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The Influence of Changes in the Internal Energy of the Interface on a Two-Layer Flow in a Cylinder

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The exact solution of the equations of the creeping flow model with the Himentsa type velocity field is considered in this paper. The solution describes thermocapillary convection in layers. It is interpreted as the motion of viscous heat-conducting liquids in a cylinder with solid walls and combined movable non-deformable interface. At the same time there are no mass forces. From a mathematical point of view the resulting initial-boundary problem is inversible and nonlinear because the total energy condition at the interface is taken into account. It is established that there can be two such solutions.

Keywords: nonlinear inverse problem, Marangoni number, energy condition, creeping thermocapillary motion, Himentsa solution.

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The specificity of the phenomena occurring at the phase interface is associated with the existence of energy and entropy of the surface phase which are excessive with respect to the volumetric phases in the phase transition layer [1]. However, the energy exchange between the volumetric and surface phases is not well studied. For ordinary fluids at room temperature, the effect of changes of the internal energy of the interfacial surface on the formation of heat fluxes, temperature fields and velocities in the surface vicinity is insignificant in comparison with viscous friction and heat transfer [2]. Little attention has been given to the class of problems associated with these phenomena.

When a liquid media with an interface moves in an inhomogeneous temperature field the difference in heat fluxes is not equal to zero [3],

$$k_2 \frac{\partial \theta_2}{\partial n} - k_1 \frac{\partial \theta_1}{\partial n} = \varkappa \theta \operatorname{div}_\Gamma \mathbf{u} + \omega (\theta_t + \mathbf{u} \cdot \nabla_\Gamma \theta), \quad (1)$$

where $\varkappa = -\partial\sigma/\partial\theta$, $\omega = \partial(\sigma(\theta) + \varkappa\theta)/\partial\theta$, $\sigma(\theta)$ is the surface tension coefficient. In relation (1) k_j is the coefficient of heat conduction, θ_j is the temperature of a liquid, $j = 1, 2$; $\theta = \theta_1 = \theta_2$, $\mathbf{u} = \mathbf{u}_1 = \mathbf{u}_2$ are common temperatures and velocity vectors at the interface Γ and \mathbf{n} is the normal to Γ directed to the second fluid. For many liquids, $\sigma(\theta)$ is a linear function of temperature

$$\sigma(\theta) = \sigma_0 - \varkappa(\theta - \theta_0) \quad (2)$$

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where σ_0 , \varkappa , θ_0 are constants. In this case energy relation (1) is simplified:

$$k_2 \frac{\partial \theta_2}{\partial n} - k_1 \frac{\partial \theta_1}{\partial n} = \varkappa \theta \operatorname{div}_{\Gamma} \mathbf{u}. \quad (3)$$

The ratio of the right-hand side of (3) to the first term in the left-hand side of (3) is $E = \varkappa^2 \theta^* / k_2 \mu_2$. Here μ_j is the dynamic viscosity, and θ^* is the characteristic temperature at the interface. This value characterizes the significance of the process of releasing or absorption of heat during local increments of the interfacial surface for the development of convective motion near the interface. For ordinary liquids the value of E is small at room temperature, and changes in the characteristic velocity of convection due to increments of surface internal energy are insignificant [2]. For example, experiments for the water-ethyl alcohol system at $\theta^* = 15^\circ C$ give $E \sim 5 \cdot 10^{-4}$. Therefore, the right-hand side in (1) is often omitted, and we have the equality of heat fluxes across the interface.

However, at sufficiently high temperatures when viscosity and thermal conductivity of ordinary liquids are significantly reduced and for liquids with reduced viscosity (for example, for some cryogenic liquids, such as liquid CO_2) effects associated with the heat of formation of the interfacial surface can have a significant impact on fluid motion [4]. The maximum value of parameter E is reached near the critical points. It is known that for water $E \sim 0.02$ at $\theta = 303.15$ K; $E \sim 0.6$ at $\theta = 573.15$ K; $E \sim 0.7$ at $\theta = 623.15$ K (the critical value for water is $\theta_{\text{cr}} = 647.3$ K). Let us note that mechanism of local changes in the internal energy of an interfacial surface (IEIS) affects the convective stability of thermocapillary systems (see [3], chapter VI). This mechanism should be taken into account for liquids at elevated temperatures and low viscosity. In particular, it was shown that the increment of IEIS leads to the expansion of the stability limits at rest of a horizontal flat layer of liquid in the presence of a temperature gradient in the vertical direction. The effect of IEIS changes on the linear and weakly nonlinear stability of the two-layer Poiseuille flow and film flow of a low-viscosity fluid for an isothermal ground state shows up as the change in the phase velocity of waves. In all considered problems the ground state did not depend on the IEIS.

The exact solution for the equations of a creeping-flow model with a velocity field of the Himents type is obtained in this paper. It describes thermocapillary convection in a two-layer viscous heat-conducting fluid. The motion occurs in a cylinder with solid walls and a common movable non-deformable interface. In this case mass forces are absent. From a mathematical point of view the initial boundary-value problem is inversive and nonlinear because the total energy condition at the interface is taken into account. It is established that there can be two such solutions.

1. The problem formulation and its transformation

The equations of rotationally symmetric stationary motion of a viscous incompressible heat-conducting fluid in the absence of mass forces have the form

$$uu_r + wu_z + \frac{1}{\rho} p_r = \nu \left(\Delta u - \frac{u}{r^2} \right), \quad (4)$$

$$uw_r + ww_z + \frac{1}{\rho} p_z = \nu \Delta w, \quad (5)$$

$$u_r + \frac{1}{r} u + w_z = 0, \quad (6)$$

$$u\theta_r + w\theta_z = \chi\Delta\theta. \quad (7)$$

Here $u(r, z)$, $w(r, z)$ are components of the velocity vector on the axes r and z of the cylindrical coordinate system, $p(r, z)$ is pressure, $\theta(r, z)$ is fluid temperature; ρ , ν , χ are the density, the kinematic viscosity and the thermal diffusivity, respectively; $\Delta = \partial^2/\partial r^2 + r^{-1}\partial/\partial r + \partial^2/\partial z^2$ is the Laplace operator.

The solution of system (4)–(7) is taken in the form

$$u = u(r), \quad w = v(r)z, \quad p = p(r, z), \quad \theta = \theta(r, z). \quad (8)$$

This is an axisymmetric analog of the Himenz solution. The substitution of expressions for u , w and p from (8) in the equations of motion (4), (5) leads to the following relations

$$\begin{aligned} \frac{1}{\rho} p_r &= \nu \left(u_{rr} + \left(\frac{1}{r} u \right)_r \right) - uu_r, \\ \frac{1}{\rho} p_z &= \left(uv_r + v^2 - \nu \left(v_{rr} + \frac{1}{r} v_r \right) \right) z. \end{aligned}$$

It follows from the analysis of the obtained expressions that p_r is a function of variable r , and p_z linearly depends on variable z . Taking into account (8), we obtain from equations of motion (4)–(6) that

$$\begin{aligned} u_r + \frac{1}{r} u + v &= 0, \quad uv_r + v^2 = \nu \left(v_{rr} + \frac{1}{r} v_r \right) + f, \\ \frac{1}{\rho} p &= d(r) - \frac{f}{2} z^2, \quad d = \nu \left(u_r + \frac{1}{r} u \right) - \frac{1}{2} u^2 + d_0, \quad d_0 = \text{const}, \end{aligned} \quad (9)$$

where f is an arbitrary constant that is the pressure gradient along the axis of the cylinder.

Equation for temperature (7) can be rewritten as

$$u(r)\theta_r + v(r)z\theta_z = \chi\Delta\theta.$$

One of the solutions of this equation is quadratic with respect to the variable z function of the form

$$\theta(r, z) = a(r)z^2 + b(r). \quad (10)$$

Thus the temperature takes an extreme value at point $z = 0$. It has maximum value when $a(r) < 0$, and it is minimum value when $a(r) > 0$.

Let us apply solutions (9), (10) to describe the two-layer motion of viscous heat-conducting fluids in the cylinder $0 \leq r \leq R_2$ with the solid wall at $r = R_2 = \text{const}$ and with the cylindrical interface at $r = R_1$, $0 < R_1 < R_2$. The fluid 1 occupies the region $0 \leq r \leq R_1$, and the fluid 2 occupies the cylindrical layer $R_1 \leq r \leq R_2$ (Fig. 1). Parameters of fluids are ρ_j , ν_j , χ_j , $j = 1, 2$.

Substituting (9), (10) into equations of motion (4)–(6) and heat transfer equation (7), we can obtain that $u_j(r)$, $v_j(r)$, $a_j(r)$, $b_j(r)$ are solutions of the following systems of equations ($j = 1, 2$)

$$\begin{aligned} u_j v_{jr} + v_j^2 &= \nu_j \left(v_{jrr} + \frac{1}{r} v_{jr} \right) + f_j, \\ u_{jr} + \frac{1}{r} u_j + v_j &= 0. \end{aligned} \quad (11)$$

$$\begin{aligned} 2v_j a_j + u_j a_{jr} &= \chi_j \left(a_{jrr} + \frac{1}{r} a_{jr} \right), \\ u_j b_{jr} &= \chi_j \left(b_{jrr} + \frac{1}{r} b_{jr} \right) + 2\chi_j a_j, \end{aligned} \quad (12)$$

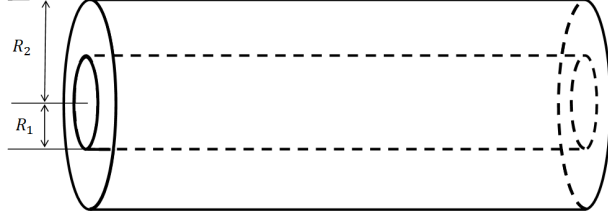


Fig. 1. Cylindrical domain with interface

The pressure obeys the following equation

$$\frac{1}{\rho_j} p_j(r, z) = d_j(r) - \frac{f_j}{2} z^2, \quad d_j = \nu \left(u_{jr} + \frac{1}{r} u_j \right) - \frac{1}{2} u_j^2 + d_{j0}, \quad d_{j0} = \text{const.} \quad (13)$$

On solid wall at $r = R_2$ we have conditions

$$u_2(R_2) = 0, \quad v_2(R_2) = 0, \quad (14)$$

$$a_2(R_2) = \alpha, \quad b_2(R_2) = \beta, \quad (15)$$

with given constants α, β . Let us note that for $\alpha < 0$ the temperature on the wall has maximum value at $z = 0$ and for $\alpha \geq 0$ it has minimum value at $z = 0$.

Taking into account the temperature dependence of the surface tension coefficient ($\sigma = \sigma_0 - \alpha(\Theta - \Theta_0)$) and (10), we obtain at the interface $r = R_1$ the following conditions

$$u_1(R_1) = u_2(R_1) = 0, \quad v_1(R_1) = v_2(R_1), \quad (16)$$

$$\mu_2 v_{2r}(R_1) - \mu_1 v_{1r}(R_1) = -2\alpha a_1(R_1), \quad (17)$$

$$a_1(R_1, t) = a_2(R_1), \quad k_2 a_{2r}(R_1) - k_1 a_{1r}(R_1) = \alpha a_1(R_1) v_1(R_1), \quad (18)$$

$$b_1(R_1, t) = b_2(R_1), \quad k_2 b_{2r}(R_1) - k_1 b_{1r}(R_1) = \alpha b_1(R_1) v_1(R_1), \quad (19)$$

where $\mu_j = \rho_j \nu_j$ is the dynamic viscosity coefficient. Moreover, functions $u_1(r), v_1(r), a_1(r)$ and $b_1(r)$ are bounded at $r = 0$.

Let us note that the problem is non-linear and inverse problem because along with $v_j(r), a_j(r), b_j(r)$, constants f_j (pressure gradients along the layers) are also unknown. Excluding functions $u_j(r)$ from the second equations (11) and taking into account adhesion conditions on the wall, we obtain the adjoint problem for functions $v_j(r), a_j(r)$. For known $u_j(r), a_j(r)$ the problem for functions $b_j(r)$ is separated. Functions $d_j(r)$ are found upon integrating (13).

We introduce dimensionless functions and parameters

$$V_j(\xi) = \frac{R_1^2}{M \chi_1} v_j(r), \quad A_j(\xi) = \frac{a_j(r)}{\alpha}, \quad F_j = \frac{R_1^4}{M \chi_1^2} f_j, \quad M = \frac{\alpha \alpha R_1^3}{\mu_2 \chi_1},$$

$$\xi = \frac{r}{R_1}, \quad \text{Pr}_j = \frac{\nu_j}{\chi_j}, \quad \chi = \frac{\chi_1}{\chi_2}, \quad \mu = \frac{\mu_1}{\mu_2}, \quad k = \frac{k_1}{k_2}, \quad \gamma = \frac{R_2}{R_1},$$

where parameter M is the Marangoni number. Then the nonlinear conjugate inverse boundary

value problem takes the form in dimensionless variables

$$\begin{aligned} \text{Pr}_1 \left(V_{1\xi\xi} + \frac{1}{r} V_{1\xi} \right) + \frac{M}{\xi} V_{1\xi} \int_0^\xi x V_1(x) dx - M V_1^2 + F_1 &= 0, \\ A_{1\xi\xi} + \frac{1}{\xi} A_{1\xi} + \frac{M}{\xi} A_{1\xi} \int_0^\xi x V_1(x) dx - 2V_1 A_1 &= 0, \quad 0 < \xi \leq 1; \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{\text{Pr}_2}{\chi} \left(V_{2\xi\xi} + \frac{1}{\xi} V_{2\xi} \right) - \frac{M}{\xi} V_{2\xi} \int_\xi^{R_2} x V_2(x) dx - M V_2^2 + F_2 &= 0, \\ \frac{1}{\chi} \left(A_{2\xi\xi} + \frac{1}{\xi} A_{2\xi} \right) - \frac{M}{\xi} A_{2\xi} \int_\xi^{R_2} x V_2(x) dx - 2V_2 A_2 &= 0, \quad 1 \leq \xi \leq \gamma, \end{aligned} \quad (21)$$

$$V_2(\gamma) = 0, \quad A_2(\gamma) = 1, \quad (22)$$

$$\int_0^1 x V_1(x) dx = 0, \quad \int_1^\gamma x V_2(x) dx = 0, \quad (23)$$

$$V_{2\xi}(1) - \mu V_{1\xi}(1) = -2A_1(1), \quad (24)$$

$$V_1(1) = V_2(1), \quad |V_1(0)| < \infty, \quad (25)$$

$$A_{2\xi}(1) - k A_{1\xi}(1) = E A_1(1) V_1(1), \quad (26)$$

$$A_1(1) = A_2(1), \quad |A_1(0)| < \infty, \quad (27)$$

where parameters Pr_j are Prandtl numbers, $E = \alpha^2 R_1^2 / \mu_2 k_2$ is the parameter that determines the influence of internal interfacial energy on the motion of fluids inside the layers. Integral conditions (23) allow us to find unknown constants (pressure gradients along the layers) F_j , $j = 1, 2$.

2. Solution of model problems

Let us consider the obtained nonlinear conjugate inverse boundary value problem (20)–(27). Equations (20), (21) contain the Marangoni number. Let us assume that $M \ll 1$, that is, we consider the creeping thermocapillary motion. First we exclude all nonlinear terms in equations (20), (21). Then we leave unchanged the only nonlinear boundary condition (26) and assume that $E = O(1)$. After some transformations we get

$$V_1(\xi) = -\frac{1}{4} \frac{F_1}{\text{Pr}_1} \xi^2 + C_1 \ln(\xi) + C_2, \quad (28)$$

$$A_1(\xi) = C_3 \ln(\xi) + C_4, \quad 0 \leq \xi \leq 1;$$

$$V_2(\xi) = -\frac{1}{4} \frac{\chi F_2}{\text{Pr}_2} \xi^2 + C_5 \ln(\xi) + C_6, \quad (29)$$

$$A_2(\xi) = C_7 \ln(\xi) + C_8, \quad 1 \leq \xi \leq \gamma,$$

where C_i ($i = 1, \dots, 8$) are unknown constants. Substituting solutions (28), (29) into the system of boundary conditions (22)–(27), we obtain an algebraic system of equations for unknown C_i and F_j . Thus, boundedness of functions $V_1(\xi)$ and $A_1(\xi)$ at $\xi = 0$ ((25), (27)) implies that $C_1 = C_3 = 0$. Functions $A_j(\xi)$ are equal on the interface (27) then $C_4 = C_8$. Taking into

account conditions (22)–(24) and condition of equality of functions $V_j(\xi)$ on the interface, we obtain from (25) that

$$\begin{aligned} C_2 &= -\frac{2(S \ln(\gamma) - 1)}{4\mu(S \ln(\gamma) - 1) + 2L + 1} C_8, & C_5 &= -\frac{2}{4\mu(S \ln(\gamma) - 1) + 2L + 1} C_8, \\ C_6 &= \frac{2(L\gamma^2 + \ln(\gamma))}{4\mu(S \ln(\gamma) - 1) + 2L + 1} C_8, \\ F_1 &= -\frac{16\text{Pr}_1(S \ln(\gamma) - 1)}{4\mu(S \ln(\gamma) - 1) + 2L + 1} C_8, & F_2 &= \frac{8\text{Pr}_2 L}{\chi(4\mu(S \ln(\gamma) - 1) + 2L + 1)} C_8, \end{aligned} \quad (30)$$

where

$$L = \frac{1 + 2 \ln(\gamma) - \gamma^2}{(\gamma^2 - 1)^2}, \quad S = \frac{\gamma^2 + 1}{\gamma^2 - 1}, \quad (31)$$

and constant C_8 is the root of the quadratic equation

$$\frac{2E \ln(\gamma)(S \ln(\gamma) - 1)}{4\mu(S \ln(\gamma) - 1) + 2L + 1} C_8^2 + C_8 - 1 = 0. \quad (32)$$

It is clear that when there is no effect of internal interfacial energy on the motion of fluids inside the layers ($E = 0$), that is, the heat fluxes on the interface are equal to each other or $\gamma \rightarrow 1$ (radii of the outer and inner cylinders coincide) equation (32) has one root $C_8 = 1$. In other cases when $\gamma > 1$ and $E \neq 0$ ($\alpha \neq 0$) equation (32) has two real roots. In general, the analysis depends on the sign of the expression

$$D = 1 + \frac{8E \ln(\gamma)(S \ln(\gamma) - 1)}{4\mu(S \ln(\gamma) - 1) + 2L + 1}. \quad (33)$$

For $D > 0$ equation (32) has two roots and for $D = 0$ it has one root. When $D < 0$ problem (20)–(27) has no solutions.

Taking into account (28), (29), unknown dimensionless velocity functions have the form

$$\begin{aligned} U_1(\xi) &= \frac{1}{16} \frac{F_1}{\text{Pr}_1} \xi^3 - \frac{1}{2} C_2 \xi, \\ U_2(\xi) &= -\frac{1}{16} \frac{F_2}{\text{Pr}_2} \xi^3 + \frac{1}{4} ((2 \ln(\xi) - 1) C_5 + 2C_6) \xi - \\ &\quad - \frac{1}{16} \left(-\frac{\chi F_2}{\text{Pr}_2} \gamma^2 + 4(2 \ln(\gamma) - 1) C_5 + 8C_6 \right) \gamma^2 \xi^{-1}, \end{aligned} \quad (34)$$

where constants C_2, C_5, C_6, F_j are from (30), (31). Taking into account (28)–(32), the remaining required functions $b_j(r), p_j(r, z), d_j(r)$ are determined from equations (12), (13).

We study the effect of changes in the internal energy of the interface on the velocity profiles $U_j(\xi)$ and functions $V_j(\xi)$. We choose water H_2O and liquid CO_2 as working medium. Parameters of liquids are given in the order " $\text{H}_2\text{O}, \text{CO}_2$ ": $\rho = \{0.998 \cdot 10^3, 1.977\}$ kg/m³, $\nu = \{1.004 \cdot 10^{-6}, 7.386 \cdot 10^{-6}\}$ m²/s, $\chi = \{1.442 \cdot 10^{-7}, 9.828 \cdot 10^{-6}\}$ m²/s, $k = \{0.5984, 1.643 \cdot 10^{-2}\}$ Wt/(m·K), $\alpha = 1.989 \cdot 10^{-4}$ N/(m·K), $\sigma_0 = 72.86 \cdot 10^{-3}$ N/m. Velocity profiles $\bar{U}_j(\xi)$ and functions $\bar{V}_j(\xi)$ are shown in Fig. 2 for various values of parameter $E = \{0.01, 0.02, 0.1, 0.7\}$, where $\bar{U}_j(\xi) = M^{-1}U_j(\xi)$ and $\bar{V}_j(\xi) = M^{-1}V_j(\xi)$, $M = E/M_0$, $M_0 = \alpha\chi_1/(k_2R_1)$.

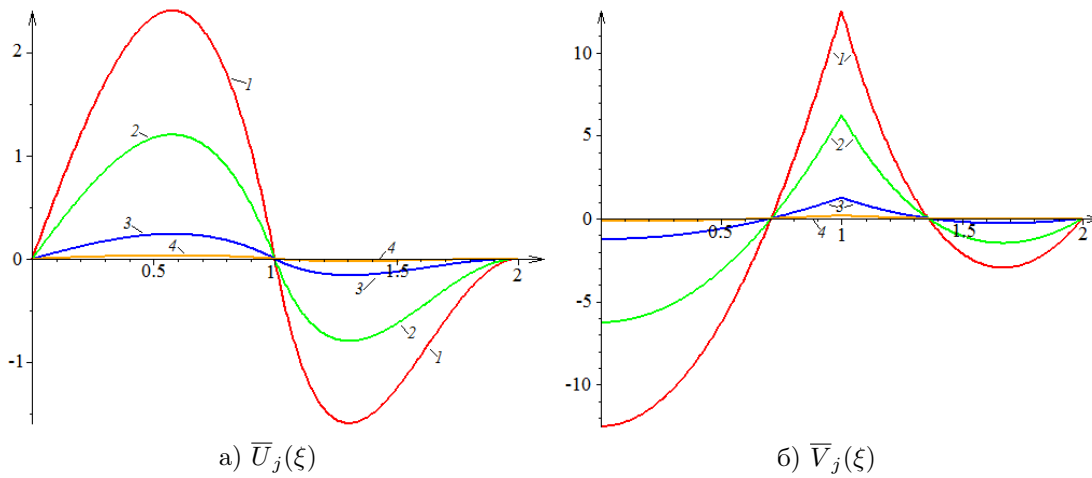


Fig. 2. The influence of parameter E on velocity profiles $\bar{U}_j(\xi)$ and functions $\bar{V}_j(\xi)$. 1 – E = 0.01, 2 – E = 0.02, 3 – E = 0.1, 4 – E = 0.7

Profiles of velocity $\bar{U}_1(\xi)$ and function $\bar{V}_1(\xi)$ are presented on the interval $0 \leq \xi \leq 1$. Profiles of velocity $\bar{U}_2(\xi)$ and function $\bar{V}_2(\xi)$ are presented on the interval $1 \leq \xi \leq \gamma$. It is also assumed that $R_1 = 10^{-10}$ m, and the ratio of the outer radius and the inner radius of the cylinder remains constant ($\gamma = 2$). The Fig. 2 shows that with the increase of the internal energy parameter of the interphase boundary values of functions $\bar{U}_j(\xi)$ and $\bar{V}_j(\xi)$ decrease. In Fig. 3 profiles of velocity $\bar{U}_j(\xi)$ and functions $\bar{V}_j(\xi)$ are presented for various values of the internal radius of the cylinder ($R_1 = \{10^{-10}, 2 \cdot 10^{-10}, 5 \cdot 10^{-10}, 10^{-9}\}$ m) for $\gamma = 2$ and E = 0.7. Here the Marangoni number is determined as $M = \alpha \Theta R_1 / (\mu_2 \chi_1)$, where $\Theta = \alpha R_1^2 = 623.15$ K that corresponds to the critical temperature for water. The Fig. 3 shows that functions $\bar{U}_j(\xi)$ and $\bar{V}_j(\xi)$ decrease with increasing R_1 .

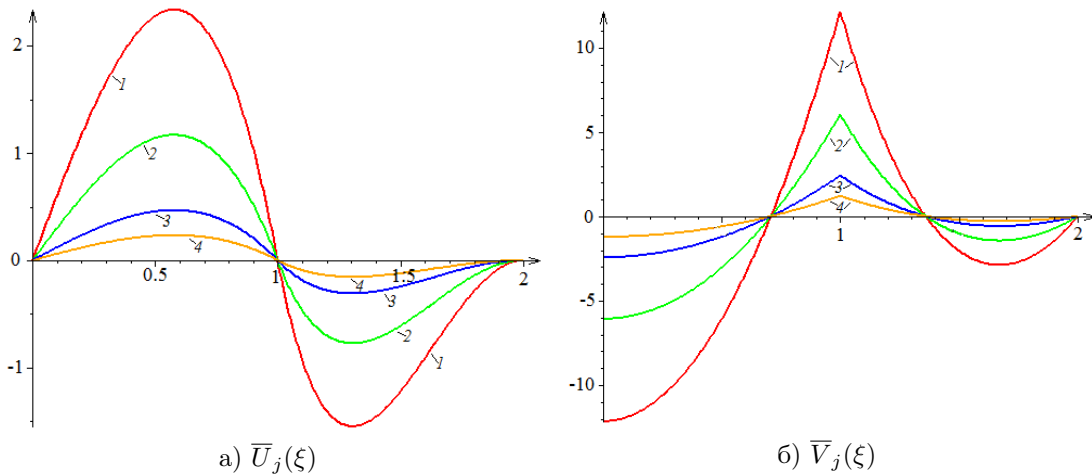


Fig. 3. The influence of parameter R_1 on profiles of velocity $\bar{U}_j(\xi)$ and functions $\bar{V}_j(\xi)$. 1 – $R_1 = 10^{-10}$ m, 2 – $R_1 = 2 \cdot 10^{-10}$ m, 3 – $R_1 = 5 \cdot 10^{-10}$ m, 4 – $R_1 = 10^{-9}$ m

This effect is due to the fact that the increase of the inner radius of the cylinder R_1 for fixed

$\gamma = 2$ the influence of the constant temperature defined on the outer surface of the cylinder weakens. Fig. 4 shows a relationship between profiles of velocity $\bar{U}_j(\xi)$, functions $\bar{V}_j(\xi)$ and (γ) for $E = 0.7$, $M = 0.04$, $R_1 = 10^{-10}$ m.

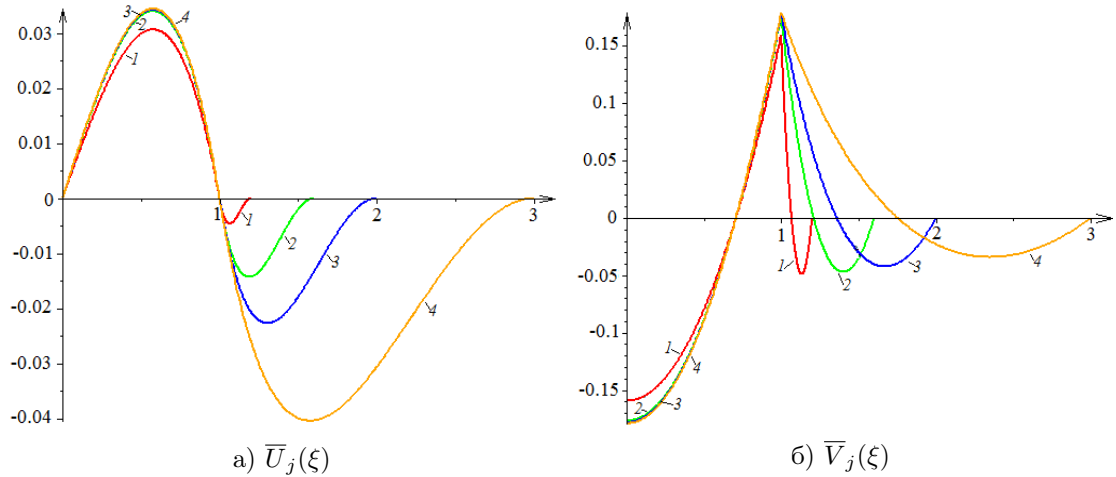


Fig. 4. The influence of parameter γ on profiles of velocity $\bar{U}_j(\xi)$ and functions $\bar{V}_j(\xi)$. 1 — $\gamma = 1.2$, 2 — $\gamma = 1.6$, 3 — $\gamma = 2$, 4 — $\gamma = 3$

It is clear that the increase of parameter $\gamma = \{1.2, 1.6, 2, 3\}$ strongly affects profiles of velocity $\bar{U}_2(\xi)$ and function $\bar{V}_2(\xi)$. Values of these functions are significantly increased. The increase of values of functions $\bar{U}_1(\xi)$ and $\bar{V}_1(\xi)$ is not so significant. This can be explained by the fact that for a fixed value of R_1 the radius of the outer cylinder increases because $\gamma = R_2/R_1$.

Thus, the influence of the internal energy of the interface on the two-layer flow in the cylinder was studied. It was found that with the increase of parameter E values of functions $\bar{U}_j(\xi)$ and $\bar{V}_j(\xi)$ decrease.

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Влияние изменений внутренней энергии поверхности раздела на двухслойное течение в цилиндре

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В данной работе исследуется точное решение для уравнений модели ползущего течения с полем скоростей типа Хименца, описывающее термокапиллярную конвекцию в слоях. Оно интерпретируется как движение вязких теплопроводных жидкостей в цилиндре с твердыми стенками и общей подвижной недеформируемой поверхностью раздела. При этом массовые силы отсутствуют. С математической точки зрения возникающая начально-краевая задача является обратной и нелинейной, так как учитывается полное энергетическое условие на границе раздела. Установлено, что может существовать два таких решения.

Ключевые слова: нелинейная обратная задача, число Марангони, энергетическое условие, ползущее термокапиллярное движение, решение Хименца.