Mathematical models of extended objects used for planning submeter resolution satellite imagery

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Abstract. This paper presents mathematical models describing extended objects with complex configuration used for planning satellite imagery with optoelectronic scanners of submeter spatial resolution. The most efficient spline approximation methods were used for mathematical description of extended objects with complex configuration. The proposed method ensures obtaining maximum coverage of the extended object in one-orbit period by choosing an appropriate spline smoothing coefficient, shifting and turning coverage contour. The results of computer simulation of extended object imagery are given, taking into account the swath width of the imaging instrument, the required scanning direction and limitations on retargeting angles range and satellite angular velocities.

1. Introduction

When planning satellite imagery of lengthy territories with complex configuration (borders, roads, rivers, coastlines, etc.) which are not located in the satellite-covered area, there is a problem connected with a narrow (usually from 5 to 20 km) swath width of ultra-high resolution optical scanners, which does not allow to take images of arbitrarily-spaced lengthy territories in one-orbit period [1, 2]. As a rule, in such cases it is necessary to take images of several scenes from different orbits, but for solar-synchronous orbits which are characteristic for Earth remote sensing satellites with optical-electronic scanners, the same territory can be imaged only in one orbit during the day. Therefore, even double-orbit imagery may require several days in the absence of clouds and limitations on the minimum scanner angles (depending on the scanner's swath width and the retargeting angles range). It may take even longer time due to cloudiness, which is unacceptable for most tasks.

Some foreign satellites equipped with high resolution optoelectronic scanners allow to take images of arbitrarily-spaced straight-line lengthy territories [1]. However, to cover a lengthy object with complex configuration, it is necessary to take images from different orbits.

An advanced technique implemented using automatic satellite attitude program control in the process of retargeting (i.e., with non-zero angular velocities) may significantly improve the efficiency of satellite imagery of lengthy territories with complex configuration [2-7].

The main stages of planning such imagery are:

- approximation of an extended object, defined by separate nodal points on a digital map (linear, quadratic, spline, etc.);
- imagery simulation to determine the optimal coverage of the extended object, taking into account the swath width of the imaging instrument, the required scanning direction and limitations of the satellite attitude system.

The aim of the study is to develop mathematical models describing extended objects in order to improve the efficiency of satellite imagery of lengthy territories having complex configuration with optoelectronic scanners of submeter spatial resolution.

The main objectives of the research are:

- to analyze and choose the most efficient method for mathematical description of extended objects in satellite imagery;
- to estimate quantitatively the effectiveness of the proposed method using computer simulation of satellite imagery of extended territories with complex configuration.

2. Research methods

Spline approximation is the most efficient method for mathematical description of extended objects with complex configuration. The main stages and methods of mathematical calculations that were used in this study are discussed below.

2.1. The method of spline interpolation

For mathematical description of extended objects with the function $S(x)$, a natural cubic interpolating spline is used, where $S''(x_0) = 0$ and $S''(x_n) = 0$, then smoothed with the least squares method. The object to be imaged is defined on the map by the nodal points xi with arbitrary latitude and longitude steps (the number of nodal points of the object $n = 6\ldots9$).

The function $S(x)$ is interpolated by a polynomial

$$S_i(x) = \omega y_i + \bar{\omega} y_{i-1} + h_i^2\left[(\omega^3 - \omega)\delta_i + (\bar{\omega}^3 - \bar{\omega})\delta_{i-1}\right]$$

where $h_i = x_{i+1} - x_i$, $\omega = \dfrac{x - x_i}{h_{i+1}}$, $\bar{\omega} = 1 - \omega$.

Spline coefficients for $\delta_0 = 0$, $\delta_n = 0$, $\delta_1 \ldots \delta_{n-1}$ are defined by a system of linear equations

$$\begin{pmatrix} 2[h_1 + h_2] & h_2 & 0 & \ldots & 0 \\ h_2 & 2[h_2 + h_3] & h_3 & \ldots & 0 \\ 0 & h_3 & 2[h_3 + h_4] & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & h_{n-1} & 2[h_{n-1} + h_n] \end{pmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \ldots \\ \delta_{n-1} \end{pmatrix} = \begin{pmatrix} \Delta_2 - \Delta_1 \\ \Delta_3 - \Delta_2 \\ \Delta_4 - \Delta_3 \\ \ldots \\ \Delta_n - \Delta_{n-1} \end{pmatrix} \qquad (1)$$

Its matrix is tridiagonal, symmetrical, with a strict diagonal predominance.

A given class of systems is effectively solved using sweep method.

2.2. The sweep method

Write the following system of equations (1) to calculate cubic interpolating spline coefficients in the form

$$\begin{pmatrix} a_0 & b_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ c_1 & a_1 & b_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & c_2 & a_2 & b_2 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & c_{n-2} & a_{n-2} & b_{n-2} & 0 \\ 0 & 0 & 0 & \cdots & 0 & c_{n-1} & a_{n-1} & b_{n-1} \\ 0 & 0 & 0 & \cdots & 0 & 0 & c_n & a_n \end{pmatrix} \times \begin{pmatrix} m_0 \\ m_1 \\ m_2 \\ \vdots \\ m_{n-2} \\ m_{n-1} \\ m_n \end{pmatrix} = \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_{n-2} \\ d_{n-1} \\ d_n \end{pmatrix} \tag{2}$$

where:

$$a_0 = \frac{h_1}{3}, \quad b_0 = \frac{h_1}{6}, \quad d_0 = -p_0 + \frac{y_1 - y_0}{h_1}, \quad c_n = \frac{h_n}{6}, \quad a_n = \frac{h_n}{3},$$

$$d_n = p_n - \frac{y_n - y_{n-1}}{h_n}, \quad i = 1,\dots,n-1, \quad c_i = \frac{h_i}{6},$$

$$a_i = \frac{h_i + h_{i+1}}{3}, b_i = \frac{h_{i+1}}{6}, \quad d_i = \frac{y_{i+1} - y_i}{h_{i+1}} - \frac{y_i - y_{i-1}}{h_i}.$$

Solution of tridiagonal system (2) is in the form

$$m_i = \lambda_i\, m_i + 1 + \mu_i, \quad i = 0,\dots, n-1,$$

where λ_i, μ_i – sweep coefficients ($m_n = \mu_n$ if $b_n = 0$).

Recurrence formulas for sweep coefficients λ_i, μ_i are as follows

$$\lambda_0 = -\frac{b_0}{a_0}, \quad \mu_0 = \frac{d_0}{a_0}, \quad \lambda_i = \frac{-b_i}{a_i + c_i\lambda_{i-1}}, \quad \mu_i = \frac{d_i - c_i\mu_{i-1}}{a_i + c_i\lambda_{i-1}}, \quad i = 1,\dots,n \; .$$

2.3. The least squares method

When using the method of least squares, the best coefficients $a_1, a_2,\dots, a_m$ of the approximating function S are those for which the sum of squared deviations from the given empirical values will have the minimal value.

Consequently, we should define the coefficients (that is, choose one curve from the set) to get the least sum of squared deviations.

$$S(a_1, a_2,\dots,a_m) = \sum_{i=1}^{n} [f(x_i; a_1, a_2,\dots,a_m) - y_i]^2 \to \min, \tag{3}$$

where $a_1, a_2,\dots,a_m$ – are the approximation coefficients.

To find a set of coefficients that ensure the minimum of the function S, defined by the formula (3), we use the necessary condition for the extremum of a function of several variables - partial derivatives equal zero. As a result, we obtain a normal system for defining the coefficients

$$\frac{\partial S}{\partial a_1} = 0; \quad \frac{\partial S}{\partial a_2} = 0; \quad \frac{\partial S}{\partial a_m} = 0. \tag{4}$$

This system is simplified, if the empirical formula (3) is linear with respect to the parameters ai, then the system (4) will be linear.

In the case of a linear relationship $y = a_1 + a_2 x$, the system (4) takes the form

$$\begin{cases} a_1 \sum\limits_{i=1}^{n} x + a_2 \sum\limits_{i=1}^{n} x_i^2 = \sum\limits_{i=1}^{n} x_i y_i \\ a_2 \sum\limits_{i=1}^{n} x_i + a_1 \cdot n = \sum\limits_{i=1}^{n} y_i \end{cases}$$

The coefficients a_i are defined by the formulas

$$a_0 = \frac{\sum\limits_{i=1}^{n} y_i \sum\limits_{i=1}^{n} x_i^2 - \sum\limits_{i=1}^{n} x_i \sum\limits_{i=1}^{n} x_i y_i}{\sum\limits_{i=1}^{n} x_i^2 - (\sum\limits_{i=1}^{n} x_i)^2}, \qquad a_1 = \frac{\sum\limits_{i=1}^{N} x_i y_i - \sum\limits_{i=1}^{N} x_i \sum\limits_{i=1}^{N} y_i}{\sum\limits_{i=1}^{N} x_i^2 - (\sum\limits_{i=1}^{N} x_i)^2}.$$

2.4. The software used

To perform computer simulation of the satellite imagery planning process, special application was developed with the C++ programming language. In this application, the mathematical models for the approximation of extended objects with complex configuration described above were implemented, as well as mathematical models of various imaging systems, mathematical models of satellite orbital motion, and support for working with digital terrain maps.

3. Results and discussion

The nodal points of the imaged object are selected on the map. Figure 1 shows an example of a piecewise linear approximation of the nodal points of the imaged object.

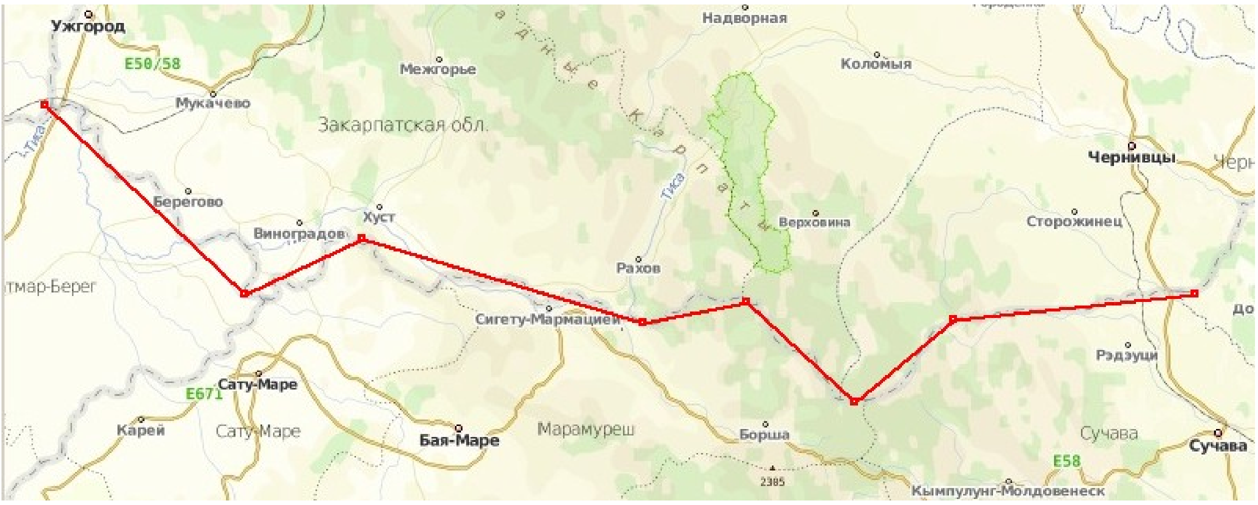

Figure 1. Piecewise linear approximation of the imaged object.

Figure 2 shows the coverage contour with a linear approximation of a given lengthy territory with complex configuration (the scanner swath width at nadir - 40 km).

Figure 2. Coverage contour with linear approximation of the imaged object.

As can be seen from Figure 2, the linear approximation does not provide full coverage of the object when the scanner swath width is about 40 km at nadir.

Figure 3 shows the coverage contour with spline interpolation of a given lengthy territory with complex configuration.

Figure 3. Coverage contour with spline interpolation of the imaged object.

As can be seen from Figure 3, spline interpolation has significant curvature, which requires dynamic satellite retargeting.

To determine optimal coverage of the extended object taking into account the swath width of the imaging instrument and the required scanning direction we have to choose the spline smoothing coefficient and, if necessary, shifts and turns of the coverage contour. Figure 4 shows the coverage contours with spline approximation of the given lengthy territory having complex configuration.

Figure 4. Coverage contours with spline approximation of the imaged object.

As can be seen from Figure 4, the spline approximation provides full coverage of a lengthy territory having complex configuration (the scanner swath width is about 40 km at nadir) with smaller curvature of the scene contour.

Computer simulation of satellite imagery performed for various types of on-board scanners, space imagery modes and types of extended objects with complex configuration (borders, roads, rivers, coastlines, etc.) have confirmed high efficiency of the technique proposed. For instance, when taking satellite imagery of lengthy territories with complex configuration (scanner swath width is about 40 km at nadir), the angular velocities of satellite retargeting were on average less than 0.5 deg/s. Spline approximation of the object appeared to be more efficient both in terms of agility (imagery can be performed in on-orbit period) and the percentage of coverage (100% of the effective area).

4. Conclusion

A technique has been developed for mathematical description of extended objects with complex configuration using spline approximation, which allows to improve significantly the efficiency of satellite imagery of lengthy territories with complex configuration performed using optical-electronic scanners of sub-meter spatial resolution. This technique ensures obtaining optimum (the percentage of effective area imaged in one-orbit period) coverage of the extended object by choosing an appropriate spline smoothing coefficient and, if necessary, shifts and turns of the coverage contour. The results of computer simulation of satellite imagery, taking into account the swath width of the imaging instrument, the required scanning direction and limitations on the retargeting angles range and angular velocities of the satellite, have confirmed high efficiency of the technique proposed.

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