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Plastic bending of the waveguide tubes with rectangular cross-section

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Plastic bending of the waveguide tubes with rectangular cross-section

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Abstract. This paper focuses on a new mathematical model of plastic bending of waveguide tubes with rectangular cross-section. The analysis of the known approaches and decisions is carried out, their shortcomings are shown and a new model of a thin-walled waveguide on the basis of the theory of plates and shells is offered. The solution is obtained by combining the semi-inverse Saint-Venant method and Airy functions. The received solution allows to generalize the available private solutions and have good agreement with known private empirical dependences for a plastic bending of tubes with rectangular cross-section.

1. Introduction

Plastic bending of tubes is one of the most common operations in the field of metal forming to produce curved details for various purposes. Consider a common method of manufacturing curved waveguide sections, which are a thin-walled tube with a rectangular cross-section. A workpiece of a straight-line waveguide is winding onto a rotating cam using the following tools: a bending head, front and rear clamps, a blank holder. At this point, a workpiece material undergoes plastic deformation and it takes a given curved shape [1-3].

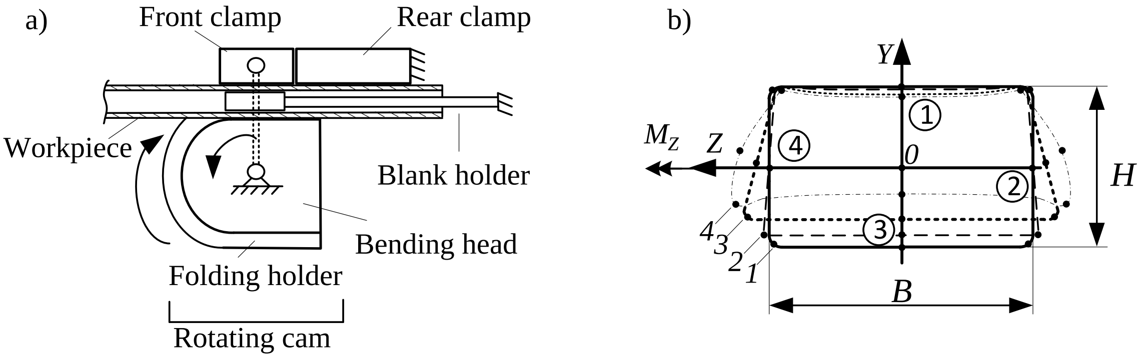

Figure 1. Plastic bending process on the tube-bending machine.

If plastic bending is realized without prevent the undesirable deformations, a cross-section of the waveguide tubes in the area of bending might experience distortion, which means unacceptable change of size and shape of cross-section (figure 1, b). Shape distortion of the cross-section is also enhanced by large angles of bending and very small bending radii of the workpiece [4]. In order to limit distortion of profile and dimensions of cross section, a blank holder or a filler of corresponding dimensions is introduced into a blank before bending. Also, in order to prevent bulging of walls of a cross section outside, a bending machine has bending track in the form of slots and guides. Thanks to such bending track, the shape and the dimensions of the cross section retain the original geometry. However, there remains the problems, which include unacceptable thinning, cracks and wrinkles at the waveguide workpiece.

A literature review in the field of plastic metal forming showed that experimental data and empirical values prevailed in this area until the early 20th century [5]. The pioneering scientific research in the field of metal forming in the 20th century. Firstly, scientists have developed a theory of plastic bending of thin metal sheets. Secondly, solutions for plastic bending of solid section rods of simple shape have been obtained. Finally, the plastic bending of the thin-walled tubes of rectangular cross-section began to be studied in the second half of the 20 century [6].

In recent years, the number of publications on plastic bending of thin-walled tubes with a non-axisymmetric cross-sectional shape has increased significantly [6-9]. This indicates that there are unsolved problems in this task. The main disadvantages of recent works are excessively simple mathematical models of bending bodies, as well as excessive use of numerical solutions. Most of the works are devoted to solving individual specific problems for a thin-walled tube of rectangular section, without obtaining a new common method.

The literature review shown that the theoretical relationships given in [10] do not always provide the required quality of the product and need to be refined using more precise models, since even recent work in this field [11-12] is based on very rough assumptions. For example, in [11], the stress state of the outer and inner walls of the waveguide is evaluated only by the scheme of pure bending of the wide sheet. The same assumption is repeated in [12]. However, during the bending process the outer wall undergoes an out-of-center tension deformation (inner-out-of-center compression), which is a combination of central tension (compression) and bending that significantly complicates their stress state and corresponding theoretical dependencies.

Thus, it is necessary to develop a more accurate mathematical model of the waveguide bending process, which will allow to investigate their refined stress-strain state during plastic bending taking into account all peculiarities of loading conditions. In order to overcome the disadvantages of previous research and obtain a more accurate solution, it is necessary to model the thin-walled tube not by the theory of beams, but by the more accurate theory of shells and plates. As is known, the shell model requires solving a complex system of differential equations, which makes it difficult to obtain an analytical solution and restrain this approach [13].

In this paper proposes a new shell model of a thin-walled beam of rectangular cross-section and an example of the obtained analytical solution for the case of pure bending. The shell model of the thin-walled waveguide tubes allows rather to estimate its stresses and deformations during plastic bending, and the proposed method of obtaining an analytical solution opens a new approach to solving similar problems of shell theory.

2. Shell model of waveguide

According to the proposed approach, we simulated a rectilinear waveguide by the theory of shells and plates. The presence of right angles in the geometry of the cross section (figure 1,b) leads to the difficulty in forming and solving constitutive equations of the waveguide state, composed by the theory of shells. To overcome this obstacle, the waveguide geometry is divided into 4 separate plates with boundary conditions that establish the conditions for their joint work. In figure 2 is shown a partition of the waveguide geometry in a bent state. This presentation of geometry facilitates stress-state analysis, since each plate is subjected to its unique case of loading when waveguide is bent.

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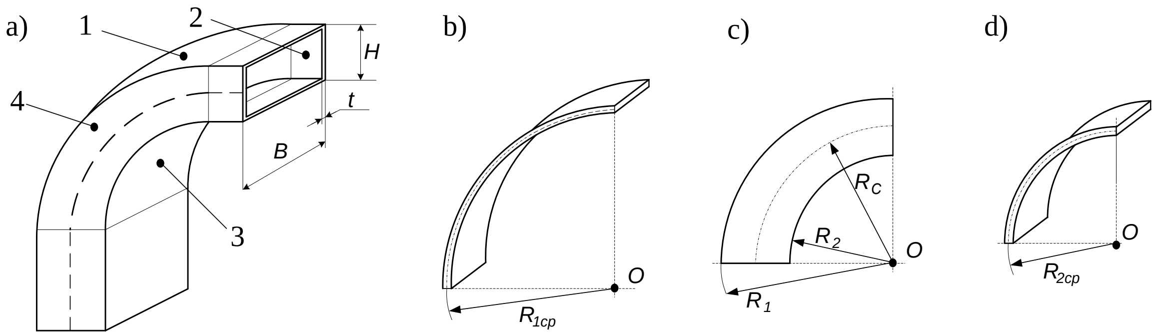

Figure 2. Elements of thin-walled waveguide.

By the theory of shells, the complete system of the waveguide constitutive equations consists of 4 subsystems for each of its plates and have the form of nonlinear partial differential equations in the following form [13]:

$$\left.\begin{aligned}&\frac{\partial^4\varphi_i}{\partial\alpha_i^4}+2\frac{\partial^4\varphi_i}{\partial\alpha_i^2\partial\beta_i^2}+\frac{\partial^4\varphi_i}{\partial\beta_i^4}=Et\cdot\left[\left(\frac{\partial^2\omega_i}{\partial\alpha_i\partial\beta_i}\right)^2-\frac{\partial^2\omega_i}{\partial\alpha_i^2}\cdot\frac{\partial^2\omega_i}{\partial\beta_i^2}\right];\\&\frac{\partial^4\omega_i}{\partial\alpha_i^4}+2\frac{\partial^4\omega_i}{\partial\alpha_i^2\partial\beta_i^2}+\frac{\partial^4\omega_i}{\partial\beta_i^4}=\frac{1}{D}\left[\frac{\partial^2\varphi_i}{\partial\beta_i^2}\cdot\frac{\partial^2\omega_i}{\partial\alpha_i^2}-2\frac{\partial^2\varphi_i}{\partial\alpha_i\partial\beta_i}\cdot\frac{\partial^2\omega_i}{\partial\alpha_i\partial\beta_i}+\frac{\partial^2\varphi_i}{\partial\alpha_i^2}\frac{\partial^2\omega_i}{\partial\beta_i^2}\right].\end{aligned}\right\}\quad(1)$$

where i= 1,2,3,4 – plate numbers; E – Young's module; D – cylindrical rigidity; α_i, β_i – coordinates of the i-th plate; ω_i, ϕ_i - deformation and stress function of the i-th plate; t - thickness of the wall.

The solution of the constitutive equations (1) allow to determine stresses and strains in the waveguide.

3. Results

When loaded, the waveguide material will first experience elastic deformations, then plastic ones. Let's consider the solution of system (1) separately for the elastic and plastic ranges.

3.1. Solution in the elastic range

In the elastic range, solution of the constitutive equations (1) has the following form [13]:

$$\left.\begin{aligned}&\varphi_1=\beta_1^2\cdot\frac{M_Z}{J_Z}\cdot\frac{h'}{4};\quad w_1=\frac{h'}{2}-\frac{M_Z}{2\cdot EJ_Z}\left[\alpha_1^2+\mu\cdot\frac{h'^2}{4}\right];\quad \varphi_2=\frac{M_Z}{J_Z}\cdot\frac{\beta_2^3}{6};\qquad w_2=0;\\&\varphi_3=-\beta_3^2\cdot\frac{M_Z}{J_Z}\cdot\frac{h'}{4};\quad w_3=-\frac{h'}{2}+\frac{M_Z}{2\cdot EJ_Z}\left[\alpha_3^2+\mu\cdot\frac{h'^2}{4}\right];\quad \varphi_4=-\frac{M_Z}{J_Z}\cdot\frac{\beta_4^3}{6};\qquad w_4=0.\end{aligned}\right\}.\quad(2)$$

The elastic solution (2) shows [13] that the outer wall 1 undergoes out-of-center stretching, that is a combination of axial stretching and bending (figure 2). The tensile stresses with respect to the bending components will be in the ratio B/t (or H/t). This means that tensile stresses values are an order of magnitude greater than bending stresses. The inner wall 3 (figure 2) undergoes out-of-center compression during bending, i.e. a combination of axial compression and bending. The values of compressive stresses also exceed the bending stresses by an order of magnitude. The side walls 2 and 4 (figure 2) of the waveguide are bent in their plane only.

The total elastic stresses in the plates 1-4 are determined according to the equations [10,13]:

$$\sigma_{\alpha i} = \sigma_{\phi i} + \sigma_{\omega i} = \frac{\partial^2 \phi_i}{\partial \beta_i^2} - \frac{E \cdot z_i}{1-\mu^2}\left(\frac{\partial^2 \omega_i}{\partial \alpha_i^2} + \mu \frac{\partial^2 \omega_i}{\partial \beta_i^2}\right); \tag{3}$$

$$\sigma_{\beta i} = \sigma_{\phi i} + \sigma_{\omega i} = \frac{\partial^2 \phi_i}{\partial \alpha_i^2} - \frac{E \cdot z_i}{1-\mu^2}\left(\frac{\partial^2 \omega_i}{\partial \beta_i^2} + \mu \frac{\partial^2 \omega_i}{\partial \alpha_i^2}\right); \tag{4}$$

The Analysis of numerical values of full stresses (3,4) shows that values of bending stresses are less than 1% of the total values and they can be ignored in the full solution.

According to the elastic solution (3,4), the stress distribution over the volume of the bending waveguide is shown in figure 3, a.

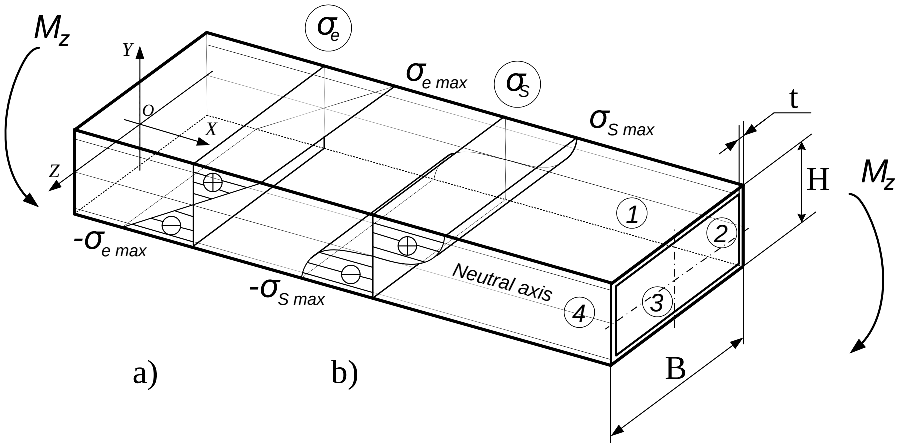

Figure 3. Stress distribution by volume of the waveguide at bending.

3.2. Solution in the plastic range

Let's waveguide material during plastic deformation has a hardening effect according to the Holomon's equation [14] as follows:

$$\sigma_S = \sigma_{S1} \cdot \varepsilon^k \ , \tag{5}$$

where σ_{S1} и k – constant for the selected material.

Substituting the equation (5) into equations (3,4) gives the distribution of plastic stresses over the waveguide shown in the figure 3, b.

4. Discussion

The analysis of the stress-strain state of the waveguide workpiece at bending (1) showed that the most critical factor limiting the value of the minimum radius of bending from below is not stress, but the value of the maximum permissible deformation of the waveguide material $\varepsilon_{\max}^P$ as well as the permitted thinning of the outer wall. Maximum tensile deformations act on the outer surface of the outer plate 1 and are equal to:

$$\varepsilon_{\max}^{P}=\ln\left(\frac{R_1}{R_C}\right)=\ln\left(\frac{H(n+1)}{H(n+0,5)}\right)=\ln\left(\frac{n+1}{n+0,5}\right). \tag{6}$$

For example, an extreme deformation $\varepsilon_{\max}^{P}$ = 25% according to the equation (7) gives $n = 1.26$, that is, the minimum allowable relative bend radius is $R_{2\,MIN}=1.26H$.

The thinning of the material on the outer wall of the waveguide can be calculated by the stresses acting in it through the generalized Hooke's law [13]:

$$\varepsilon_{Z1}=-\frac{\sigma_{\alpha 1}}{E}\mu\ . \tag{7}$$

The resulting expressions (6) for determining the minimum radius of bending and thinning of the outer wall (5) are given for the uniaxial loading pattern of the plates 1 and 3 (figure 2) but under other loading conditions will vary. For example, by using a filler or blank holder (figure 1,a), there will be a restriction in the free deformation of the plates 1 and 3 in their transverse direction, which will result in transverse tensile normal stresses and accordingly biaxial stress state. In this case, the thinning of the outer plate will result from the combined action of longitudinal $\sigma_{\alpha 1}$ and transverse $\sigma_{\beta 1}$ stresses:

$$\sigma_{\beta 1}=\frac{E}{1-\mu^2}\left(\mu\cdot\varepsilon_{\alpha 1}+\varepsilon_{\beta 1}\right)=\frac{E}{1-\mu^2}\mu\cdot\varepsilon_{\alpha 1}=\frac{\mu}{1-\mu^2}\cdot\sigma_{\alpha 1}. \tag{8}$$

The lateral deformation in this case is equal to:

$$\varepsilon_{Z1}=\frac{1}{E}\left(-\mu\cdot\sigma_{\alpha 1}-\frac{\mu^2}{1-\mu^2}\cdot\sigma_{\alpha 1}\right)=-\frac{\sigma_{\alpha 1}}{E}\mu\cdot\left(1+\frac{\mu}{1-\mu^2}\right)=-\varepsilon_{\alpha 1}\cdot\mu\cdot\left(1+\frac{\mu}{1-\mu^2}\right). \tag{9}$$

The analysis of the equation (7) and (9) shows that using a filler leads to great value of the waveguide lateral deformation (figure 2). In the most common case $\mu=0.3$ we get $k\approx 0.4$. This value is in good agreement with experimental data. As established in [15], when using a filler, the thinning of the waveguide wall increases by 30-50%. Double-sided stretching, in addition to greater thinning, results in significantly larger (5-6 times) stress concentrations at the points of different inhomogeneities of the material structure.

It is possible to increase plasticity by adding compressive axial forces to the workpiece or by switching to hydrostatic molding, which, however, leads to a significant increase in the cost of the technology. According to paper [16] heating increases plastic properties of the material by 10%.

A more promising direction is the creation of special modes of superplasticity of the material [17-23], in which the limit deformation can reach 500% or more, though this method still requires additional research.

5. Conclusion

Based on the theory of plates and shells, a new waveguide model has been proposed, which has made it possible to obtain a refined solution combining known empirical dependencies and numerical analytical solutions. The developed model has made it possible to clarify the stress state of the waveguide during plastic bending and to give recommendations to the bending technology so that products with the required quality could be produced.

Acknowledgements

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