This approach takes into account the fundamental principles of system perspective on the role of information entropy in animate and inanimate nature [12-13].

3. The construction model of a distribution network

At the initial stage of solving the optimization problem one involves the preparation of input data (building structure, determining of a set of indicators, which characterize the states of network elements). In addition, while determining the amount of entropy one should observe a set of conditions: to consider only statistical data about the state of a network structure; to consider the simplest discrete states; to use the logarithmic measure based on the classical K. Shannon's approach [14-16].

The structure of a distribution network has a tree view, which is formed as follows: the constructing of branches (links) from the energy source to the end consumer or consumers. Such a tree can be represented as a cyclic graph $G = (U, X)$, where U – a set of vertices (which simulates the only source and consumers), X – a set of edges (which simulate the connecting elements of a network). The set U consists of: u_0 – an initial vertex ("source"); $u_j \in U_1$ – a set of the end-vertices (end consumers in a network tree); $u_k \in U_2$ – a set of fictitious vertices (an attachment point of network elements (edges) to each other). All sets of tree branches (constructed connections) $M_j \subseteq G$. In addition, each j -th branch (a connection between the source and the consumer) is a set of edges m_j , $(u_0 = 1, u_j = 1, u_k) \in M_j$. For all edges: $x_i \in X$ ($i=1, 2, \dots, n$). Each edge x_i is assigned: c_i – cost of i -th element and the probability of an event occurrence (changing an element state). Each vertex $u_j \in U_1$ ($j=1, 2, \dots, m$) is assigned the boundary value of entropy, its value is a set of constructing of an optimization problem.

Having a graph of a network, it is necessary to determine the amount of an entropy for operable and non-operable states of all elements. In addition, the C. Shannon's classical model [17] is used. This model allows to determine the information I according to a qualitative character:

$$I = -\left(\sum_{i=1}^{N_1} p_i \log p_i + \sum_{i=1}^{N_2} q_i \log q_i\right), \text{ by } \sum_{i=1}^{N_1} p_i + \sum_{i=1}^{N_2} q_i = 1, \quad (1)$$

where p_i and $q_i=1-p_i$ – the probability of the operable and non-operable states of the element i (determined based on statistical processing of data: failure rate; time to failure; recovery time, etc.), N_1 and N_2 – a number of operable and non-operable states of an element.

The expression (1) is valid provided if the events (e.g., failures of network elements) have a stochastic nature, independent from each other and most of them are subjects of statistical laws of

distribution. Under this condition, one can determine the information entropy for one element i , and the total entropy of all network elements:

$$H_{\Sigma} = \sum_{i=1}^n [H(p_i) + H(q_i)] = - \sum_{i=1}^n (p_i \log_2 p_i + q_i \log_2 q_i), \quad (2)$$

where $H(p_i) = -p_i \log_2 p_i$ and $H(q_i) = -q_i \log_2 q_i$ – respectively, the entropy of operable and non-operable states of an element i , n – a number of elements in a network structure. Here the logarithm base equaled 2 demonstrates two different opposite states. The expression $p_i + q_i = 1.0$ is observed for all elements i . If for all elements: $p_i = q_i = 0.5$, then the maximum entropy: $H_{\Sigma} = n$. In the process of finding the optimal solution, value of H_{Σ} will vary and depend not on a number of network elements but on a number of redundant elements. While a number of redundant elements is increasing, H_{Σ} is increasing too.

The solution of the optimization problem using the graph comes down to determining the value of an objective linear function: $\min \langle c_i, x_i \rangle$. As for constraints, to construct them is a complex problem. As the graph has a tree shape, then each of its branch is a parallel-series connection of elements (edges) and this connection shows redundancy. In fact, a branch is a separate graph consisting of paths and sections. The entropy is determined through the probability of operable and non-operable element states. To construct the problem constraint equations one can use mathematical equations offered in [18]. Based on these equations and according to the theorems of addition and multiplication of probabilities, let's construct a mathematical expression of the entropy of a branch.

Let's consider the j -th branch u_0-u_j , which connects the source u_0 with the consumer u_j . This branch with redundancy has a parallel-serial structure, in which parallel-connected elements – the section i , and series-connected elements – the path j . Let's introduce notations: n – a number of sections on the path from the source to the consumer (for networks without reserve, n – a number of path elements); $i = \overline{1, n}$ – a sequence number of a section (or an element); m_i – a number of redundant elements in a section i ; $k = \overline{1, m_i}$ – a number of element in a section i . Let's construct a mathematical expression of a constraint problem according to the considered j -th branch.

Considering the path, one can determine:

- the probability of the non-operable state of a section i :

$$Q_i = \prod_{k=1}^{m_i} q_k, \quad (3)$$

where k – a numerical number of an element of a section i ; q_k – the probability of an non-operable state of an element k of a section i ;

- the probability of an operable state of a section i :

$$P_i = 1 - Q_i; \quad (4)$$

- an entropy of an non-operable state of a section i :

$$H(Q_i) = \sum_{k=1}^{m_i} [(\prod_{\substack{k=1 \\ k \neq i}}^{m_i} q_k) H(q_i)] = - \sum_{k=1}^{m_i} (Q_i \cdot \log_2 q_i); \quad (5)$$

- an entropy of an operable state of a path j with the redundancy:

$$H(P_j) = \sum_{i=1}^n [\prod_{\substack{k=1 \\ k \neq i}}^n P_k H(P_i)], \quad (6)$$

where k – a numerical number of an element of a section i ; P_i or P_k – the probability of a non-failure operation of a section i ;

- entropy of a non-operable state of a path j with the redundancy:

$$H(Q_j) = \sum_{i=1}^n [H(Q_i) + (1 - \prod_{\substack{k=1 \\ k \neq i}}^n P_k) H(P_i)] \quad (7)$$

where k – a numerical number of an element of a section i on a path j .

When considering the loaded redundancy, let's note the following: elements (reserving each other are in the same operating conditions) have approximately equal parameters and properties, therefore, the probabilities of a operable state of elements of a section i are taken equal: $q_1 = \dots = q_k = \dots = q_m$. Then a mathematical model of the entropy of the path j with the redundancy one is going to introduce further.

In expressions (3) and (5) a number of redundant elements m_i in the section i is unknown and is indicated as x_i , then the expression (3) is:

$$Q_i = q_i^{x_i} \quad (8)$$

where q_i – a probability of a non-operable state of an element k in a section i ; x_i – a number of redundant elements in a section i , which must be determined by the condition of an optimization problem.

According to (8):

$$P_i = 1 - Q_i = 1 - q_i^{x_i} \quad (9)$$

For section i :

- an entropy of a non-operable state of a section i according to the substitution (8) into (5):

$$H(Q_i) = x_i q_i^{x_i - 1} H(q_i) = -x_i q_i^{x_i} \cdot \log_2 q_i \quad (10)$$

- the same for an operable state:

$$H(P_i) = x_i [H(p_i) + H(q_i) - q_i^{x_i - 1} H(q_i)] \quad (11)$$

The entropy of section I according to (10) and (11): $H_i = H(P_i) + H(Q_i) = x_i [H(p_i) + H(q_i)]$.

With regard to the optimization problem, when performing series of transformations (considering the substitution of expressions (8)-(11) into (6) and (7)), one can obtain equations with an unknown x to determine $H(P_j)$ and $H(Q_j)$. However, such equations in the expanded form are very cumbersome, that complicates their implementation as constraints of the problem of finding optimal solutions. Nevertheless, they can be reduced under certain conditions: in particular, the second term in (7) will have a small quantity (according to the first term) if the probability of a non-failure operation P_i is closed to 1. For example, if all $p_i = 0.95$, the first and the second terms differ from each other more than 30 times, and for all $p_i = 0.99$ more than 200 times. As the solution of redundancy problems requires a high reliability level of a distribution network, then according to statistics probabilities of non-failure operation of the elements they are ranged $0.99 \div 0.999$. Therefore, one can neglect the second term in (7) and write the equation for determining the entropy of a non-operable state of the path with redundancy as:

$$H(Q_j) \approx \sum_{i=1}^n H(Q_i) = \sum_{i=1}^n x_i q_i^{x_i} \cdot (-\log_2 q_i) \quad (12)$$

This expression can be considered as constraints to search an optimal solution of a redundancy

problem. The mathematical model is:

$$\sum_{j=1}^L \sum_{i=1}^n c_{ji} x_j \rightarrow \min, \tag{13}$$

$$\begin{aligned} \sum_{i=1}^{n_1} x_{1i} q_{1i}^{x_{1i}} \cdot (-\log_2 q_{1i}) &\leq H^0(Q_1); \\ \dots \dots \dots & \\ \sum_{i=1}^{n_L} x_{Li} q_{Li}^{x_{Li}} \cdot (-\log_2 q_{Li}) &\leq H^0(Q_L), \end{aligned} \tag{14}$$

In the given model: L – a number of the paths of a distribution network; n_j – a number of sections (elements) of a branch j ; $H^0(Q_j)$ – a boundary value of entropy of a non-operable state of a branch j ; c_{ji} – a cost of an element i , which is included in the path j .

Let’s come to the question how to determine the value $H^0(Q_j)$. According to the reliability level of a network, one can determine this value if the designer admits the probable break (pause) in the power supply of the group or the individual consumer. The boundary entropy of a path is determined by the expression (according to [19]):

$$H^0(Q_j) = q_j^0 \log_2 q_j^0, \tag{15}$$

where $q_j^0 = M_{qj}/T$ – the probability of a possible break in power supply of consumers on a path j ; M_{qj} – an average time (in hours) for a possible break of power supply on a path j is allowed; T – a number of hours of a year.

The solution (13)-(14) refers to a nonlinear programming problem and is not considered in the given paper. However, let’s consider an example how to find a number of redundant elements for any branch of a network structure.

4. Example

The distribution network as a graph consisting of $n=4$ branches (elements) connected in series is considered. It is necessary to determine a number of redundant elements for each section of a path (a branch): x_1, x_2, x_3 and x_4 . The index j is ignored in calculations. The source data of a task: the probabilities of an operable state of elements – $p_1= 0.94, p_2=0.95, p_3= 0.97, p_4= 0.99$; the cost of all elements of $i - c_1=c_2=c_3=c_4$.

Solution. Let’s determine the probabilities of a non-operable state of elements: $q_1= 0.06, q_2=0.05, q_3= 0.03, q_4= 0.01$.

The entropy of the elements according to (2): $H(p_1)=0.084; H(q_1)=0.243; H(p_2)=0.07; H(q_2)= 0.116; H(p_3)= 0.042; H(q_3)= 0.151; H(p_4)=0.014; H(q_4)= 0.066$.

The value of a boundary entropy for the path is determined according to the expression (15): let’s accept the annual average duration for a non-operable state of the path $M_q= 80$ hours $T = 8760$ hours; the probability of an allowable failure in power supply of the path – $q^0 = M_q/T=80/8760=0.009$; a boundary entropy:

$$H^0(Q) = -q^0 \log_2 q^0 = -0.011 \cdot \log_2 0.011 = 0.061 \text{ bit.}$$

Let’s find the solution by the expression according to (12):

$$H(Q) \approx -x_1 q_1^{x_1} \cdot \log_2 q_1 - x_2 q_2^{x_2} \log_2 q_2 - x_3 q_3^{x_3} \log_2 q_3 - x_4 q_4^{x_4} \log_2 q_4. \tag{16}$$

If there are no redundant elements ($x_i = 1$):

$$\begin{aligned} H(Q) &\approx -q_1 \cdot \log_2 q_1 - q_2 \cdot \log_2 q_2 - q_3 \cdot \log_2 q_3 - q_4 \cdot \log_2 q_4 = \\ &= -0.06 \cdot \log_2 0.06 - 0.05 \cdot \log_2 0.05 - 0.03 \cdot \log_2 0.03 - 0.01 \cdot \log_2 0.01 = 0.677 \text{ bit.} \end{aligned}$$

The condition $H(Q) \leq H^0(Q)$ is not met: $0.677 \gg 0.061$. When substituting in (16) required values, performing the substitutions of integer values x and the condition is met:

$$H(Q) \leq H^0(Q) \Rightarrow 0.060 \leq 0.061,$$

One obtains the results: $x_1=2$, $x_2=2$, $x_3=2$ and $x_4=2$. Thus, each element in the path requires one redundant element.

5. Conclusion

The structure construction problem of a distribution network with the redundancy can be solved if one takes into account the measure of information uncertainty. The proposed mathematical model is quite simple and applicable for networks of a tree structure of any complexity. An important place in the optimization model is given to the calculation of the information entropy, which has been differentiated into two components: the entropy of the operable and non-operable states of a technical system. The search of minimum costs for the constructing of a network with the redundancy is connected with the accounting of information measure, observing the reliability of a network structure. It is possible to construct a distribution network with minimal costs if one has determined the boundary entropy of operable and non-operable states, compared it with the appropriate value of the considered part of a network and applied the method of nonlinear programming.

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