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Temperature influence on dynamic behavior of spacecraft waveguides

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Abstract. The lowest eigenfrequency and critical force are important parameters which defines structure behavior under dynamic loading. The paper describes the influence of temperature on the lowest eigenfrequency and critical force of a waveguide straight section in the spacecraft. The straight waveguide modeled according to the beam theory. It has been shown that the temperature change is equivalent an appearance and action of the axial force which makes the structure prestressed and significantly modifies its lowest eigenfrequency. An example of calculation of dependence of the waveguide straight section lowest eigenfrequency on temperature was considered, and it showed the existence of a critical temperature value, where the vibration frequency equals to zero. This critical temperature value corresponds to the loss of waveguide structure stability which is extremely undesirable and requires special methods to avoid such a situation. Stress state of waveguide in case of its loss of stability is also estimated.

1. Introduction

One of the most important requirements to dynamically loaded structures is ensuring their rigidity controlled, in particular, by eigenfrequency, the minimum value of which shall not be less than a certain allowable value [1-5] to avoid the resonance:

$$f_{\min} \geq \left[f \right]. \tag{1}$$

The common deficiency of calculation methods existing until recently for static, dynamic and thermoelastic states of structures is that these types of calculation are performed independently from each other which does not correspond to their actual operation conditions. In particular, the assessment of structures rigidity is performed with no regard to contributing factors, for example, their temperature, which changes the object geometry and leads to occurrence of thermal stresses, which change their expected static and dynamic states [6-9].

An example of such structures is spacecraft antenna-feeder devices' waveguides, which are designed to transmit signals between transmit-receive antennas and UHF blocks. In the course of manufacturing, testing, space launch and on-station operation within the set period of active lifetime, waveguides are subject to action of different static and dynamic loads: power, deformation and temperature loads. In this regard, waveguide structures shall meet stringent requirements for their strength and rigidity [7-9].

This paper covers investigation of influence of waveguide straight section temperature on its dynamic state determined by its lowest eigenfrequency. We are going to consider such temperature changes which may occur during the space launch and spacecraft on-station operation. During orbit

insertion, the source of heating is temperature differentials in the near-Earth space, and on orbit the source is sun rays and heat production as a result of strength losses of signals transmitted via a waveguide up to +120^{0}C. Waveguide cooling occurs in the shade down to the value of the space cold temperature -120^{0}C [10].

2. Mathematical model of waveguide

A waveguide is a thin-walled elongated structure with a rectangular cross section which satisfies the conditions of the shell theory [11,12]. However, as a first approximation, application of the shell theory is needless and requires complex mathematical calculations with numerical methods. For this reason, we consider rather elongated waveguides which allow to find solutions in analytical form according to known conditions of the beam theory and the vibration theory [1-5,13-16]. Therefore, the found solution be in an explicit form and allow to assess the influence of not only temperature but also other structure parameters on the value of its lowest eigenfrequency.
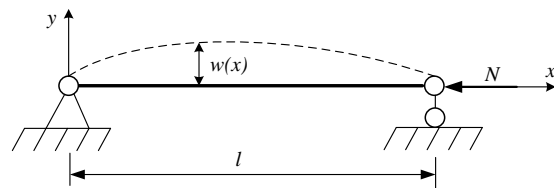
2.1. Waveguide beam model

As a first approximation, the design model for the extended waveguide may be considered as a hinged beam with equivalent geometrical characteristics, supports and loading requirements (figure 1). The hinge support simulates the actual design of the most types of real waveguide supports.

Let us consider that a waveguide is heated up evenly during spacecraft irradiation with sun rays. This holds true as the waveguide material has high thermal conductivity and it is placed in vacuum where there are no convection and other opportunities for heat removal. Additional heating of the waveguide structure occurs in the process of microwave signals transmission inside a waveguide, which may be considered according to the paper [10].

As a result of the temperature change, the waveguide are subjecting to thermal expansion and compression which can be replaced by the equivalent axial force N in the design model. During the heating of the waveguide, it expands and due to the presence of fixed supports, a compressive axial force N occurs, while when cooled, the waveguide compresses and the axial force will be tensile. It is obvious that during beam compression eigenfrequency decrease with further possible loss of stability. This state is the most dangerous, for this reason we consider the compression direction of the axial force N for the design model, as shown in figure 2. Here one of the fixed supports is replaced with a movable one.
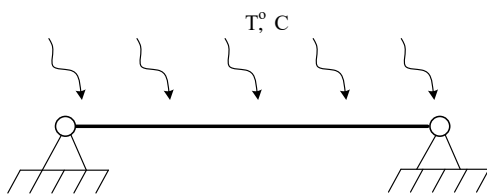
Figure 1. Sun rays action on a waveguide.   **Figure 2.** Design model of a waveguide.

The dotted line shows the waveguide deflection x-axis corresponding to its first mode of vibration. Such beam deformation also corresponds to the first mode of its loss of stability during compression during waveguide heating.

2.2. Analytical solution of free vibrations equation

According to the vibration theory [6-8], the dynamic state of the waveguide model is described by the differential equation:

$$\frac{\partial^2}{\partial x^2}\left(EJ \frac{\partial^2 w}{\partial x^2} \right) + \frac{\partial}{\partial x}\left(N \frac{\partial w}{\partial x} \right) + \rho S \frac{\partial^2 w}{\partial t^2} = q(x,t),\tag{2}$$

where: $w=(w,t)$ – deflection of waveguide longitudinal axis;
E – Young's modulus of the waveguide material;
J – waveguide inertia moment;
ρ – waveguide material density;
S – waveguide cross sectional area;
$q(x,t)$ – external force action of the waveguide.

According to the accepted design model, the waveguide geometry and its material properties, as well as axial force N do not change along the length, and external force action $q(x,y)$ is absent, in such a case equation (2) take the following simplified form:

$$EJ\frac{\partial^4 w}{\partial x^4} + N\frac{\partial^2 w}{\partial x^2} + \rho S\frac{\partial^2 w}{\partial t^2} = 0 . \tag{3}$$

For solution of equation (3), it is necessary to set 4 boundary conditions which reflect conditions of waveguide hinged supports. The absence of deflections and bending moments in hinged supports can be written as:

$$w = w(x=0,t) = w(x=l,t) = 0 ; \qquad \frac{\partial^2 w(x=0,t)}{\partial x^2} = \frac{\partial^2 w(x=l,t)}{\partial x^2} = 0 ; \tag{4}$$

According to the accepted first mode of vibrations, we use following equation as a decision function [1-3]:

$$w(x,t) = A\sin\left(\frac{x\pi}{l}\right)\sin(\omega t) , \tag{5}$$

where: A is vibration amplitude;
ω is angular vibration frequency.

After inserting decision function (5) in free vibrations equation (3), we get:

$$EJA\sin\left(\frac{x\pi}{l}\right)\frac{\pi^4}{l^4}\sin(\omega t) + NA\sin\left(\frac{x\pi}{l}\right)\frac{\pi^2}{l^2}\sin(\omega t) - \rho SA\sin\left(\frac{x\pi}{l}\right)\sin(\omega t)\omega^2 = 0 . \tag{6}$$

From equation (6), after simplification and conversion, we get vibration frequency equal to [1-3]:

$$\omega_{\min} = \frac{\pi^2}{l^2}\sqrt{\frac{EJ}{m}\left(1 - \frac{Nl^2}{\pi^2 EJ}\right)} , \tag{7}$$

where: m is the mass per unit length of the beam.

Value of angular frequency of vibration (7) can be easily converted to value of vibration frequency:

$$f = \frac{\omega}{2\pi} . \tag{8}$$

The obtained solution (7) determines the first natural eigenfrequency of the waveguide having bending stiffness EJ, length l, specific weight m and subjected to the action of axial force N.

2.3. Influence of temperature on waveguide eigenfrequency

Now we consider the effect of temperature. The value of the axial force N can be determined by Hook 's law and the coefficient of thermal expansion of the waveguide material [17-20] by equation:

$$N = \alpha \cdot \Delta T \cdot ES , \tag{9}$$

where: α – coefficient of thermal expansion of the material;
ΔT – change of the waveguide temperature.

Considering the expressions (7-9) we get an equation determining the dependence of the first eigenfrequency on temperature for the considered waveguide model (figure 1):

$$f_{\min}\left(\Delta T\right) = \frac{\pi}{2l^2}\sqrt{\frac{EJ}{m}\cdot\left(1-\frac{\alpha\cdot\Delta T\cdot S\cdot l^2}{\pi^2\cdot J}\right)}. \qquad (10)$$

Obtained resolving equation (10) determines the explicit analytical dependence of the beam lowest eigenfrequency from temperature, as well as from its geometry, material and weight.

2.4. Thermal stress
The changing in temperature reduces or increases the size of the waveguide, in particular its length. If the waveguide supports prevent a free change in its length, thermal stresses appear in this direction. In the present case of fixing the waveguide in hinged supports, the change in temperature lead to normal stresses, which, according to Hook's law, are determined by the equation [17-20]:

$$\sigma\left(\Delta T\right) = \alpha\cdot\Delta T\cdot E, \qquad (11)$$

The thermal stresses are inevitable when a spacecraft is exposed in orbit due to sunlight and can reach dangerous values for waveguide material.

3. Results
For the purpose of calculation, we consider two waveguide with typical cross-section size of 35x15x1.2 mm, inertia moment $J=6.6*10^{-9}$ m^4, having two lengths: $l=0.25$ m and $l=0.5$ m. The waveguide material is duralumin with properties as $E=7.1*10^5$ MPa, density $\rho=2.77$ kg/m^3 . The temperature change is considered within the range from –120^{0}C to +120^{0}C.

After insertion of these data in equation (10), we obtain values which are graphically shown in figure 3 in the form of two curves.
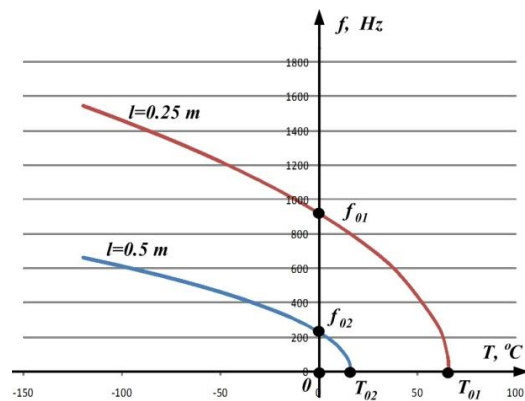
Figure 3. Dependency of the waveguide lowest eigenfrequency value on temperature (hinge supports)
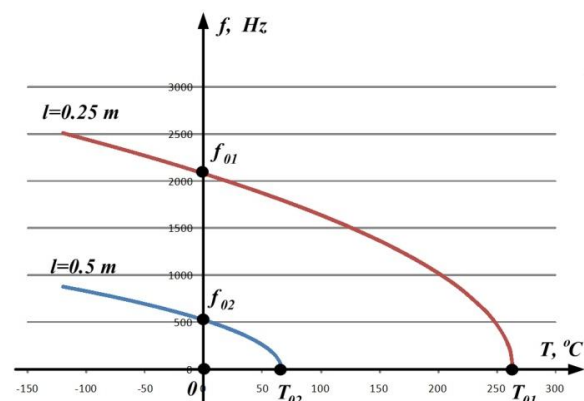
Figure 4. Comparative solutions for fixed supports

Two curves in figure 3 represent dependencies of the lowest eigenfrequency value on temperature for the two waveguide lengths with hinge supports: 0.25m and 0.5m. Characteristic points in this graph have following values: $f_{01}=$ 920 Hz; $f_{02}=$ 230 Hz; $T_{01}=$ 66 ºC; $T_{02}=$ 16 ºC.

The thermal stresses in the waveguide calculated by equation (11) show that a temperature change of $\Delta T=1$ºC results in normal stress equal 8.875 MPa. So, for the characteristic values of temperatures in figure 3 we obtain the following values of the corresponding normal stresses: $\sigma_{01}=$ 586 MPa, $\sigma_{02}=$ 142 MPa. These values are dangerous and exceed the yield strength of some waveguide material.

4. Discussion

Obtained equation (7) is perfectly compliant with known formulas obtained by another authors. For example, if we accept that there is no axial force (N=0), we obtain a well known expression of the lowest eigenfrequency for a hinged beam [1]:

$$\omega = \pi^2 \sqrt{\frac{EJ}{m \cdot l^4}} \ . \tag{12}$$

Solution (10) shows that a situation is possible in equation (10) when a term of a fraction under the radical turns into zero. This critical case corresponds to such a value of axial force N_{cr} at which vibration frequency of a waveguide become equal to zero and the loss of stability occur [21-24]:

$$N_{cr} = \frac{\pi^2 EJ}{l^2} \ . \tag{13}$$

Expression (13) fully coincides with the Euler formula for the critical compressive force in case of the loss of stability of a hinged beam. Resolving equation (10) and a diagram in figure 3 show non-linear and significant dependence of the lowest eigenfrequency of the waveguide on temperature. In figure 3 values f_{01}=233.4 Hz and f_{02}=933.6 Hz correspond to the lowest eigenfrequencies for waveguides with lengths of 0.25 and 0.5 m respectively in case of absence of axial force N. Values T_{01}=66 ºC and T_{02}=16ºC in figure 3 determine temperatures, which correspond to the loss of construction stability.

With the aim of comparative assessment of results, calculations of the waveguide shell model were performed FEM analysis in the Ansys software, and in this case these calculations showed decrease of frequencies by 5-7% in comparison with those obtained in accordance with the beam theory. For this reason, results obtained as per equation (10) for critical structures require clarification by means of calculations with more precise procedures [25-27]. For example, it is obvious that the actual waveguide supports have some bending stiffness and conditions (4) not be exactly met. Consider the extreme case of fixed waveguide supports at its both edges. In this case the solution of the equation (3) give more higher frequencies: f_{01} = 2085 Hz; f_{02} = 521 Hz; T_{01} = 264 ºC; T_{02} = 66 ºC (figure 4).

In order to increase eigenfrequency, as well as the stability of a waveguide, a simple and efficient method is changing the supports design in such a way as to compensate temperature changes of waveguide section lengths. For example, one of the supports may be made sliding to compensate for thermal expansion along the longitudinal axis of the waveguide.

5. Conclusions

In this paper, we considered the influence of the waveguide temperature on its dynamic behavior which was characterized with the lowest eigenfrequency and critical compressive force. For this purpose, the beam deflection curve vibration equation was solved taking into account the action of the axial force occurring as a result of thermal deformations.

An analytical solution was obtained for a case of the waveguide hinge support which determines the eigenfrequency dependence on the temperature. The solution showed significant dependence of the waveguide first eigenfrequency and stress on the temperature, and even probability of the loss of its stability even at relatively small temperature changes which is an unacceptable situation. A way to decrease the temperature impact has been suggested which is based on changing the design of waveguide supports. The obtained results may be used for assessment of dynamic behavior of any extended structures which are subject to temperature changes, for example, steam piping, oil piping, pipelines and others.

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