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The logarithmic basis to measure the amount of information related to the assessment of reliability of elements of the technical system

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Abstract. The possibilities of application of logarithmic measure in the problem of reliability evaluation of elements of a technical system are considered. The article presents a brief analysis of the processes of additive and multiplicative growth of system indicators, among which the probability of failure-free operation and failure rate are considered. These indicators were considered in relation to the exponential distribution law of random variables associated with multiplicative growth. The growth of reliability indicators is expressed through the natural logarithm, which allows to determine the amount of information belonging to the element of the system. Considering the additive growth and multiplicative growth in the aggregate, the mathematical expressions of the determination of information entropy in the case of operation of the technical system are presented. The obtained quantitative entropy values are the basis for assessing the state and level of reliability of the system. An example of the calculation is given, confirming the importance of the participation amount of entropy in the problem of estimating the reliability of systems.

1. Introduction

One of the essential factors for the development of technical systems (energy, transport, informational and other) is reliability [1]. Its connection with the appearance of random events, such as failures, stops, shutdowns, accidents, etc. leaves its mark on the development of the system, forcing operators to maintain the required level of its reliability. However, the growth of unwanted random events leads to a decrease in the level of reliability, since it is caused not only by external stochastic processes, but also by the internal state of each element of the system. In connection with the problem of assessing the degree of reliability, the logarithmic basis for measuring information is also based on probabilistic indicators [2].

Since it is not necessary to consider all the characteristics to assess the level of reliability of the system, it is sufficient to determine only some of them (for example, failure rate, uptime), the dynamics of which is determined by the type of distribution law of the event occurrence time (which is understood as a sudden failure) [3]. Such parameters, taking into account the probabilistic nature of their changes and patterns of distribution over time, are the basis for determining the value of statistical entropy, as one of the characteristics of the state of the technical system. One of the tools for making decisions about

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maintaining a high level of reliability can be considered the system analysis, the creation of mathematical models and the development of methods for calculating the amount of information characterizing the state of the system.

2. Analysis of the growth of events in the system

With regard to the possibilities of system analysis, additive growth (associated with the addition operation) and multiplicative growth (based on the multiplication operation) are distinguished. Growth can be characterized by a variety of indicators, one of which will be considered an event that translates an element from one state to another – the opposite. In fact, the event is a sudden failure, disabling the element. Considering the growth associated with the addition of the number of emerging events, we are talking about a streaming process, regardless of whether it is an additive or multiplicative process. Such a streaming process determines the growth rate of the number of events N, which is directly proportional to its current value: $\Delta N = k \cdot N$, where k is some constant. From the point of view of dynamics of change of this indicator, we will have:

$$N(t) = N(0)e^{Ct}, \tag{1}$$

where $N(0)$ – a quantitative value at the initial moment of time, C – an indicator related to the dynamics of the occurrence of events.

Consider this growth in order to obtain a probabilistic characteristic. Let $p(x) \cdot dx$ be the share or probability of events/failures between x and $x+dx$. If its histogram is a straight line in double logarithmic coordinates, then $\ln p(x) = - \alpha \ln x + C$, where α and C are constants. In this mathematical representation, the following regularity is visible: a histogram constructed in such coordinates is a straight line. This statement is associated to the name of Zipf [4]. Zipf's law is one of the basic laws used in measuring the quantitative characteristics of information, when the probability is determined by the frequency of occurrence of events and the total number of events. Thus, we can move from equation (1) to the probabilistic characteristic:

$$p(x) = Cx^{-a}, \tag{2}$$

where $C = e^C$.

The distribution of the form (2) is a power law. Here, the constant α is an exponent of the power law and has a fixed value, whereas the constant C doesn't play a significant role, since it is determined from the requirement: the sum of the distribution $p(x)$ must be equal to 1 [5]. The exponent (2) is consistent with the exponent of the probability distribution of failure-free operation of element i of the system:

$$p_i(t) = e^{-\lambda_i t}, \tag{3}$$

where λ_i is the failure rate of element i. The quantitative reliability characteristics of this element are known reliably.

The value of $p_i(t)$ can be determined based on the data obtained as a result of testing, operation and processing expert assessments of the reliability of new equipment. The expression (3), reflecting the validity of the exponential law of reliability, is based on the assumptions that failures are random and independent events, failure of any element leads to failure of the entire system, and the failure rate is a constant. If it is known that the failures of elements appear sequentially, the probability of failure-free operation of the test system is determined by the expression:

$$P(t) = \prod_i p_i(t). \tag{4}$$

In turn, the failure rate is proportional to the size of the system, and then the failure rate of the system can be calculated by the formula:

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$$\Lambda = \sum_i \lambda_i. \tag{5}$$

According to (3), the probability of a non-failure operation of the system:

$$P(t) = e^{-\Lambda t}. \tag{6}$$

The third statistical calculation parameter is the average uptime of an element i:

$$T_i = 1/\lambda_i, \tag{7}$$

then the average system uptime:

$$T = 1/\Lambda \text{ or } T = \frac{\prod_{i=1}^{n} T_i}{\sum_{i=1}^{n}(\prod_{\substack{j=1 \\ j\neq i}}^{n} T_j)}, \tag{8}$$

where n is the number of considered elements.

According to [6, 7, 8], parameters (3)-(8) can be directly taken into account when measuring information about the elements state. The nature of changes in many random variables corresponds to the power distribution laws [9, 10, 11]. The role of growth over time should be emphasized.

3. Logarithmic measure of information measurement

The possibilities of applying the logarithmic measure of information measurement of additive and multiplicative growth can be discussed in terms of information theory [12, 13]. It substantiates the principles of calculating the amount of information using the Shannon's model [14, 15], which is a generalization of the works of Ralph Hartley [16]. Here the questions of the application of information uncertainty measure, which is quantitatively expressed through the entropy of elements states and the system as a whole, are touched upon. The entropy of a system (in case, element i of this system can be in alternative binary states with probabilities p_i and q_i) is calculated by Shannon's formula:

$$H = -\sum_{i=1}^{n}(p_i \log_2 p_i + q_i \log_2 q_i) \text{ bit, by } \sum_1 p_i + \sum_1 q_i = 1, \tag{9}$$

where n – the number of elements in the system. Next, omit the designation "bit", implying that this unit of measurement is present in mathematical expressions.

Expression (9) is valid in the case of additive growth, since at least 2 conditions must be fulfilled: the sum of the probabilities of events is equal to one; independence of the occurrence of events, meaning the absence of accounting for the connection between the elements of the system [17].

The division of entropy in (9) according to the qualitative attribute into two components is due to the need to measure the level of system reliability through indicators associated with the appearance of independent events leading to the failure of the element [18]. The base of the logarithm in the expression (9) is assumed to be 2, since in the theory of reliability [1] two states of the element are considered: operable and non-operable. In the case of multiplicative growth, the entropy H is calculated in the nat (nepit or unit) and in fact according to the same Shannon's formula:

$$H = -\sum_{i=1}^{n} p_i \ln p_i, \text{ by } \sum_{i=1}^{n} p_i = 1. \tag{10}$$

There is no fundamental difference between the expressions (9) and (10), which was also confirmed in [19]. Let's show this using the following reasoning. As the base of the logarithm (2 or the Neper's number e), a number convenient for the analysis is taken. The base of the logarithm has no effect on the result of entropy value. For a set of probabilities of events, entropy can be calculated by formulas:

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$$H = -\int_0^1 p \log_2 p \, dp = -(\frac{p^2 \ln p}{2} - \frac{p^2}{4}) / \ln 2; \tag{11}$$

$$H = -\int_0^1 p \ln p \, dp = -(\frac{p^2 \ln p}{2} - \frac{p^2}{4}). \tag{12}$$

The difference between expressions (11) and (12) allows us to establish equality between entropies:

$$1.443 \cdot p \ln p \approx p \log_2 p. \tag{13}$$

As for the e (Napier's number), it is considered convenient for analysis, since it doesn't have a qualitative effect on entropy, which is also evidenced by equality (13).

Based on [20], let's present the role of the integral part of the expression (10):

$$h_i = -\ln p_i(t), \tag{14}$$

where h_i – the partial entropy inherent in element i.

Quantitative characteristic h_i in the exponential law reflects the number of events per unit time t (since if we substitute formula (3) into (14) for $t = 1$, we obtain the failure rate of the element – λ_i). A function $h_i(\lambda)$ defined on some set Λ that has a limiting value equal to 0: $\lim_{\lambda \to 0} h_i(\lambda) = 0$. This means that with increasing reliability the partial entropy tends to zero. In practice, for example, when testing equipment or its long-term operation, it is possible to determine the failure rate λ_i based on the recorded number of failures (in the allocated time interval) and further according to (3) to calculate the probability of failure-free operation of element i. For the system as a whole (under the condition of exponential distribution $p_i(t)$), the partial entropy is determined by the expression:

$$h = -\sum_{i=1}^{n} \ln p_i(t) = -\ln \prod_{i=1}^{n} p_i(t) = \Lambda. \tag{15}$$

With growth $p_i(t)$ the partial entropy decreases, that is, the frequency of states leading to system failures decreases.

Next, we denote the role of the probability facing the logarithm in (10). Since the failure rates of elements (for example, during operation) depend on many unpredictable factors, the fixed values of events will be within certain limits. Typically, such distribution of events is possible at time intervals of intensive aging and initial equipment operation. In this case, it is necessary to analyze the reliability state of the element in order to determine its probability p_i. For this the failures of each element are fixed, i.e., record the number of failures ω_i in time interval t (e.g. 1 year). For the entire system, the number of failures is $\Omega = \sum_{i=1}^{n} \omega_i$. The probability of finding element i in the operable state:

$$p_i = \frac{\omega_i}{\Omega}, \text{ by } \sum_{i=1}^{n} p_i = 1 \tag{16}$$

If in the considered time interval t the value of ω_i=0, then to determine the probability according to (16) we can take $\omega_i = \lambda_i$. Knowing the role of the probabilities p_i and $p_i(t)$ in the measurement of information, expression (10) will make it possible to determine the entropy of the system as the sum of the entropies of its elements:

$$H = \sum_{i=1}^{n} H_i = -\sum_{i=1}^{n} p_i \ln p_i(t), \text{ by } \sum_{i=1}^{n} p_i = 1. \tag{17}$$

If additive growth is associated with a change in the structure of the system, the multiplicative growth means the immutability of the structure of organized systems. Expression (17) implies consideration of both additive and multiplicative growth.

4. Example of determining the amount of information

Suppose there is a system containing n=5 elements. Based on the condition of the absence of consequences and ordinariness, the failures of the elements will be considered as random and independent events. Thus, the system (without taking into account the connections between the elements) is considered as a "black box" at the entrance of which we have a statistical series of indicators. It is necessary to determine the entropy of the elements and the system.

There is a statistical series of data (obtained during tests or based on expert estimates), including failure rates λ_i distributed in selected time interval t (year or more) (table 1).

Table 1. Statistical data.

№ element	1	2	3	4	5
Failure rate, λ_i	0.1	0.4	1.0	1.5	2.0
Time interval, t	1	1	1	1	1

Based on the statistical series using formulas (3) and (14) we calculate the probability and partial entropy of each element i (table 2).

Table 2. Probability of failure-free operation and partial entropy.

Probability of failure-free operation, $p_i(t)$	0.905	0.670	0.368	0.223	0.135
Partial entropy, h_i	0.1	0.4	1.0	1.5	2.0

During the operation of the system for a time t statistical data on the number of failures ω_i of each of the elements i (which are presented in table 3) are obtained.

Table 3. Statistical data on the number of failures during operation.

Number of failures, ω_i	1	2	4	3	5
Time interval, t	1	1	1	1	1

According to (16) and (17) determine the probability and entropy values for each element (table 4).

Table 4. Probability and entropy of occurrence of element i events.

The probability of a operable state (p_i) of the element i,	0.067	0.133	0.267	0.20	0.333
Entropy of the element, H_i	0.007	0.053	0.267	0.30	0.666

Total entropy of the system according to (17) - $H = 1.293 nat$. The resulting entropy value H_i is related to the number of failures: the more the element is subject to failures, the higher its entropy. Comparing the entropy of the element H_i with the entropy of the system H through the index H_i/H, one can judge the role of each element in ensuring the reliability of the system.

5. Conclusion

The process of operating the technical system can be considered from the perspective of a system analysis of its reliability. The indicators of this system are changing and correlating with additive and multiplicative growth. A number of variables, for example, such as failure rate, are subject to the exponential distribution law of a random variable, which allows the use of information theory to determine the amount of information entropy in order to assess the state of the system. The exponential law served as the basis for the creation of a mathematical model describing the connection between reliability indicators and entropy, which allows to evaluate the states of the elements and the system as a whole. The model provides a logarithmic basis for measuring entropy, taking into account the features: the division of entropy into two components. The first includes a particular entropy, the second - the distribution of particular entropies between events related to each of the elements. This approach to the determination of information allows, through the amount of entropy, to identify system growth trends, assess the degree of participation of each element in the reliability and build simple trends, which in turn contributes to economic efficiency by saving money on the costs associated with identifying and eliminating of undesirable consequences. The presented example confirms the role of the adequacy of the model in the problem of assessing the reliability of technical facilities. The practical application of this approach is possible to assess the reliability of technical systems in order to calculate the future financial risks associated with the failure of the technical system. Regulation on accounting "Estimated liabilities, contingent liabilities and contingent assets" (PBU 8/2010) (approved by order of the Ministry of Finance of the Russian Federation dated 13.12.2010 № 167n) formulate disclosure requirements in future financial statements. Estimated liabilities, contingent liabilities and contingent assets – forecast estimates. The effectiveness of the standards of the PBU is limited by the difficulty of assessing future events in conditions of uncertainty. The use of the model in the task of assessing the reliability of technical objects will contribute to the introduction of the most accurate estimates of future events in accounting practice.

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