Method to improve the accuracy of navigation definitions

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Abstract. The article deals with the method of building a multipath portrait of stationary objects, which allows increasing the reliability of navigation definitions when using satellite radio-navigation systems GLONASS and GPS.

1. Introduction

Global navigation satellite systems are widely used in various technical systems, domestic life, science, military industry and other areas of human activity. Therefore, in modern conditions, the task of precise positioning and spatial orientation of objects is of great importance.

In real conditions, the accuracy of the location of an object is affected by many factors, one of which is the re-reflection of navigation signals from local objects and surfaces, the so-called multipath [1], [2].

Solving the problem of eliminating the effect of multipath effect will reduce the error, and therefore improve the reliability of navigation definitions [3].

This paper proposes a method to exclude the effect of multipath for stationary objects, provided that the spatial location of the antenna system of the navigation receiver relative to the shielding surfaces does not change.

To estimate the influence of errors caused by the refraction of navigation signals from local objects, allows the portrait of multipath object, based on mathematical modeling or experimental measurements using a simulator of navigation signals [4],[5],[6]. Using this method, it is possible to determine which of the satellites involved in determining the coordinates of the consumer should be excluded from the calculations to improve the accuracy of the navigation problem.

2. Structures of a mathematical model of multipath portrait

For the convenience of modeling, let us imagine that the re-reflection of navigation signals comes only from a vertical reflective screen (RS) of finite dimensions, having the correct geometric shape (figure 1).

Let's set two Cartesian coordinate systems (SC):

- SC $x_E y_E z_E O_E$ associated with the Earth's centre (sphere);
- SC $xyzO$ related to SC $x_E y_E z_E O_E$ conversion:

$$x = x_E\ ;\ y = y_E\ ;\ z = z_E - R_E\ , \tag{1}$$

where R_E the average radius of the Earth is 6,371 km.

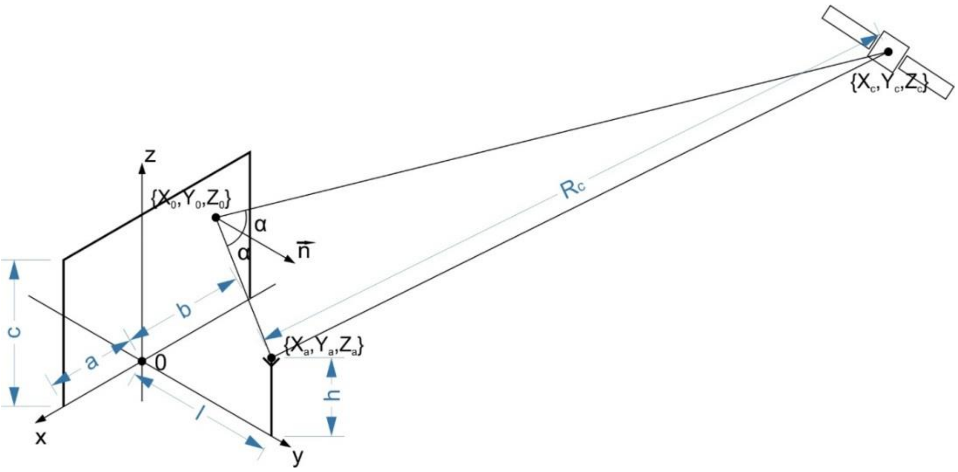

Figure 1. Multipath signal propagation with reflection from the finite size screen.

Let us know the screen height c and width equal to the sum of distances a (from the beginning of the RS to the centre of the SC) and (from the centre of the SC to the end of the RS). Then, in the SC, the plane of the reflecting screen is described by the equation $y=0$, and its points satisfy the relations:

$$y=0;\ a\geq x\geq b;\ c\geq z\geq 0. \tag{2}$$

Let, at some distance l from the screen, considerably smaller Earth radius, the receiving antenna lifted over a surface on height is located h. Then, the point $\{X_a, Y_a, Z_a\}$ or its radius vector $\vec{r}_a$ where acts as model of the phase center of the consumer in SK $xyzO$:

$$X_a=0;\ Y_a=l;\ Z_a=h. \tag{3}$$

Let's accept a mirroring hypothesis from the screen. Then, the incidence angle of a signal is equal to the angle of its reflection:

$$\frac{(\vec{r}_a-\vec{r}_0)\cdot\vec{n}}{|\vec{r}_a-\vec{r}_0|}=\frac{(\vec{r}_c-\vec{r}_0)\cdot\vec{n}}{|\vec{r}_c-\vec{r}_0|}, \tag{4}$$

where – $\vec{n}=(0,1,0)$ a vector of a normal to the screen.

Let's enter vectors:

$$\vec{r}_{a0}=\vec{r}_a-\vec{r}_0;\ \vec{r}_{c0}=\vec{r}_c-\vec{r}_0. \tag{5}$$

Then expression (4) will be transformed to a look:

$$\vec{r}_{a0}\cdot\vec{n}\cdot|\vec{r}_{c0}|=\vec{r}_{c0}\cdot\vec{n}\cdot|\vec{r}_{a0}|, \tag{6}$$

what in a type of the entered definition leads to expression:

$$Y_a=Y_c\cdot\frac{|\vec{r}_{a0}|}{|\vec{r}_{c0}|}, \tag{7}$$

from where follows:

$$Y_a^2\cdot\left(\frac{|\vec{r}_{c0}|^2}{Y_c^2}-1\right)=(X_a-X_0)^2+(Z_a-Z_0)^2. \tag{8}$$

The normal, the falling and reflected beams lie in one plane:

$$\frac{\vec{r}_{c0}}{|\vec{r}_{c0}|}+\frac{\vec{r}_{a0}}{|\vec{r}_{a0}|}=\alpha\cdot\vec{n}=(0;\alpha;0), \tag{9}$$

what for a components X and Z will also be transformed to expression:

$$\frac{X_c - X_0}{X_a - X_0} = -\frac{\left|\vec{r}_{c0}\right|}{\left|\vec{r}_{a0}\right|};\ \frac{Z_c - Z_0}{Z_a - Z_0} = -\frac{\left|\vec{r}_{c0}\right|}{\left|\vec{r}_{a0}\right|} \qquad (10)$$

from (10) follows:

$$X_0 = \frac{X_c + X_a \cdot \frac{\left|\vec{r}_{c0}\right|}{\left|\vec{r}_{a0}\right|}}{1 + \frac{\left|\vec{r}_{c0}\right|}{\left|\vec{r}_{a0}\right|}};\ Z_0 = \frac{Z_c + Z_a \cdot \frac{\left|\vec{r}_{c0}\right|}{\left|\vec{r}_{a0}\right|}}{1 + \frac{\left|\vec{r}_{c0}\right|}{\left|\vec{r}_{a0}\right|}} \qquad (11)$$

expression (8) can be transformed to a look:

$$Y_a^{\,2} \cdot \left(\frac{\left|\vec{r}_{c0}\right|^2}{Y_c^{\,2}} - 1 \right) + Y_a^{\,2} = \left|\vec{r}_{a0}\right|^2, \qquad (12)$$

then:

$$\frac{\left|\vec{r}_{c0}\right|}{\left|\vec{r}_{a0}\right|} = \left|\frac{Y_c}{Y_a}\right|. \qquad (13)$$

Substituting expression (13) in (11), we receive reflection point coordinates on the infinite screen:

$$X_0 = \frac{X_c + X_a \cdot \left|\frac{Y_c}{Y_a}\right|}{1 + \left|\frac{Y_c}{Y_a}\right|};\ Y_0 = 0;\ Z_0 = \frac{Z_c + Z_a \cdot \left|\frac{Y_c}{Y_a}\right|}{1 + \left|\frac{Y_c}{Y_a}\right|}. \qquad (14)$$

Let's calculate artificial satellite signal viewing angles from a point of reflection (figure 2).

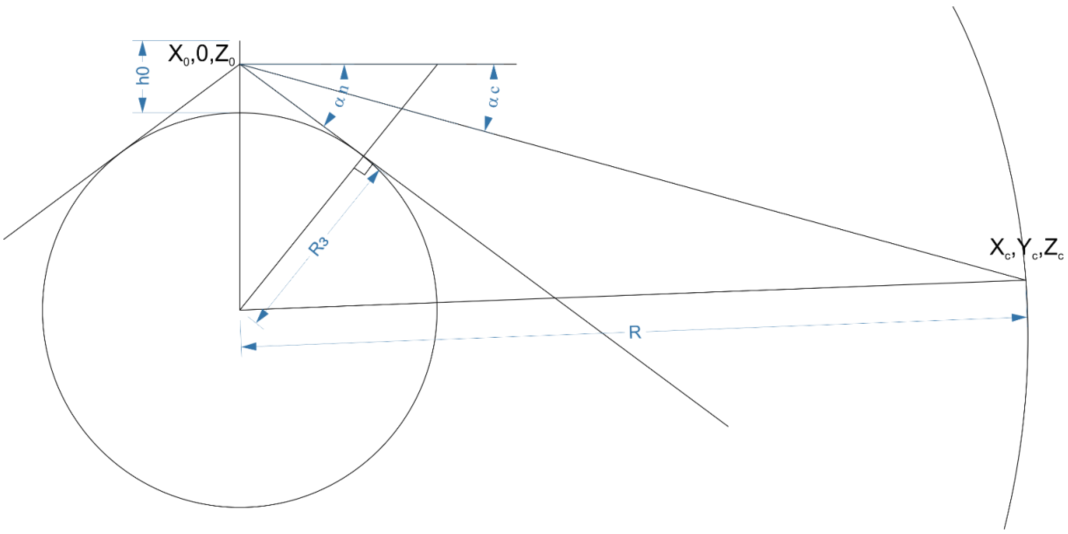

Figure 2. Viewing angles of a signal from the artificial satellite in a reflection point.

Tangent of angle of the place under which of a point of reflection the horizon is visible:

$$tg(\alpha_n) = \frac{-\sqrt{2 \cdot R_3 \cdot Z_0^{\,2}}}{R_3}. \qquad (15)$$

Tangent of angle of the place under which the artificial satellite is visible from a reflection point:

$$tg(\alpha_c) = \frac{Z_c - Z_0}{\sqrt{(X_c - X_0)^2 + (Y_c - Y_0)^2}} . \quad (16)$$

Condition of finding of the artificial satellite over the horizon for a reflection point:

$$tg(\alpha_c) > tg(\alpha_n) . \quad (17)$$

At the eminence of the artificial satellite over the horizon, at observations from a point of the phase center of the consumer, inequality is carried out:

$$tg(\alpha_{c1}) > tg(\alpha_{n1}) . \quad (18)$$

$$tg(\alpha_{c1}) = \frac{Z_c - Z_a}{\sqrt{(X_c - X_a)^2 + (Y_c - Y_a)^2}} . \quad (19)$$

$$tg(\alpha_n) = \frac{-\sqrt{2 \cdot R_3 \cdot h + h^2}}{R_3} . \quad (20)$$

Let's calculate the angles of shadowing of a signal from the artificial satellite the reflecting screen.

The angles of shadowing of a signal of the artificial satellite the reflecting screen:

$$\phi_1 = arctg\left(\frac{c-h}{l}\right); \ \phi_2 = arctg\left(\frac{a}{l}\right); \ \phi_3 = arctg\left(\frac{b}{l}\right) . \quad (21)$$

Then conditions of shadowing of a signal of RS:

$$\pi > \alpha_i > \pi - \phi_1 \text{ and } \frac{\pi}{2} - \phi_2 > \gamma_i > \phi_3 + \frac{\pi}{2} . \quad (22)$$

For creation of a portrait of multipath we need to define the relation of distances of a forward signal and reflected. We will define distance from the artificial satellite to the consumer the next way:

$$R_{ca} = \sqrt{(X_a - X_c)^2 + (Y_a - Y_c)^2 + (Z_a - Z_c)^2} . \quad (23)$$

Let's similarly find distance from the artificial satellite to a point of reflection R_{ct} and from a reflection point to the consumer R_{ta}. The distance which was overcome by a signal in case of reflection from RS will be their sum:

$$R_{ct} = \sqrt{(X_0 - X_c)^2 + (Y_0 - Y_c)^2 + (Z_0 - Z_c)^2} ; \quad (24)$$

$$R_{ta} = \sqrt{(X_a - X_0)^2 + (Y_a - Y_0)^2 + (Z_a - Z_0)^2} ; \quad (25)$$

$$R_{sum} = R_{ct} + R_{ta} . \quad (26)$$

Then the relation of distances of a forward signal and reflected equally:

$$R_i = \frac{R_{ca}}{R_{sum}} . \quad (27)$$

As the distance from a reflection point up to the consumer is much less than distance to the artificial satellite from RS and the consumer, for more evident representation of a phase portrait we will increase R_{sum} twice. Then expression (27) will take a form:

$$R_i = \frac{R_{ca}}{2 \cdot R_{sum}} . \quad (28)$$

In case there is no rereflected signal, the relation of distances of a forward signal and reflected R_i will be equal 1, under shadowing conditions – 0. Conditions of definition of presence of a point of reflection at borders of RS will be defined as follows:

$$a \geq X_0 \geq -b ; \ c \geq Z_0 \geq 0 ; \ Y_c > 0 . \quad (29)$$

As artificial satellites move on a circle of the orbits, at draw of a portrait of multipath in the three-dimensional plane the dome with a radius equal 1, in the conditions of rereflection by radius R_i, in the conditions of shadowing of a signal of RS – 0 should turn out. For this purpose we will set conditions of change of coordinates of the artificial satellite at change of slope angle and azimuth:

$$\begin{gathered} X_c = R \cdot \cos(\alpha) \cdot \cos(\gamma); \\ Y_c = R \cdot \cos(\alpha) \cdot \sin(\gamma); \, , \\ Z_c = R \cdot \sin(\alpha) \end{gathered} \qquad (30)$$

where – R distance from a point of the center of Earth to the satellite.

Then the portrait of multipath will be defined as follows:

$$K_{i,j} = R_i \cdot \begin{bmatrix} \cos(\alpha_i) \cdot \cos(\gamma_i) \\ \cos(\alpha_i) \cdot \sin(\gamma_i) \\ \sin(\alpha_i) \end{bmatrix}. \qquad (31)$$

3. Results of multi-beam portrait modeling

In figure 3 results of modeling phase an object portrait are given in the Cartesian SK.

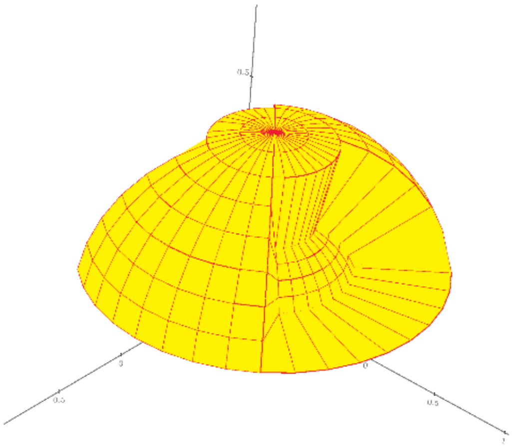

Figure 3. A phase portrait of an object in the Cartesian SK (area of points of reflection)

In figure 4 results of modeling of a phase portrait of an object are given in polar SK.

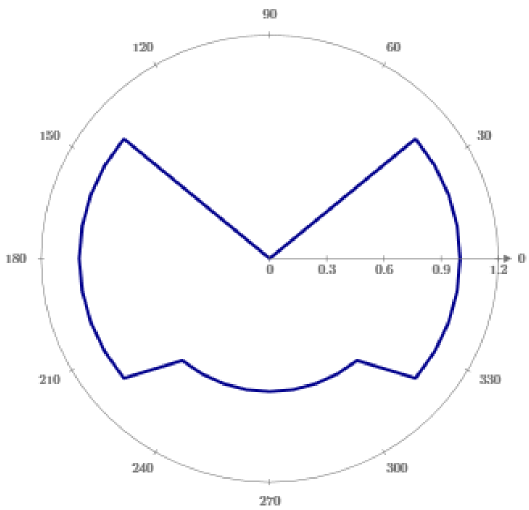

Figure 4. The phase portrait of an object in polar SK (the azimuthal plane), a corner of the place is equal 160 degree.

From Figure 4 it can be concluded that at the location of the navigation satellite on the azimuth in the range from 40 to 140 degrees from the antenna there is complete shading, which leads to a lack of signal. When the positioning of the navigation satellite on azimuth in the range from 220 to 320 degrees there is an effect of signal re-reflection.

4. Conclusions

1. The developed mathematical model allows modeling the multipath portrait of the object in conditions of re-reflection of the navigation signal from local objects and surfaces with stationary position of the reflecting screen relative to the navigation receiver antenna system.
2. The developed mathematical model of multipath portrait construction by azimuth and angle of a place allows to define the area of applicability of navigation satellites to exclude the effect of re-reflection.
3. The exclusion of navigation satellites outside the scope of application improves the validity of navigation definitions.

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