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Removable Singularities of Separately Harmonic Functions

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Abstract. Removable singularities of separately harmonic functions are considered. More precisely, we prove harmonic continuation property of a separately harmonic function $u(x, y)$ in $D \setminus S$ to the domain D, when $D \subset \mathbb{R}^n(x) \times \mathbb{R}^m(y)$, $n, m > 1$ and S is a closed subset of the domain D with nowhere dense projections $S_1 = \{x \in \mathbb{R}^n : (x, y) \in S\}$ and $S_2 = \{y \in \mathbb{R}^m : (x, y) \in S\}$.

Keywords: separately harmonic function, pseudoconvex domain, Poisson integral, $\mathcal{P}$-measure.

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The theorem on removal of compact singularities (see [1, 2]) is one of the most important results in the theory of functions in several complex variables: *if a function f is holomorphic everywhere in the domain $\Omega \subset \mathbb{C}^n$ $(n > 1)$ except a set $K \Subset \Omega$, which does not divide the domain (i.e. such that $\Omega \setminus K$ is connected), then f can be extended holomorphically to whole domain Ω.* In the work [3], an analogue of this theorem was proved for separately harmonic functions, i.e. for functions which are harmonic in each variable separately: *let D be a domain in $\mathbb{R}^n(x) \times \mathbb{R}^m(y)$, $n, m > 1$, $K \Subset D$ a compact set such that $D \setminus K$ is connected. If the function $u(x, y)$ is separately harmonic in $D \setminus K$, then it harmonically continues to D.*

1. Separately harmonic functions

Definition 1. *If a function $u(x, y)$ is defined in the domain $D \subset \mathbb{R}^n(x) \times \mathbb{R}^m(y)$ and satisfies the following properties:*

1) *for any fixed $x^0 : \{x = x^0\} \cap D \neq \emptyset$, a function $u(x^0, y)$ is harmonic in y on $\{x = x^0\} \cap D$;*

2) *for any fixed $y^0 : \{y = y^0\} \cap D \neq \emptyset$, a function $u(x, y^0)$ is harmonic in x on $\{y = y^0\} \cap D$, then it is called a separately harmonic function in the domain D.*

One of the main methods of studying extension of harmonic functions is the transition to holomorphic functions, and then using the principles of holomorphic extensions.

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Lemma 1 ([5])**.** *For any domain $D \subset \mathbb{R}^n(x) \subset \mathbb{C}^n$ there is a domain of holomorphy $\widehat{D} \subset \mathbb{C}^n(z)$ such that $D \subset \widehat{D}$ and any harmonic function $u(x)$ in D holomorphically extends into the domain $\widehat{D}$, i.e. there is a holomorphic function $f_u(z)$ in $\widehat{D}$ such that $f_u|_D = u$.*

The existence of the domain $\widehat{D}$ follows easily from the representation of harmonic functions by the Poisson integral. Indeed, let $B = B(x^0, R) \Subset D$ be an arbitrary ball in D, and $u(x)$ be a harmonic function in D. Then the following formula holds

$$u(x) = \frac{1}{\sigma_n} \int\limits_{\partial B} \frac{R^2 - |x - x^0|^2}{R|x - y|^n} u(y) d\sigma(y),$$

where σ_n is the surface area of the unit sphere. It is clear that the Poisson kernel

$$P(x, y) = \frac{1}{\sigma_n} \frac{R^2 - |x - x^0|^2}{R|x - y|^n}$$

for any fixed $y \in \partial B$ holomorphically extends to some domain $\widehat{B} \in \mathbb{C}^n$, $\widehat{B} \supset B$. Eventually, $\widehat{B}$ is a Lie ball centered at $x^0 = (x_1^0, x_2^0, \ldots, x_n^0)$ with the radius R (see [11])

$$\widehat{B} = \left\{ z \in \mathbb{C}^n : \sqrt{|z - x^0|^2 + \sqrt{|z - x^0|^4 - \left| \sum_{j=1}^{n} \left(z_j - x_j^0 \right)^2 \right|^2}} < R \right\}.$$

Consequently, every harmonic in B function holomorphically extends to $\widehat{B}$, which implies the existence a domain $\widehat{D}$, $D \subset \widehat{D} \subset \mathbb{C}^n$ satisfying the above properties.

It can be seen from the construction that for each fixed $z^0 \in \widehat{D}$ there is a constant M_{z^0} such that

$$|f_u(z^0)| \leqslant M_{z^0} \|u\|_D, \tag{1}$$

nevertheless, M_{z^0} is bounded on compact subsets of $\widehat{D}$ and

$$\lim_{z \to x \in D} M_z = 1.$$

2. Separately analytic functions

Let two domains $\mathbb{D} \subset \mathbb{C}^n$, $\mathbb{G} \subset \mathbb{C}^m$ and two subsets, $E \subset \mathbb{D}$, $F \subset \mathbb{G}$ be given. Assume that a function $f(z, w)$, determined firstly on the set $E \times F$, has the following properties:

a) for any fixed $w^0 \in F$, a function $f(z, w^0)$ holomorphically extends to the domain $\mathbb{D}$;

b) for any fixed $z^0 \in E$, a function $f(z^0, w)$ holomorphically extends to the domain $\mathbb{G}$.

In this case $f(z, w)$ defines some function on the set $X = (\mathbb{D} \times F) \cup (E \times \mathbb{G})$ and it is called *a separately-analytic* function on X.

We will use the following theorem on analytic continuation of separately-analytic functions (V. Zakharyuta [8], J. Sichak [9], and see also [7]): *let two domains $\mathbb{D} \subset \mathbb{C}^n$, $\mathbb{G} \subset \mathbb{C}^m$ be strongly pseudoconvex and two subsets $E \subset \mathbb{D}$, $F \subset \mathbb{G}$ be non-pluripolar Borel sets. If $f(z, w)$ is a separately analytic function on the set $X = (\mathbb{D} \times F) \cup (E \times \mathbb{G})$, then it extends holomorphically to the domain*

$$\widehat{X} = \{(z, w) \in \mathbb{D} \times \mathbb{G} : \omega^*(z, E, \mathbb{D}) + \omega^*(w, F, \mathbb{D}) < 1\}.$$

Here $\omega^*(z,E,\mathbb{D})$ is the $\mathcal{P}$-measure of the set E with respect to the domain $\mathbb{D}$ (see [7,8,10]). It is defined as an extremal plurisubharmonic function

$$\omega^*(z,E,\mathbb{D}) = \overline{\lim_{\zeta\to z}}\, \omega(\zeta,E,\mathbb{D}),$$

where

$$\omega(z,E,\mathbb{D}) = \sup\{u(z) : u \in psh(\mathbb{D}), u|_{\mathbb{D}} \leqslant 1, u|_E \leqslant 0\}.$$

3. On Lelong's theorem

P. Lelong [4] proved the following analogue of the fundamental theorem of Hartogs (see [1], Ch. 1): *if $u(x,y)$ is separately harmonic in the domain $D \subset \mathbb{R}^n(x)\times\mathbb{R}^m(y)$, then it is harmonic in D in both variables.*

The proof of Lelong's theorem can be obtained easily if we use the above theorem of V. Zakharyuta and J. Sichak: if $u(x,y)$ is separately harmonic in the domain $D\subset\mathbb{R}^n\times\mathbb{R}^m$ and $B_1\subset\mathbb{R}^n$, $B_2\subset\mathbb{R}^m$ are arbitrary balls such that $B_1\times B_2\subset D$, then by Lemma 1 it extends to the set $X = (\widehat{B}_1\times B_2)\cup(B_1\times\widehat{B}_2)$ as a separately analytic function. Therefore, $u(x,y)$ extends holomorphically to the domain

$$\widehat{X} = \left\{(z,w)\in\widehat{B}_1\times\widehat{B}_2 : \omega^*(z,B_1,\widehat{B}_1)+\omega^*(w,B_2,\widehat{B}_2)<1\right\}.$$

Since $B_1\times B_2\subset\widehat{X}$, the function $u(x,y)$ is infinitely differentiable in $B_1\times B_2$ and therefore, harmonic in both variables. Since the balls are arbitrary, it follows that $u(x,y)$ is harmonic in both variables in the domain D.

4. The main results

Now we are ready to prove the main results of this paper.

Theorem 1. *Let S be a closed subset of the domain $D\subset\mathbb{R}^n(x)\times\mathbb{R}^m(y)$, $n,m>1$, and its orthogonal projections $S_1=\{x\in\mathbb{R}^n:(x,y)\in S\}$ and $S_2=\{y\in\mathbb{R}^m:(x,y)\in S\}$ are nowhere dense. Then any function $u(x,y)$ which is separately harmonic in the domain $D\setminus S$ extends harmonically to the domain D.*

Proof. Let $u(x,y)$ be a separately harmonic function in the domain $D\setminus S$ and the projections of the closed set S:

$$S_1=\{x\in\mathbb{R}^n:(x,y)\in S\},\ S_2=\{y\in\mathbb{R}^m:(x,y)\in S\},$$

are nowhere dense. We denote by $\tilde{S}\subset S$ the set of non-removable singularities for the function $u(x,y)$. Suppose that $\tilde{S}\neq\varnothing$. We take arbitrary balls $B_1\subset\mathbb{R}^n(x)$ and $B_2\subset\mathbb{R}^m(y)$ such that $B_1\times B_2\subset D$ and $(B_1\times B_2)\cap\tilde{S}\neq\varnothing$. We denote by

$$\tilde{S}_1=\{x\in B_1:(x,y)\in(B_1\times B_2)\cap\tilde{S}\},\ \tilde{S}_2=\{y\in B_2:(x,y)\in(B_1\times B_2)\cap\tilde{S}\}.$$

Since $(B_1\times B_2)\cap\tilde{S}\subset\tilde{S}_1\times\tilde{S}_2$, we have

$$(B_1\times B_2)\setminus(\tilde{S}_1\times\tilde{S}_2) = \Big(B_1\times(B_2\setminus\tilde{S}_2)\Big)\cup\Big((B_1\setminus\tilde{S}_1)\times B_2\Big)\subset(B_1\times B_2)\setminus\tilde{S}.$$

Hence, by Lemma 1, the function $u(x,y)$ can be extended analytically to the set $X = \left(\widehat{B}_1 \times (B_2 \setminus \tilde{S}_2)\right) \cup \left((B_1 \setminus \tilde{S}_1) \times \widehat{B}_2\right)$ as a separately analytic function. Consequently, $u(x,y)$ extends holomorphically to the domain

$$\widehat{X} = \left\{(z,w) \in \widehat{B}_1 \times \widehat{B}_2 : \omega^*(z, B_1 \setminus \tilde{S}_1, \widehat{B}_1) + \omega^*(w, B_2 \setminus \tilde{S}_2, \widehat{B}_2) < 1\right\}.$$

Since the sets $B_1 \setminus \tilde{S}_1$, $B_2 \setminus \tilde{S}_2$ are locally pluri-regular, we get

$$X \subset \widehat{X}, \text{ i.e. } (B_1 \times B_2) \setminus (\tilde{S}_1 \times \tilde{S}_2) \subset \widehat{X}.$$

(About pluri-regular sets and their properties, see [6,12]). Now we take an arbitrary point $a \in \tilde{S}_1$ and $x^0 \in U(a,\varepsilon) \setminus \tilde{S}_1$, where $U(a,\varepsilon) = \{x : |x-a| < \varepsilon\}$, $0 < \varepsilon < \frac{1}{2}\, dist(a, \partial B_1)$. For the point x^0 there is a point $a^0 \in \tilde{S}_1$ such that

$$d = |x^0 - a^0| = \inf\left\{|x^0 - x| : x \in \tilde{S}_1\right\}.$$

It is clear that the intersection $B_1 \cap \{x : |x^0 - x| < d\} \subset B_1 \setminus \tilde{S}_1$ contains the interval (x^0, a^0), which is not pluri-thin at the point $a^0 \in \tilde{S}_1$ (see [6], Proposition 4.1). Hence, it follows that

$$\omega^*(a^0, B_1 \setminus \tilde{S}_1, \widehat{B}_1) = 0.$$

On the other hand, there is a point $b^0 \in \tilde{S}_2$ such that $(a^0, b^0) \in \tilde{S}$ and by the definition of $\mathcal{P}$-measure there is also some number $\delta_2 : \omega^*(b^0, B_2 \setminus \tilde{S}_2, \widehat{B}_2) < \delta_2 < 1$. Now we take some number $\delta_1 > 0$ so that $\delta_1 + \delta_2 < 1$. Hence, an open neighborhood of the point

$$(a^0, b^0) \in \tilde{S} : \left\{z : \omega^*(z, B_1 \setminus \tilde{S}_1, \widehat{B}_1) < \delta_1\right\} \times \left\{w : \omega^*(w, B_2 \setminus \tilde{S}_2, \widehat{B}_2) < \delta_2\right\},$$

is contained in $\widehat{X}$, i.e. the point $(a^0, b^0) \in \tilde{S}$ is a removable singularity and this contradicts our assumption concerning $\tilde{S}$. Thus $\tilde{S} = \varnothing$. The theorem is proved. □

Using methods of V. Zahariuta on analytic extension of separately analytic functions we get the following result which generalizes Hamano's theorems [3].

Theorem 2. *Let two domains $D \subset \mathbb{R}^n$, $G \subset \mathbb{R}^m$ and two sets $E \subset D$, $F \subset G$ be given. If $E \Subset D$ is compact and F is a closed subset of G with nonempty complement $G \setminus F \neq \emptyset$, then any separately harmonic function $u(x,y)$ in $(D \times G) \setminus (E \times F)$ harmonically extends to the domain $D \times G$.*

Proof. According to Lemma 1 there is a pseudoconvex domain $\widehat{G} \subset \mathbb{C}^m$ such that $G \subset \widehat{G}$ and for each fixed $x \in D \setminus E$ a function $u(x,\cdot)$ holomorphically extends to $\widehat{G}$. Moreover, there is a sequence of strongly pseudoconvex domains $\widehat{G}_j, j = 1, 2, \ldots$ such that $\widehat{G}_j \Subset \widehat{G}_{j+1} \Subset \widehat{G}$, $\widehat{G} = \bigcup_{j=1}^{\infty} \widehat{G}_j$ and $(G \cap \widehat{G}_1) \setminus F \neq \emptyset$. According to (1) for the set

$$K_\varepsilon = \{z \in D : dist(z, E) \leqslant \varepsilon\} \Subset D,$$

where $\varepsilon > 0$ is a small enough number, there is a sequence of positive real numbers M_j such that

$$|u(x,w)| \leqslant M_j \ \forall (x,w) \in \partial K_\varepsilon \times \widehat{G}_{j+1}.$$

Consequently, for any $l \in N$ there is a sequence of positive numbers $N_j^{(l)}$ such that the inequality

$$\sum_{|\alpha|\leqslant l}\left(\int_{\widehat{G}_j}\left|\frac{\partial^{|\alpha|}u(x,w)}{\partial w^{\alpha}}\right|^2 dV\right)^{\frac{1}{2}} \leqslant N_j^{(l)} \ \forall x \in \partial K_\varepsilon \tag{2}$$

holds.

Now we take a closed ball $\overline{B} \Subset (G \cap \widehat{G}_1) \setminus F$ and for a fixed j and a sequence of sets $\overline{B} \Subset \widehat{G}_j$ we consider a Hilbert space $H_0 \subset H_1$. For H_0 we take the closure of the space

$$\mathcal{O}(\widehat{G}) \cap h(G) \cap W_2^l(\widehat{G}_j), \ l > m.$$

(Here $\mathcal{O}(\widehat{G})$ is the space of holomorphic functions on $\widehat{G}$, $h(G)$ is the space of harmonic functions on G and $W_2^l(\widehat{G}_j)$ is the Sobolev space.) For H_1 we take the closure of the space $h(G) \cap L_2(\overline{B}, \sigma)$, where

$$L_2(\overline{B}, \mu) = \left\{ f : \left(\int_{\overline{B}} |f(w)|^2 d\sigma\right)^{\frac{1}{2}} \leqslant \infty \right\}$$

and $d\sigma = \left(dd^c\omega^*(w, \overline{B}, \widehat{G}_j)\right)^m$ (see [7, 8, 10]). Let $\{e_k(w)\}_{k=1}^{\infty}$ be the common orthogonal basis for spaces $H_0 \subset H_1$ such that $\|e_k\|_{H_0} = \mu_k$, $\|e_k\|_{H_1} = 1$, $\dfrac{1}{M}k^{\frac{1}{m}} \leqslant ln\mu_k \leqslant Mk^{\frac{1}{m}}$, and M is a constant, $k = 1, 2, \dots$ (see [8, 13]).

From the continuous embedding of $H_0 \subset C(\overline{\widehat{G}_j}) \cap \mathcal{O}(\widehat{G}_j)$ it follows that

$$|e_k(w)| \leqslant C\|e_k\|_{H_0} = C\mu_k, \ \ w \in \widehat{G}_j, \tag{3}$$

where C is a constant.

We consider the set $A_k = \{z \in \overline{B} : |e_k(y)| > k\}$. By Chebyshev's inequality we have

$$\sigma(A_k) \leqslant \frac{1}{k^2}\int_{\overline{B}} |e_k(y)|^2 d\sigma(y) = \frac{1}{k^2}\|e_k\|_{H_1} = \frac{1}{k^2}, \ \ k = 1, 2, \dots .$$

Consequently, $\sum\limits_{k=1}^{\infty}\sigma(A_k) < \infty$ and $\lim\limits_{s\to\infty}\sigma\left(\bigcup\limits_{k=s}^{\infty}A_k\right) = 0$. We let $U_s = \overline{B} \setminus \bigcup\limits_{k=s}^{\infty}A_k$, $U = \bigcup\limits_{s=1}^{\infty}U_s$. Then $\sigma(\overline{B} \setminus U) = 0$. Therefore, $\omega^*(w, \overline{B}, \widehat{G}_j) = \omega^*(w, U, \widehat{G}_j)$, i.e. $\omega^*(w, U_s, \widehat{G}_j) \downarrow \omega^*(w, \overline{B}, \widehat{G}_j)$, $w \in \widehat{G}_j$ (see [7, 10]). Since $|e_k(y)| \leqslant k$, $w \in E_s$, $k \geqslant s$, taking into account (3), by two constants theorem we obtain the following estimation

$$|e_k(w)| \leqslant c(s)k\mu_k^{\omega^*(w, U_s, \widehat{G}_j)}, \ \ k \geqslant s, \ \ w \in \widehat{G}_j, \tag{4}$$

where $c(s)$ is a constant independent of k.

Now we compare the formal Fourier–Hartogs series to the function $u(x, w), (x, w) \subset D \times \widehat{G}_j$,

$$u(x, w) \sim \sum_{k=1}^{\infty} a_k(x)e_k(w), \tag{5}$$

where the coefficients are defined by the usual formulas of the space H_1:

$$a_k(x) = \int_{\overline{B}} u(x, w)\overline{e_k(w)}d\sigma, \ \ k = 1, 2, \dots .$$

We show that the series (5) converges locally uniformly in the set $K_\varepsilon \times \widehat{G}_j$.

Since the function $u(x,y)$ is continuous and separately harmonic on the set $D \times \overline{B}$, it follows that $a_k(x)$ is harmonic on D. Moreover, for any fixed $x \in \partial K_\epsilon$ the function $u(x,w) \in H_0$, then $|a_k(x)| = (u(x,\cdot), e_k)_{H_1} = \mu_k^{-2}(u(x,\cdot), e_k)_{H_0}$. Consequently,

$$|a_k(x)| \leqslant \frac{1}{\mu_k^2}\|u(x,\cdot)\|_{H_0}\|e_k\|_{H_0} \leqslant \frac{\|u(x,\cdot)\|_{H_0}}{\mu_k}, \quad x \in \partial K_\varepsilon.$$

Hence, by the estimation (2) and the maximum principle we get the following estimation

$$|a_k(x)| \leqslant \frac{N_j^l}{\mu_k}, \; k = 1, 2, \ldots, \quad x \in K_\varepsilon. \tag{6}$$

Comparing the estimates (4) and (6), we obtain

$$|a_k(x)e_k(w)| \leqslant c(s)N_j k \mu_k^{\omega^*(w,U_s,\widehat{G}_j)-1} \leqslant c(s)N_j k e^{Mk^{\frac{1}{m}}(\omega^*(w,U_s,\widehat{G}_j)-1)},$$

$k \geqslant s$, $(x,w) \in K_\varepsilon \times \widehat{G}_j$, where $U_s \subset \overline{B}$, $\sigma(U_s) > 0$. The last estimation shows that the series (5) converges locally uniformly on the set $K_\epsilon \times \widehat{G}_j$ and its sum $\widetilde{u}(x,w)$ coincides with $u(x,w)$ on the set $\partial K_\varepsilon \times \widehat{G}_j$, i.e. $\widetilde{u}(x,w)$ is an analytic continuation of $u(x,w)$. Finally, letting j tend to infinity we obtain an analytic continuation of the function $u(x,w)$ on the set $K_\varepsilon \times \widehat{G}$ which contains the set $E \times F$, that is the function $u(x,y)$ can be separately harmonically extended to $D \times G$. The proof of Theorem 2 is completed. □

Comparing the ideas of proof of theorems above, one can easily prove the following theorem:

Theorem 3. *Let two domains $D \subset \mathbb{R}^n$, $G \subset \mathbb{R}^m$ and two sets $E \subset D$, $F \subset G$ be given. If E is a nowhere dense closed subset of the domain D and F is a closed subset of the domain G with a non-empty complement $G \setminus F \neq \emptyset$, then any separately harmonic function $u(x,y)$ on the domain $(D \times G) \setminus (E \times F)$ can be extended harmonically to the domain $D \times G$.*

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Стираемые особенности сепаратно-гармонических функций

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Аннотация. В работе рассматриваются устранимые особенности сепаратно-гармонических функций. Точнее, доказана теорема о гармоническом продолжении сеператно-гармонической в $D \setminus S$ функции $u(x, y)$ в область D, где $D \subset \mathbb{R}^n(x) \times \mathbb{R}^m(y)$, $n, m > 1$ и S — замкнутое подмножество области D, а ее проекции $S_1 = \{x \in \mathbb{R}^n : (x, y) \in S\}$ и $S_2 = \{y \in \mathbb{R}^m : (x, y) \in S\}$ нигде не плотны.

Ключевые слова: сепаратно-гармоническая функция, псевдовыпуклая область, интеграл Пуассона, $\mathcal{P}$-мера.