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Local Automorphisms of Nil-triangular Subalgebras of Classical Lie Type Chevalley Algebras

Igor N. Zotov*
Institute of Mathematics and Computer Science
Siberian Federal University
Svobodny 79, Krasnoyarsk, 660041
Russia

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We study the problem of describing local automorphisms of nil-triangular subalgebra of the Chevalley algebra over an associative commutative ring with identity.

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Introduction

A local automorphism of an algebra A is arbitrary modular automorphism which acts on each element $\alpha \in A$ as suitable automorphism of this algebra. The local automorphisms of an algebra A form a group under the composition of mappings (Lemma 1 in Section 1). Automorphisms of an algebra are its trivial local automorphisms.

Local automorphisms and local derivations of an algebra are systematically studied since the 1990s. According to [1], local automorphisms of the algebra $M(n, \mathbb{C})$ of complex $n \times n$ matrices exhausted by automorphisms and anti-automorphisms. See also [2] for local automorphisms of the simple Lie algebra $\mathfrak{sl}_n$ over a field of characteristic zero. R. Crist [3] constructed first example of a nontrivial local automorphism for subalgebra of triangular matrices in $M(3, \mathbb{C})$ with pairwise coincide elements on each diagonal.

Let K be an associative commutative ring with identity. In [4] and [5] local automorphisms of the algebra $NT(n, K)$ of nil-triangular $n \times n$ matrices over K and associated Lie algebra are investigated; they are described for $n = 3$ and, when K is a field, for $n = 4$. In this article we study the more general problem of describing local automorphisms of nil-triangular subalgebra $N\Phi(K)$ of the Chevalley algebra over K associated with a root system Φ. The main result is a reduction Theorem 1 in Section 1. The proof of the theorem is devoted to Section 2. See also the remarks in Section 3.

1. Remarks and the main theorem

Further K is an arbitrary associative commutative ring with identity, unless specified otherwise.

*zotovin@rambler.ru

According to [5] and [6], *a local automorphism* of an arbitrary K-algebra A is an automorphism of the K-module A which acts on each element $\alpha \in A$ as some automorphism depending, in general, on the choice of α. (Definition in [1] has certain difference.) Local ring automorphisms are defined analogously. Denote by $Laut\ A$, the set of all local automorphisms of algebra A.

Lemma 1. *The local automorphisms of any algebra A (similarly for the ring) form a group under the composition of mappings.*

Proof. Choose an arbitrary $\phi, \psi \in Laut\ A$. They acts on each element $x \in A$ as some automorphism ϕ_x, ψ_x of an algebra A. It is necessary to show that

$$(\phi\psi)_x = \phi_{\psi(x)}\psi_x, \quad (\phi^{-1})_x = (\phi_{\phi^{-1}(x)})^{-1}, \quad x \in A.$$

It's evident that, $x = \phi(z) = \phi_z(z)$ with uniquely $z \in A$. Hence

$$\phi^{-1}(x) = z = \phi_z^{-1}(\phi_z(z)) = (\phi_z)^{-1}(x) = (\phi^{-1})_x(x),$$

$$(\phi\psi)(x) = \phi(\psi_x(x)) = \phi_{\psi_x(x)}(\psi_x(x)) = (\phi_{\psi(x)}\psi_x)(x) = (\phi\psi)_x(x).$$

This completes the proof for local ring automorphisms. From here lemma follows easily. □

We investigate local automorphisms of a nil-triangular subalgebra in Chevalley K-algebras.

A Chevalley algebra over a field K is associated with each indecomposable root system Φ in the Euclidean space and characterized by Chevalley base consisting of generating elements e_r ($r \in \Phi$) [7, Sec. 4.4]. We fix a base Π in Φ. Positive system of roots $\Phi^+ \supseteq \Pi$ in Φ is unique [7]. The subalgebra $N\Phi(K)$ with the base $\{e_r \mid r \in \Phi^+\}$ is said to be *a niltriangular subalgebra.*

According to Chevalley's theorem on base [7, Sec. 4.2], if $r, s \in \Phi^+$, then

$$e_r * e_s = N_{r,s}e_{r+s} = -e_s * e_r \ (r + s \in \Phi), \quad e_r * e_s = 0 \ (r + s \notin \Phi),$$

where either $N_{r,s} = \pm 1$ or $|r| = |s| < |r + s|$ and $N_{r,s} = \pm 2$ or Φ is of type G_2 and $N_{r,s} = \pm 2$ or ± 3. The signs of the structure constants $N_{r,s}$ may be chosen arbitrarily (up to isomorphisms $N\Phi(K)$) for extraspecial pairs $(r, s) \in \Phi^+$, [7, Proposition 4.2.2].

The height of the root r is the sum $ht(r)$ of the coefficients in the expansion of r in the base Π in Φ. The Coxeter number $h = h(\Phi)$ of the system Φ equals $ht(\rho) + 1$, where ρ is a maximal root in Φ^+, [7, 8]. Subalgebras L_m with base $\{e_r \mid r \in \Phi^+,\ ht(r) \geqslant m\}$ form in the algebra $L_1 = N\Phi(K)$ *the standard central series*

$$L_1 \supset L_2 \supset \cdots \supset L_{h-1} = Ke_\rho \supset L_h = 0,\ h = ht(\rho) + 1. \tag{1}$$

We now may to formulate our main theorem.

Theorem 1. *The ideal L_2 of Lie algebra $N\Phi(K)$ of classical type of rank > 4 is characteristic and any local automorphism of $N\Phi(K)$ acts as its suitable automorphism, modulo L_2.*

2. Proof of the main theorem

It is well known that the standard central series (1) of the algebra $N\Phi(K)$ is an upper central (or hypercentral) and lower central, except the cases $2K \neq K$, when the system Φ has no roots of different lengths and, also, the case $6K \neq K$ for type G_2. Thus, all ideals L_m in the Lie algebra

$N\Phi(K)$ are characteristic when all roots of the system Φ have the same length or $2K = K$ for the types B_n, C_n and F_4.

The Lie algebra $N\Phi(K)$ of type A_{n-1} is associated to the algebra $NT(n, K)$ of all lower nil-triangular (with zeros on and above the main diagonal) $n \times n$ matrices over K. Usual matrix units e_{ij} $(1 \leqslant j < i \leqslant n)$ gives Chevalley base $\{e_r \mid r \in \Phi^+,\ e_r = e_{ij}\}$ after the corresponding numbering of the roots.

The Lie algebras $N\Phi(K)$ of type B_n, C_n and D_n are given in [9] similarly in the base of matrix units e_{iv}, respectively

$$-i < v < i \leqslant n, \quad -i \leqslant v < i \leqslant n,\ v \neq 0, \quad 1 \leqslant |v| < i \leqslant n.$$

We assume that $r = r_{iv}$ as $e_r = e_{iv}$. The sums of two roots that are the root, in addition to the standard $r_{ij} + r_{jv} = r_{iv}$, as for the type A_n, here also $r_{kv} + r_{m,-v} = r_{k,-m}$ $(k > m > |v|)$ and for type C_n, moreover, $r_{kv} + r_{k,-v} = r_{k,-k}$ $(k > |v|)$. Any element of the Lie algebra $N\Phi(K)$ here is represented by a Φ^+-matrix $||a_{iv}| = \sum a_{iv}e_{iv}$ for corresponding type. Thus, the B_n^+-matrix has the form

$$\begin{matrix} a_{10} \\ a_{2,-1}\ a_{20}\ a_{21} \\ \dots\ \ \dots\ \ \dots \\ a_{n,-n+1}\ \dots\ a_{n,-1}\ a_{n0}\ a_{n1}\ \dots\ a_{n,n-1}. \end{matrix}$$

If we cancel zeros column, then we obtain D_n^+-matrix.

Let T_{im} be the ideal of all Φ^+-matrices of $||a_{uv}||$ with the condition $a_{uv} = 0$ for $u < i$ or $v > m$. We assume that $T_{1m} := T_{im}$ if for the selected Φ and m the number i is the smallest. In the Lie algebra $N\Phi(K)$ of type B_n (or $NB_n(K)$), we select submodules $R_j := \sum_{i=j}^{n} Ke_{i0}$, $1 \leqslant j \leqslant n$, and also select the submodule $L_j^{[0}$ with base $\{e_{uv} \mid 0 \leqslant v < u \leqslant n,\ u - v \geqslant j\}$.

We need the following two lemmas from [10].

Lemma 2. *Let $2K \neq K$ and $n \geqslant 2$. Then the Lie rings $NB_n(K)$ and $NC_n(K)$ generate*

$$\{Ke_{ii-1}\ (1 \leqslant i \leqslant n);\quad Ke_{2,-1}\},$$

$$\{Ke_{ii-1}\ (2 \leqslant i \leqslant n);\quad Ke_{i,-i}\ (1 \leqslant i \leqslant n)\},$$

respectively, and no Ke_{iv} can be dropped in them.

Denote by $\mathcal{A}_2$, the annihilator of the element 2.

Lemma 3. *Hypercenters of the Lie algebra $NC_n(K)$ $(n \geqslant 2)$ are written as*

$$Z_i = L_{2n-i} + \mathcal{A}_2 L_{2n-i-1}\ (1 \leqslant i < 2n-1), \quad Z_{2n-1} = L_1.$$

For the algebra $NB_n(K)$ $(n \geqslant 2)$ we have

$$Z_i = L_{2n-i} + \mathcal{A}_2 R_{n+1-i}\ (1 \leqslant i \leqslant n-2), \quad Z_{n-1} = L_{n+1} + \mathcal{A}_2 R_2 + \mathcal{A}_2 e_{n1},$$

$$Z_{n+i} = L_{n-i} + \mathcal{A}_2 R_1 + \mathcal{A}_2 L_{n-i-2}^{[0}\ \ (0 \leqslant i \leqslant n-3), \quad Z_{2n-2} = L_2 + \mathcal{A}_2 L_1.$$

The diagonal automorphisms $h(\chi) : e_r \to \chi(r)e_r$ $(r \in \Phi^+)$ of the Lie algebra $N\Phi(K)$ correspond to each K-character χ of the root lattices to the multiplicative group $K^\sharp$ of invertible elements of the ring K [7, Sec. 7.1].

For any root r the mapping $t \to x_r(t) := exp\ (t \cdot ad.e_r)$ $(t \in K)$ generate an isomorphism of the additive group $K^+ := (K, +)$ to the automorphism group of the algebra Chevalley. Root subgroups $X_r = x_r(K)$ generate a Chevalley group, [7,11]. The restrictions of automorphisms of its unipotent subgroup $U\Phi(K) = \langle X_r\ (r \in \Phi^+)\rangle$ generate the subgroup J of *inner automorphisms* of the Lie algebra $N\Phi(K)$.

The standard automorphisms of the Lie algebra $N\Phi(K)$ include inner, diagonal, graph [7, Ch. 12] and central automorphisms, that is, the identity automorphism modulo the center.

According to [9], if the Lie ring (or group) does not coincide with its mth hypercenter, then its automorphism is said to be *hypercentral of height* m, or simply *hypercentral*, if it is the identity automorphism modulo the mth hypercenter and an outer automorphism modulo the $(m-1)$th hypercenter.

The main hypercentral automorphisms of height > 1 of Lie algebras of $N\Phi(K)$ of classical types are revealed previously, [12–14]. Let $V(\Phi, K)$ denote the subgroup generated by them.

It is well known that the adjoint group of the ring $R = NT(n, K)$ under the adjoint multiplication $a \circ b = a + b + ab$ is isomorphic to the unitriangular group $UT(n, K)$. The automorphism group of the associated Lie algebra $\Lambda(R)$ (that is, $N\Phi(K)$ of type A_{n-1}) is found in [12]:

$$Aut\ \Lambda(R) = \mathcal{Z} \cdot J \cdot V \cdot \mathcal{D} \cdot W \quad (n > 4), \tag{2}$$

where $\mathcal{Z}$, $\mathcal{D}$ and W are subgroups of central, diagonal and idempotent automorphisms, respectively. The subgroup V is generated by the main hypercentral automorphisms of height 2 and for $\mathcal{A}_2 \neq 0$ – of height 3. For the Lie ring $\Lambda(R)$ the subgroup $\simeq Aut\ K$ of induced automorphisms is added, [12, Theorem 1]. The description of automorphisms in [12] also for $n = 3, 4$ is certain difference.

In the Lie algebra $N\Phi(K)$ of type B_n, the ideal L_2 is larger than the commutant as $2K \neq K$, by Lemma 2 and Lemma 3. When $\mathcal{A}_2 \neq 0$, it less than hypercenter Z_{2n-2}. The Lie algebra $NB_n(K)$ admits hypercentral automorphisms, whose height depends linearly on the rank n. To any pair $t, d \in \mathcal{A}_2$ there corresponds such automorphism

$$\chi_{t,d} : \alpha \to \alpha + \sum_{k=2}^{n-1} a_{k,-1}(te_{k0} + de_{n,-k}),$$

which translates the (-1)th column of the B_n^+-matrix to 0th column.

The subgroup in $Aut\ NB_n(K)$ isomorphic to the adjoint group in $\mathcal{A}_2$ form *semi-diagonal automorphisms*

$$\delta_c^{(-1)} : e_{kv} \to (1+c)e_{kv}\ (0 < -v < k \leqslant n), e_{kv} \to e_{kv}\ (0 \leqslant v < k \leqslant n),$$

with invertible $1 + c \in 1 + \mathcal{A}_2$.

According to [13], for simple symmetric roots r and $\bar{r} \neq r$ $(\bar{\bar{r}} = r)$ of a system roots Φ of type D_n $(n \geqslant 4)$, an isomorphic embedding$\widetilde{\ }$of the subgroup

$$S = \{\alpha = ||a_{uv}|| \in SL(2, K)\ :\ 2a_{11}a_{12} = 2a_{21}a_{22} = 0\}$$

of the group $SL(2, K)$ into the group $Aut\ ND_n(K)$ is defined by the rule

$$\widetilde{\alpha} : \quad e_r \to a_{11}e_r + a_{12}e_{\bar{r}}, \quad e_{\bar{r}} \to a_{21}e_r + a_{22}e_{\bar{r}}, \quad e_s \to e_s \quad (s \in \Pi \setminus \{r, \bar{r}\}).$$

The description of automorphisms of Lie rings $N\Phi(K)$ of classical types is completed in [13]. It is summarized by the following theorem.

Theorem 2. *Any automorphism of the Lie ring $NC_n(K)$ ($n > 4$) is a product of the standard and hypercentral of $V(\Phi, K)$ automorphisms. For the Lie ring $NB_n(K)$ ($n > 4$), a semi-diagonal automorphism is added as a factor, and for the Lie ring $ND_n(K)$ ($n > 4$) an automorphism of $\widetilde{S}$ is added as a factor.*

Remark that the ideal L_3 is not even invariant under the hypercentral automorphism $\chi_{t,d}$. On the other hand, we have the following lemma.

Lemma 4. *The ideal L_2 in the Lie algebra $N\Phi(K)$ of rank > 4 of the classical type is always characteristic.*

Proof. Obviously, the ideal L_2 in the Lie algebra $NB_n(K)$ is $\chi_{t,d}$-invariant. Its $V(\Phi, K)$-invariance follows directly from the definitions of the other main hypercentral automorphisms of height > 1 in all cases with regard to the restriction on Lie rank, [13, 14].

Graph automorphisms of Lie algebras $N\Phi(K)$ of rank > 4 are defined only for root systems of the same length (more precisely, for A_n, D_n and E_n types, $n = 6, 7, 8$). As noted above, in these cases all ideals of L_m are characteristic. With respect to diagonal (and ring) automorphisms, all one-dimensional subalgebras Ke_r are invariant, and therefore any L_m ideals are invariant. This is also true for semi-diagonal automorphisms of the Lie algebra $NB_n(K)$.

The inner automorphisms act on L_m identically modulo L_{m+1}, which also implies J-invariance of all L_m. Taking into account that under the conditions of the lemma the center Z_1 always is in L_2, we obtain the characteristic of the ideal L_2 with respect to any standard automorphism. This completes the proof of the lemma. □

The first statement of main theorem 1 is given by Lemma 4. Let us prove the second statement.

The Lie algebra $N\Phi(K)$ of type A_{n-1} is represented, as above, by the algebra $\Lambda(R)$ for $R = NT(n, K)$ ($n > 4$). The group automorphisms $Aut\ \Lambda(R)$ is factorized by the product (2), and its normal subgroup $\mathcal{Z} \cdot J \cdot V$ acts identically modulo L_2.

Let $i' := n + 1 - i$. According to [12], any idempotent g of the ring K defines a Lie automorphism

$$\tau_g : e_{ij} \to ge_{ij} + (-1)^{i-j-1}(1-g)e_{j'i'},\ 1 \leqslant j < i \leqslant n,$$

called *idempotent* and $W =< \tau_g \mid g \in K,\ g^2 = g >$. In particular, for the case $i' = i + 1$ we obtain the characteristic ideal

$$T_{i'i'-1} = \tau_0(T_{i+1i}) = T_{i+1i}$$

of the Lie algebra $\Lambda(R)$.

Any local automorphism φ of the Lie algebra $\Lambda(R)$ acts on an arbitrary element modulo L_2 as a suitable automorphism of $\mathcal{D} \cdot W$. Taking into account that φ is K-module automorphism, we have, up to its multiplication by an automorphism of $\mathcal{D} \cdot W$,

$$\varphi(e_{21}) = e_{21}, \quad \varphi(xe_{21}) = x\varphi(e_{21}) = xe_{21} \quad mod\ L_2 \quad (x \in K).$$

We need the following lemma (cf. [15, Lemma 1.3.5]).

Lemma 5. *If $\varphi(e_{21}) = e_{21}$ for a local automorphism φ of Lie algebra $\Lambda(R)$ ($n > 4$), then*

$$\varphi(e_{i+1i}) \in T_{i+1i} + L_2, \quad 1 \leqslant i < n. \tag{3}$$

Proof. Assume that the inclusion (3) is violated for some number i, $1 < i < n$ and $i' \neq i+1$. Taking into account the condition $n > 4$, for some idempotent $f \neq 1$ we obtain, up to a multiplication of φ by a diagonal automorphism, the following equalities:

$$\varphi(e_{21}) = e_{21}, \quad \varphi(e_{i+1i}) = \tau_f(e_{i+1i}) \mod R^2, \quad 1 - f \neq 0.$$

Since there exists automorphism $\psi \in Aut\ \Lambda(R)$ acting on $e_{21} + e_{i+1i}$ similarly to φ, so

$$e_{21} + fe_{i+1i} + (1-f)e_{i'i'-1} = \psi(e_{21}) + \psi(e_{i+1i}) \mod L_2. \tag{4}$$

We can assume that $\psi \in D \cdot W$ and hence $\psi = \delta\tau_g$ for a suitable idempotent g and diagonal automorphism δ. If $i < n-1$, then $(n, n-1)$-projection of the element (4) on the right is in $(1-g)K^\sharp$, and on the left is zero. Hence $g = 1$ and $\psi = \delta$. Comparing now $(i', i'-1)$-projections of matrices in (4) on the right and left, we obtain the equality $1 - f = 0$, that gives a contradiction.

It remains to investigate the case $i = n-1$. Using (4), we find invertible elements $c, d \in K$ such that

$$(2-f)e_{21} + fe_{nn-1} = (ge_{21} + (1-g)e_{nn-1})^\delta + (ge_{nn-1} + (1-g)e_{21})^\delta,$$

$$(2-f)e_{21} + fe_{nn-1} = (cg + c(1-g))e_{21} + (d(1-g) + dg)e_{nn-1}.$$

The last equality gives $d = f$. It is easily follows $f = 1$. This contradicts to the condition $f \neq 1$. □

As corollary of the proved lemma we obtain the equalities for some elements $c_i \in K$

$$\varphi(e_{i+1i}) = c_i e_{i+1i} \quad (mod\ L_2).$$

Now the equality (2) shows that φ acts on each element of e_{i+1i} modulo L_2 as a suitable automorphism of $\mathcal{D}$. It follows that all elements of c_i are invertible in K. This completes the proof of the theorem for the type A_n.

Any local automorphism of the φ Lie algebra $ND_n(K)$ acts on arbitrary element, as a suitable automorphism. Up to multiplication by an automorphism, one can even assume that the equality $\varphi(e_{2,-1}) = e_{2,-1}$ is satisfied.

Then φ acts modulo L_2 on each element e_{i+1i} as a suitable automorphism from the product $\mathcal{D} \cdot \widetilde{S}$. Therefore

$$\varphi(e_{21}) = a_{21}e_{2,-1} + c_1 e_{21} \quad mod\ L_2,$$

$$\varphi(e_{i+1i}) = c_i e_{i+1i} \quad mod\ L_2, \quad 2 \leqslant i < n,$$

where all elements of c_i $(1 \leqslant i < n)$ are invertible in K and $2a_{21} = 0$. Up to multiplication of φ by a diagonal automorphism, we can assume that $c_i = 1$ $(1 \leqslant i < n)$. Then φ acts modulo L_2 as an automorphism $\widetilde{\alpha}$ with matrix

$$\alpha = \begin{pmatrix} 1 & 0 \\ a_{21} & 1 \end{pmatrix}.$$

Further we note that if an ideal of an algebra is characteristic, then it is invariant with respect to any local automorphism of the algebra. From the Theorem 2 easily implies the following lemma

Lemma 6. *In the Lie ring $NC_n(K)$ $(n > 4)$, the ideals T_{ij} for $i < n$ and the ideals T_{iv} for $v < 0$ are characteristic.*

It follows from lemma that the ideal $T_{2,-2}$ is characteristic in the Lie algebra $NC_n(K)$ $(n > 4)$ and this shows that any local automorphism of φ induces a local automorphism of the factor algebra

$$NC_n(K)/T_{2,-2} \simeq NA_n(K) \simeq NT(n+1, K),$$

moreover, $\varphi(T_{1,-1}) = T_{1,-1}$. Applying the proved case for the type A_n, we obtain the statement of the theorem for the type C_n.

In the Lie ring $NB_n(K)$ $(n \geqslant 5)$ the ideals T_{10} and $T_{10} + T_{21}$ are always characteristic and

$$NB_n(K)/T_{10} \simeq NA_{n-1}(K) \simeq NT(n, K).$$

Applying the proved case for the type A_n, we obtain the statement of theorem for the type B_n.

The theorem is proved. □

3. Some remarks

The algebra R is called *enveloping* for the Lie algebra L if replacing the multiplication in R with new $a * b := ab - ba$ gives algebra $R^{(-)}$ isomorphic to L. It's obvious that $Aut\ R \subseteq Aut\ R^{(-)}$, [16]. Unlike the Lie algebras $N\Phi(K)$ the enveloping algebras (in general, non-associative) that are constructed for them in [16, 17] depend on the choice of signs of the constants $N_{r,s}$.

When the choice of signs of the constants of the Lie algebra $N\Phi(K)$ of the classical type corresponds to its representation in [9], the enveloping algebra is denoted by $R\Phi(K)$. The developed methods are applicable for transferring the main theorem to algebras $R\Phi(K)$.

The restrictions in the main theorem on the rank of n are related to the fact that some basic hypercentral automorphisms of the Lie algebra $N\Phi(K)$ of classical Lie type for small n can remain automorphisms that are not hypercentral automorphisms. In these cases, the action of $Aut\ N\Phi(K)$, modulo L_2, becomes exceptional. See [4, 12] for the type A_n and the description of $Aut\ ND_4(K)$ in [9].

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Локальные автоморфизмы нильтреугольных подалгебр алгебр Шевалле классических типов

Игорь Н. Зотов
Институт математики и фундаментальной информатики
Сибирский федеральный университет
Свободный, 79, Красноярск, 660041
Россия

Исследуется задача описания локальных автоморфизмов нильтреугольной подалгебры алгебры Шевалле над ассоциативно-коммутативным кольцом с единицей.

Ключевые слова: автоморфизм, локальный автоморфизм, стандартный центральный ряд, характеристический идеал, алгебра Шевалле, нильтреугольная подалгебра.