УДК 517.95

On Differentiability of the Solution of the Mixed Boundary Value Problem for a Nonlinear Pseudohyperbolic Equation with Respect to Small Parameters

Tursun K. Yuldashev*
Institute of Informatics and Telecommunations,
Siberian State Aerospace University,
Krasnoyarsky rabochy, 31, Krasnoyarsk, 660014,
Russia

Received 18.09.2013, received in revised form 10.12.2013, accepted 20.02.2014

The theorems on the differentiability of the solution of the mixed boundary value problem for nonlinear partial pseudohyperbolic differential equations of the fifth order with respect to small parameters are proved in this paper

Keywords: nonlinear equation, solution differentiability, small parameters, countable system of nonlinear differential equations.

Introduction

Mixed boundary value problems arise in various structural and civil engineering problems. The study of many problems of stress concentration in the vicinity of cracks, inclusions, supporting stringers and linings also leads to mixed boundary value problems. A variety of mixed problems arise in fluid dynamics. These are nonlinear problems in the theory of wing sections and gliding, in the theories of jet streams and explosion and in the ship theory. Some mixed problems arise in the theory of filtration and hydroelasticity [1].

Great interest is shown in the study of the partial differential equations of higher orders from the point of view of practical applications. The study of many problems in gas dynamics, in the theory of elasticity, in theory of plates and shells leads to the consideration of partial differential equations of higher orders [2].

Partial differential equations of higher orders are also solved in the construction of invariant solutions of differential equations with the use of higher symmetries and conservation laws [3,4].

In the domain D we consider a nonlinear equation

$$\frac{\partial^2}{\partial t^2}\left(u(t,x)-\nu\frac{\partial^2 u(t,x)}{\partial x^2}\right)+\mu\frac{\partial^5 u(t,x)}{\partial t\,\partial x^4}+\frac{\partial^4 u(t,x)}{\partial x^4}=f\left(t,x,u(t,x)\right) \tag{1}$$

with initial conditions

$$u(t,x)_{|t=0}=\varphi_1(x),\ \frac{\partial}{\partial t}u(t,x)_{|t=0}=\varphi_2(x) \tag{2}$$

and boundary conditions

$$u(t,x)_{|x=0}=u_{x\,x}(t,x)_{|x=0}=\int_0^l u(t,y)\,dy=\int_0^l u_{y\,y}(t,y)\,dy=0, \tag{3}$$

*tursunbay@rambler.ru

where $f(t,x,u) \in C(D \times R)$, $\varphi_j(x) \in C^5(D_l)$, $\varphi_j(x)_{|x=0} = \varphi_j''(x)_{|x=0} = \int_0^l \varphi_j(y)\,dy = \int_0^l \varphi_j''(y)\,dy = 0$, $j=1,2$, $D \equiv D_T \times D_l$, $D_T \equiv [0,T]$, $D_l \equiv [0,l]$, $0<T<\infty$, $0<l<\infty$ and $0<\nu$, μ are small parameters.

It should be noted that to study the various types of linear and nonlinear partial differential equations and their systems different methods can be used [5–7]. One of the qualitatively new problems is the non-local problem for partial differential equations [8]. Nonlocal problems arise in the mathematical modeling of the various processes occurring in nature. One can exemplify such processes as moisture transfer, thermal conductivity in biological objects, control etc [9]. Mixed value problems with integral conditions were considered by many authors (for example, see [10–12]).

The description of the real process by differential equations is a transition from the object to its idealized model. Every mathematical idealization is associated with the neglect of small quantities [13]. For example, in the study of transverse vibrations of rods Rayleigh considered the equation

$$\frac{\partial^2 u}{\partial t^2} - \varepsilon\chi^2 \frac{\partial^4 u}{\partial t^2\,\partial x^2} + b^2\chi^2 \frac{\partial^4 u}{\partial x^4} = 0$$

for the cases $\varepsilon = 0$ and $\varepsilon = 1$. In particular, when $\varepsilon = 0$ he obtained a classical solution of the mixed value problem for this equation with the aid of eigenfunctions of the following spectral problem

$$y^{IV}(x) = \lambda y(x),\ y''(0) = y''(l) = y'''(0) = y'''(l) = 0.$$

It is easy to see that this spectral problem is self-adjoined, positive definite and it has a pure point spectrum.

In this paper we consider the questions of differentiability of solutions of mixed value problem (1)–(3) with respect to small parameter. In this case we use the method of separation of variables based on finding a solution of mixed value problem (1)–(3) as a Fourier series [14]

$$u(t,x) = \sum_{i=1}^{\infty} a_i(t)\, b_i(x),$$

where $b_i(x) = \sqrt{\dfrac{2}{l}} \sin \lambda_i x,\ \lambda_i = \dfrac{2i\pi}{l}$.

We note that the equation (1) for $\nu = 0$ and $\mu = 1$ takes the form

$$\frac{\partial^2 u(t,x)}{\partial t^2} + \frac{\partial^5 u(t,x)}{\partial t\,\partial x^4} + \frac{\partial^4 u(t,x)}{\partial x^4} = f(t,x,u(t,x)),$$

while for $\nu = 0$ and $\mu = 0$ it takes the form

$$\frac{\partial^2 u(t,x)}{\partial t^2} + \frac{\partial^4 u(t,x)}{\partial x^4} = f(t,x,u(t,x)). \tag{4}$$

We need to study the differentiability of solution of mixed value problem (1)–(3) with respect to small parameters for the purpose of stability analysis of the solution of this mixed problem with small parameters and in order to approximate the solution of problem (1)–(3) by the solution of problem (4), (2), (3). The questions of solvability of equation (1) have been studied previously [15, 16]. However, the study of the differentiability of the solution of this mixed value problem with respect to small parameters is being conducted for the first time.

Let us consider a set $\Big\{a(t) = (a_i(t))\,|a_i(t) \in C(D_T),\ i = 1,2,3,\ldots\Big\}$ and introduce the norm as follows

$$\|a(t)\|_{B_2(T)} = \left[\sum_{i=1}^{\infty} \max_{t\in D_T} |a_i(t)|^2\right]^{\frac{1}{2}}.$$

Then the set is a Banach space and we denote it as $B_2(T)$.

For each $a(t) \in B_2(T)$ we define the following operator

$$Qa(t) = u(t,x) = \sum_{i=1}^{\infty} a_i(t) b_i(x).$$

The set of values of this operator is denoted by $E_2(D)$. It is obvious that $Q\colon B_2(T) \to E_2(D)$ and $E_2(D) \subset L_2(D)$.

We suppose that the following conditions are fulfilled

$$\lambda_i^4 \mu^2 - 4\lambda_i^2 \nu - 4 < 0; \tag{5}$$

$$\int_0^T \| f(t,x,u) \|_{L_2(D_l)} \, dt \leqslant \Delta < \infty; \tag{6}$$

$$f(t,x,u) \in Lip\Big\{ L(t,x)_{|\,u} \Big\},\ 0 < \int_0^l \| L(t,y) \|_{L_2(D_l)} \, dy < \infty; \tag{7}$$

$$\| W(t,\nu,\mu) \|_{B_2(T)} < \infty. \tag{8}$$

In this case the one-valued solvability of mixed value problem (1)–(3) has been studied [16] and the solution of this problem in the domain D has the form

$$u(t,x,\nu,\mu) = \sum_{i=1}^{\infty} a_i(t,\nu,\mu) b_i(x),$$

where $a_i(t,\nu,\mu)$ is defined as a solution of the countable system of nonlinear integral equations (CSNIE)

$$a_i(t,\nu,\mu) = W_i(t,\nu,\mu) + \int_0^t \int_0^l f(s,y,Qa(s,\nu,\mu))\, G_i(t,s,\nu,\mu) b_i(y) dy ds, \tag{9}$$

$$W_i(t,\nu,\mu) = \exp\Big\{ -\frac{1}{2}\omega_{1i}(\nu,\mu) t \Big\} \times$$
$$\times \Big[\varphi_{1i} \cos \omega_{2i}(\nu,\mu) \frac{t}{2} + \frac{2}{\omega_{2i}(\nu,\mu)} \Big(\varphi_{2i} + \frac{\varphi_{1i}}{2} \omega_{1i}(\nu,\mu) \Big) \sin \omega_{2i}(\nu,\mu) \frac{t}{2} \Big],$$

$$G_i(t,s,\nu,\mu) = \frac{2\exp\Big\{ -\omega_{1i}(\nu,\mu)\frac{t-s}{2} \Big\} \sin \omega_{2i}(\nu,\mu)\frac{t-s}{2}}{\omega_{0i}(\nu)\Big[\omega_{2i}(\nu,\mu) + \omega_{1i}(\nu,\mu) \sin \omega_{2i}(\nu,\mu) s \Big]}, \quad \omega_{0i}(\nu) = 1 + \lambda_i^2 \nu,$$

$$\omega_{1i}(\nu,\mu) = \frac{\lambda_i^4 \mu}{\omega_{0i}(\nu)}, \quad \omega_{2i}(\nu,\mu) = \frac{\lambda_i^2 \sqrt{4\omega_{0i}(\nu) - \lambda_i^4 \mu^2}}{\omega_{0i}(\nu)}, \ \varphi_{ji} = \int_0^l \varphi_j(y) b_i(y) dy, \quad j = 1,2.$$

1. Main results

First we study the differentiability of the solution of mixed value problem (1)–(3) with respect to first small parameter ν.

Theorem 1. *Let the conditions (5)–(8) be fulfilled and*

1) $\displaystyle\sum_{i=1}^{\infty} \frac{\lambda_i^4 \mu}{\sqrt{\rho}} \Big[\lambda_i^4 + \mu^2 \Big(\frac{\omega_{0i}(\nu)}{\rho} + \frac{\lambda_i^2}{\sqrt{\rho}} \Big) \Big] \cdot |\varphi_{1i}| < \infty$, $\rho = 4 + 4\lambda_i^2 \nu - \lambda_i^4 \mu^2$;

2) $\sum_{i=1}^{\infty} \frac{\lambda_i^4 \mu}{\omega_{2i}(\nu,\, \mu)} \Big(\lambda_i^2 + \frac{\mu}{\sqrt{\rho}}\Big) \cdot |\varphi_{2i}| < \infty;$

3) $\sum_{i=1}^{\infty} \alpha_i \int_0^T |f_i\,(u,\nu)|\, dt < \infty,$

where $f_i(u,\nu) = \int_0^l f\,(t, y, Qa(t,\nu,\mu))\, b_i(y)\, dy$, $\tau_0 = \inf_{[0;\nu]} |\omega_{2i}(\nu,\, \mu) + \omega_{1i}(\nu,\, \mu) \sin \omega_{2i}(\nu,\, \mu) t|$,

$$\alpha_i = \frac{1}{\omega_{0i}(\nu)\big(\lambda_i^2\mu + \sqrt{\rho}\big)} \cdot \left[\frac{\lambda_i^4 \mu T}{2\omega_{0i}(\nu)} + \frac{1}{\big(\omega_{0i}(\nu)\big)^2} + \frac{\lambda_i^8 \mu^3 T}{\tau_0\, \sqrt{\rho}}\right] + \frac{\lambda_i^4 \mu}{\tau_0} \left[\frac{\mu T}{\omega_{0i}(\nu)\sqrt{\rho}} + \frac{\mu}{\sqrt{\rho}} + \frac{\lambda_i^2}{\big(\omega_{0i}(\nu)\big)^2}\right];$$

4) $\gamma = \max_{t \in D_T} \left\| \frac{\partial\, f(t,\, x,\, u)}{\partial\, u} \right\|_{L_2(D_l)} < \infty$.

Then there exists a derivative of solution of mixed value problem (1)–(3) with respect to small parameter ν in the class $E_2(D)$.

Proof. Since

$$\frac{\partial\, u(t,\, x,\, \nu,\, \mu)}{\partial\, \nu} = \sum_{i=1}^{\infty} \frac{\partial\, a_i(t,\, \nu,\, \mu)}{\partial\, \nu}\, b_i(x),$$

then we differentiate CSNIE (9) with respect to small parameter ν and obtain

$$\frac{\partial\, a_i(t,\, \nu,\, \mu)}{\partial\, \nu} = \overline{W}_i(t,\nu,\mu) + \\ + \int_0^t \int_0^l \frac{\partial\, f(s,\, y,\, u)}{\partial\, u}\, G_i(t,s,\nu,\mu) \left(\sum_{j=1}^{\infty} \frac{\partial\, a_j(s,\, \nu,\, \mu)}{\partial\, \nu}\, b_j(y)\right) b_i(y) dy ds, \tag{10}$$

where

$$\overline{W}_i(t,\nu,\mu) = -\big(\omega_{1i}(\nu,\mu)\big)'_\nu\, \frac{t}{2} \exp\Big\{-\omega_{1i}(\nu,\mu)\, \frac{t}{2}\Big\} \times \\ \times \left[\varphi_{1i} \cos \omega_{2i}(\nu,\mu)\, \frac{t}{2} + \frac{2}{\omega_{2i}(\nu,\mu)} \Big(\varphi_{2i} + \frac{\varphi_{1i}}{2} \omega_{1i}(\nu,\mu)\Big)\, \sin \omega_{2i}(\nu,\mu)\, \frac{t}{2}\right] + \\ + \exp\Big\{-\omega_{1i}(\nu,\mu)\, \frac{t}{2}\Big\} \left\{-\frac{\varphi_{1i} t}{2}\, \Big(\omega_{2i}(\nu,\mu)\Big)'_\nu\, \sin \omega_{2i}(\nu,\mu)\, \frac{t}{2} - \right. \\ -2 \frac{(\omega_{2i}(\nu,\mu))'_\nu}{(\omega_{2i}(\nu,\mu))^2}\, \Big(\varphi_{2i} + \frac{\varphi_{1i}}{2} \omega_{1i}(\nu,\mu)\Big)\, \sin \omega_{2i}(\nu,\mu)\, \frac{t}{2} + \\ \left. + \big(\omega_{1i}(\nu,\mu)\big)'_\nu \left[\varphi_{1i}\, \frac{\sin \omega_{2i}(\nu,\mu)\, \frac{t}{2}}{\omega_{2i}(\nu,\mu)} + \frac{\cos \omega_{2i}(\nu,\mu)\, \frac{t}{2}}{\omega_{2i}(\nu,\mu)}\, \Big(\varphi_{2i} + \frac{\varphi_{1i}}{2} \omega_{1i}(\nu,\mu)\Big)\, t\right]\right\} + \\ + \int_0^t \int_0^l \big(G_i(t,s,\nu,\mu)\big)'_\nu\, f\Big(s, y, \sum_{j=1}^{\infty} a_j(s,\, \nu,\, \mu)\, b_j(y)\Big)\, b_i(y)\, dy ds, \tag{11}$$

$$\Big(G_i(t,s,\nu,\mu)\Big)'_\nu = -G_i(t,s,\nu,\mu) \left[\frac{t-s}{2}\, \big(\omega_{1i}(\nu,\mu)\big)'_\nu + \right. \\ \left. + \frac{(\omega_{0i}(\nu))'_\nu}{\omega_{0i}(\nu)} + \frac{\big(\omega_{2i}(\nu,\mu) + \omega_{1i}(\nu,\mu) \sin \omega_{2i}(\nu,\mu) s\big)'_\nu}{\omega_{2i}(\nu,\mu) + \omega_{1i}(\nu,\mu) \sin \omega_{2i}(\nu,\mu) s}\right] + \\ + \frac{t-s}{2}\, \big(\omega_{2i}(\nu,\mu)\big)'_\nu\, \frac{2 \exp\Big\{-\omega_{1i}(\nu,\mu)\, \frac{t-s}{2}\Big\} \cos \omega_{2i}(\nu,\mu)\, \frac{t-s}{2}}{\omega_{0i}(\nu)\, \big(\omega_{2i}(\nu,\mu) + \omega_{1i}(\nu,\mu) \sin \omega_{2i}(\nu,\mu) s\big)}. \tag{12}$$

Further, taking into account the conditions of the theorem, from (11) and (12) we obtain

$$\begin{gathered}\left\|\overline{W}(t,\nu,\,\mu)\right\|_{B_2(T)}\leqslant\sum_{i=1}^{\infty}\left\|\overline{W}_i(t,\nu,\,\mu)\right\|_{C(D_T)}\leqslant\\ \leqslant\sum_{i=1}^{\infty}\frac{\lambda_i^6\mu T}{2\left(\omega_{0i}(\nu)\right)^2}\left[|\varphi_{1i}|\left(1+\frac{\omega_{1i}(\nu,\mu)}{\omega_{2i}(\nu,\mu)}+\frac{2|\varphi_{2i}|}{\omega_{2i}(\nu,\mu)}\right)\right]+\\ +\sum_{i=1}^{\infty}\left\{|\varphi_{1i}|\left[\frac{\lambda_i^4\mu^2}{\sqrt{\rho}}\left(\frac{T}{2}+\frac{\omega_{1i}(\nu,\mu)}{(\omega_{2i}(\nu,\mu))^2}+\frac{T}{2}\frac{\omega_{1i}(\nu,\mu)}{\omega_{2i}(\nu,\mu)}\right)+\right.\right.\\ \left.\left.+\frac{\lambda_i^6\mu}{(\omega_{0i}(\nu))^2}\cdot\frac{1}{\omega_{2i}(\nu,\mu)}\right]+|\varphi_{2i}|\frac{\lambda_i^4\mu^2}{\sqrt{\rho}}\cdot\frac{2+T\,\omega_{2i}(\nu,\mu)}{(\omega_{2i}(\nu,\mu))^2}\right\}+\\ +\sum_{i=1}^{\infty}\max_{t\in D_T}\int_0^t\int_0^l\left|\left(G_i(t,s,\nu,\mu)\right)'_\nu\right|\cdot\left|f\left(s,y,\sum_{j=1}^{\infty}a_j(s,\,\nu,\,\mu)\,b_j(y)\right)b_i(y)\right|dyds\leqslant\\ \leqslant A_0(\nu,\mu)+M_1\sum_{i=1}^{\infty}\sqrt{\frac{2}{l}}\,\alpha_i\int_0^T\int_0^l\max_{t\in D_T}\left|f\left(t,y,\sum_{j=1}^{\infty}a_j(t,\,\nu,\,\mu)\,b_j(y)\right)\right|\cdot|b_i(y)|\,dydt<\infty,\end{gathered}\tag{13}$$

where $M_1=\|G_i(t,s,\nu,\mu)\|_{B_2(T)}$,

$$A_0(\nu,\mu)=\sum_{i=1}^{\infty}\frac{\lambda_i^6\mu T}{2(\omega_{0i}(\nu))^2}\left[|\varphi_{1i}|\left(1+\frac{\lambda_i^2\mu}{\sqrt{\rho}}\right)+\frac{2|\varphi_{2i}|}{\omega_{2i}(\nu,\mu)}\right]+$$

$$+\sum_{i=1}^{\infty}\left\{|\varphi_{1i}|\left[\frac{\lambda_i^4\mu^2}{\sqrt{\rho}}\left(\frac{T}{2}+\frac{\omega_{0i}(\nu)\,\mu}{\rho}+\frac{T}{2}\frac{\lambda_i^2\mu}{\sqrt{\rho}}\right)+\frac{\lambda_i^4\mu}{\omega_{0i}(\nu)\sqrt{\rho}}\right]+|\varphi_{2i}|\frac{\lambda_i^4\mu^2}{\sqrt{\rho}}\cdot\frac{2+T\,\omega_{2i}(\nu,\mu)}{(\omega_{2i}(\nu,\mu))^2}\right\}\leqslant$$

$$\leqslant\sum_{i=1}^{\infty}\left\{\frac{\lambda_i^4\mu}{\sqrt{\rho}}\left[\lambda_i^4+\mu^2\left(\frac{\omega_{0i}(\nu)}{\rho}+\frac{\lambda_i^2}{\sqrt{\rho}}\right)\right]\cdot|\varphi_{1i}|+\frac{\lambda_i^4\mu}{\omega_{2i}(\nu,\,\mu)}\left(\lambda_i^2+\frac{\mu}{\sqrt{\rho}}\right)\cdot|\varphi_{2i}|\right\}.$$

Then from (10) we have

$$\begin{gathered}\left\|\frac{\partial\,a(t,\,\nu,\,\mu)}{\partial\,\nu}\right\|_{B_2(t)}\leqslant\left\|\overline{W}(t,\nu,\mu)\right\|_{B_2(T)}+\\ +M_1M_2\int_0^t\left(\int_0^l\left|\frac{\partial\,f(s,\,y,\,u)}{\partial\,u}\right|^2\cdot\left|\sum_{j=1}^{\infty}\frac{\partial\,a_j(s,\,\nu,\,\mu)}{\partial\,\nu}\,b_j(y)\right|^2dy\right)^{\frac{1}{2}}ds\leqslant\\ \leqslant\left\|\overline{W}(t,\nu,\mu)\right\|_{B_2(T)}+M_1M_2^2\int_0^t\left\|\frac{\partial\,f(s,\,y,\,u)}{\partial\,u}\right\|_{L_2(D_l)}\left\|\frac{\partial\,a(s,\,\nu,\,\mu)}{\partial\,\nu}\right\|_{B_2(s)}ds,\end{gathered}\tag{14}$$

where $M_2=\|b(x)\|_{B_2(l)}$, $\left\|\frac{\partial\,a(t,\,\nu,\,\mu)}{\partial\,\nu}\right\|_{B_2(t)}=\left[\sum_{i=1}^{\infty}\left|\frac{\partial\,a(t,\,\nu,\,\mu)}{\partial\,\nu}\right|^2\right]^{\frac{1}{2}}$.

Appling the Gronwall-Bellman inequality to (14) and taking into account (13), we derive

$$\frac{\partial\,a(t,\,\nu,\,\mu)}{\partial\,\nu}\in B_2(T).$$

Let us consider the following iteration process

$$\frac{\partial a_i^0(t,\,\nu,\,\mu)}{\partial \nu}=\overline{W}_i(t,\nu,\mu),\ \frac{\partial a_i^{k+1}(t,\,\nu,\,\mu)}{\partial \nu}=\overline{W}_i(t,\nu,\mu)+$$

$$+\int_0^t\int_0^l G_i(t,s,\nu,\mu)\,\frac{\partial f(s,\,y,\,u)}{\partial u}\left(\sum_{j=1}^{\infty}\frac{\partial a_j^k(s,\,\nu,\,\mu)}{\partial \nu}\,b_j(y)\right)b_i(y)dyds,\ k=0,1,2,\ldots.$$

It is easy to verify that there are the following estimates

$$\left\|\frac{\partial a^1(t,\,\nu,\,\mu)}{\partial \nu}-\frac{\partial a^0(t,\,\nu,\,\mu)}{\partial \nu}\right\|_{B_2(t)}\leqslant$$

$$\leqslant M_1M_2^2\int_0^t\left\|\frac{\partial f(s,\,y,\,u)}{\partial u}\right\|_{L_2(D_l)}\left\|\frac{\partial a^0(s,\,\nu,\,\mu)}{\partial \nu}\right\|_{B_2(s)}ds\leqslant$$

$$\leqslant M_1M_2^2\,\|\overline{W}(t,\nu,\mu)\|_{B_2(T)}\int_0^t\left\|\frac{\partial f(s,\,y,\,u)}{\partial u}\right\|_{L_2(D_l)}ds\leqslant M_1M_2^2\,\|\overline{W}(t,\nu,\mu)\|_{B_2(T)}\,\gamma t\,,$$

$$\left\|\frac{\partial a^{k+1}(t,\,\nu,\,\mu)}{\partial \nu}-\frac{\partial a^k(t,\,\nu,\,\mu)}{\partial \nu}\right\|_{B_2(t)}\leqslant$$

$$\leqslant M_1M_2^2\int_0^t\left\|\frac{\partial f(s,\,y,\,u)}{\partial u}\right\|_{L_2(D_l)}\left\|\frac{\partial a^k(s,\,\nu,\,\mu)}{\partial \nu}-\frac{\partial a^{k-1}(s,\,\nu,\,\mu)}{\partial \nu}\right\|_{B_2(s)}ds\leqslant$$

$$\leqslant M_1^{k+1}M_2^{2k+2}\,\|\overline{W}(t,\nu,\mu)\|_{B_2(T)}\,\frac{(\gamma t)^{k+1}}{(k+1)!}.$$

The existence of solution of countable system (10) in the space $B_2(T)$ follows from the last two estimates. The uniqueness of this solution is proven by the following estimate

$$\left\|\frac{\partial a(t,\,\nu,\,\mu)}{\partial \nu}-\frac{\partial v(t,\,\nu,\,\mu)}{\partial \nu}\right\|_{B_2(t)}\leqslant$$

$$\leqslant M_1M_2^2\int_0^t\left\|\frac{\partial f(s,\,y,\,u)}{\partial u}\right\|_{L_2(D_l)}\left\|\frac{\partial a(s,\,\nu,\,\mu)}{\partial \nu}-\frac{\partial v(s,\,\nu,\,\mu)}{\partial \nu}\right\|_{B_2(s)}ds, \tag{15}$$

if we apply the Gronwall-Bellman inequality to (15).

It follows that

$$\left|\frac{\partial u(t,x,\,\nu,\,\mu)}{\partial \nu}\right|\leqslant\sum_{i=1}^{\infty}\left|\frac{\partial a_i(t,\,\nu,\,\mu)}{\partial \nu}\right|\cdot|b_i(x)|\leqslant$$

$$\leqslant\left(\sum_{i=1}^{\infty}\left|\frac{\partial a_i(t,\,\nu,\,\mu)}{\partial \nu}\right|^2\right)^{\frac12}\cdot\left(\sum_{i=1}^{\infty}|b_i(x)|^2\right)^{\frac12}=M_2\left\|\frac{\partial a(t,\,\nu,\,\mu)}{\partial \nu}\right\|_{B_2(T)}<\infty.$$

Let us consider the following relation

$$\frac{a_i(t,\,\nu+h,\,\mu)-a_i(t,\,\nu,\,\mu)}{h}=\frac{\exp\Big\{-\omega_{1i}(\nu+h,\,\mu)\,t\Big\}-\exp\Big\{-\omega_{1i}(\nu,\,\mu)\,t\Big\}}{h}\times$$

$$\times\left[\varphi_{1i}\cos\omega_{2i}(\nu+h,\,\mu)\,\frac{t}{2}+\frac{2}{\omega_{2i}(\nu+h,\,\mu)}\left(\varphi_{2i}+\frac{\varphi_{1i}}{2}\,\omega_{1i}(\nu+h,\,\mu)\right)\sin\omega_{2i}(\nu+h,\,\mu)\frac{t}{2}\right]+$$

$$+\exp\Big\{-\omega_{1i}(\nu,\,\mu)\,\frac{t}{2}\Big\}\cdot\left\{\frac{\varphi_{1i}}{h}\left(\cos\omega_{2i}(\nu+h,\,\mu)\,\frac{t}{2}-\cos\omega_{2i}(\nu,\,\mu)\,\frac{t}{2}\right)+\right.$$

$$+\frac{1}{h}\Big(\frac{2}{\omega_{2i}(\nu+h,\,\mu)}-\frac{2}{\omega_{2i}(\nu,\,\mu)}\Big)\cdot\Big(\varphi_{2i}+\frac{\varphi_{1i}}{2}\,\omega_{1i}(\nu+h,\,\mu)\Big)\sin\omega_{2i}(\nu+h,\,\mu)\frac{t}{2}+$$
$$+\frac{2}{\omega_{2i}(\nu,\,\mu)}\Big[\frac{\varphi_{1i}}{2}\cdot\frac{\omega_{1i}(\nu+h,\,\mu)-\omega_{1i}(\nu,\,\mu)}{h}\,\sin\omega_{2i}(\nu+h,\,\mu)\,\frac{t}{2}+$$
$$+\frac{\sin\omega_{2i}(\nu+h,\,\mu)\,\frac{t}{2}-\sin\omega_{2i}(\nu,\,\mu)\,\frac{t}{2}}{h}\,\Big(\varphi_{2i}+\frac{\varphi_{1i}}{2}\,\omega_{1i}(\nu,\,\mu)\Big)\Big]\Big\}+$$
$$+\int_0^t\int_0^l\frac{G_i(t,s,\nu+h,\mu)-G_i(t,s,\nu,\mu)}{h}\,f\Big(s,y,\sum_{j=1}^{\infty}a_j(s,\,\nu+h,\,\mu)\,b_j(y)\Big)\,b_i(y)dyds+$$
$$+\int_0^t\int_0^l G_i(t,s,\nu+h,\mu)\,\frac{1}{h}\,\Big[f\Big(s,y,\sum_{j=1}^{\infty}a_j(s,\,\nu+h,\,\mu)\,b_j(y)\Big)-$$
$$-\,f\Big(s,y,\sum_{j=1}^{\infty}a_j(s,\,\nu,\,\mu)\,b_j(y)\Big)\Big]\,b_i(y)dyds,\qquad(16)$$

where

$$\frac{G_i(t,s,\nu+h,\mu)-G_i(t,s,\nu,\mu)}{h}=2\frac{\exp\Big\{-\omega_{1i}(\nu+h,\,\mu)\,\frac{t-s}{2}\Big\}-\exp\Big\{-\omega_{1i}(\nu,\,\mu)\,\frac{t-s}{2}\Big\}}{h}\times$$
$$\times\frac{\sin\omega_{2i}(\nu+h,\,\mu)\,\frac{t-s}{2}}{\omega_{0i}(\nu+h)\,\Big[\omega_{2i}(\nu+h,\,\mu)+\omega_{1i}(\nu+h,\,\mu)\,\sin\omega_{2i}(\nu+h,\,\mu)\,s\Big]}+$$
$$+2\exp\Big\{-\omega_{1i}(\nu,\,\mu)\,\frac{t-s}{2}\Big\}\Big\{\frac{\sin\omega_{2i}(\nu+h,\,\mu)\,\frac{t-s}{2}-\sin\omega_{2i}(\nu,\,\mu)\,\frac{t-s}{2}}{h}\times$$
$$\times\frac{1}{\omega_{0i}(\nu+h)\,\Big[\omega_{2i}(\nu+h,\,\mu)+\omega_{1i}(\nu+h,\,\mu)\,\sin\omega_{2i}(\nu+h,\,\mu)\,s\Big]}+$$
$$+\sin\omega_{2i}(\nu+h,\,\mu)\,\frac{t-s}{2}\,\Big[\frac{1}{h}\Big(\frac{1}{\omega_{0i}(\nu+h)}-\frac{1}{\omega_{0i}(\nu)}\Big)\times$$
$$\times\frac{1}{\omega_{2i}(\nu+h,\,\mu)+\omega_{1i}(\nu+h,\,\mu)\,\sin\omega_{2i}(\nu+h,\,\mu)\,s}+\frac{1}{\omega_{0i}(\nu)}\cdot\frac{1}{h}\times$$
$$\times\Big(\frac{1}{\omega_{2i}(\nu+h,\mu)+\omega_{1i}(\nu+h,\mu)\,\sin\omega_{2i}(\nu+h,\mu)\,s}-\frac{1}{\omega_{2i}(\nu,\mu)+\omega_{1i}(\nu,\mu)\,\sin\omega_{2i}(\nu,\mu)\,s}\Big)\Big]\Big\}.$$

Taking the limit $h\to 0$ in (16), we obtain from(10)

$$\left|\frac{u(t,x,\nu+h,\mu)-u(t,x,\nu,\mu)}{h}-\frac{\partial\,u(t,x,\nu,\mu)}{\partial\,\nu}\right|\leqslant$$
$$\leqslant M_2\,\left\|\frac{a(t,\,\nu+h,\,\mu)-a(t,\,\nu,\,\mu)}{h}-\frac{\partial\,a(t,\,\nu,\,\mu)}{\partial\,\nu}\right\|_{B_2(T)}.$$

The theorem is proved. □

The differentiability of the solution of mixed value problem (1)–(3) with respect to second small parameter μ is obtained in an analogous way.

Theorem 2. *Let conditions (5)–(8) be fulfilled and*

1) $B_0(\nu,\mu)<\infty$, where $B_0(\nu,\mu)=\sum\limits_{i=1}^{\infty}\Bigg\{|\varphi_{1i}|\cdot\left[\frac{\lambda_i^4 T}{2}\left(1+\frac{\lambda_i^2\omega_{0i}(\nu)}{\sqrt{\rho}}\right)+\frac{\lambda_i^6\mu}{\sqrt{\rho}}\cdot\left(T+\frac{2\mu\omega_{0i}^2(\nu)}{\sqrt{\rho}}+\frac{T^2\omega_{0i}(\nu)}{2}\right)+\frac{\lambda_i^2\omega_{0i}(\nu)}{\sqrt{\rho}}\right]+|\varphi_{2i}|\cdot\left[\frac{\lambda_i^2\omega_{0i}(\nu)}{\sqrt{\rho}}\left(2T+\frac{4\mu\,\omega_{0i}(\nu)}{\sqrt{\rho}}\right)\right]\Bigg\}$, $\rho=4+4\lambda_i^2\nu-\lambda_i^4\mu^2$;

2) $\sum\limits_{i=1}^{\infty}\beta_i\int_0^T|f_i(u,\nu)|\,dt<\infty$, *where* $f_i(u,\nu)=\int_0^l f\left(t,y,Qa(t,\nu,\mu)\right)b_i(y)\,dy$,

$\overline{\tau}_0=\inf_{[0;\,\mu]}|\omega_{2i}(\nu,\,\mu)+\omega_{1i}(\nu,\,\mu)\sin\omega_{2i}(\nu,\,\mu)t|$, $\beta_i=\lambda_i^4\left[\frac{T}{2}+\frac{1}{\overline{\tau}_0}+\frac{2\lambda_i^2\mu}{\sqrt{\rho}}\cdot\left(1+T+\lambda_i^4\right)\right]$;

3) $\gamma=\max_{t\in D_T}\left\|\frac{\partial f(t,\,x,\,u)}{\partial u}\right\|_{L_2(D_l)}<\infty$.

Then there exists a derivative of solution of mixed value problem (1)–(3) with respect to small parameter μ in the class $E_2(D)$.

Proof. Because

$$\frac{\partial u(t,\,x,\,\nu,\,\mu)}{\partial\mu}=\sum_{i=1}^{\infty}\frac{\partial a_i(t,\,\nu,\,\mu)}{\partial\mu}\,b_i(x),$$

we differentiate CSNIE (9) with respect to parameter μ and obtain

$$\frac{\partial a_i(t,\,\nu,\,\mu)}{\partial\mu}=W_{1i}(t,\nu,\mu)+ +\int_0^t\int_0^l\frac{\partial f(s,\,y,\,u)}{\partial u}\,G_i(t,s,\nu,\mu)\,b_i(y)\left(\sum_{j=1}^{\infty}\frac{\partial a_j(s,\,\nu,\,\mu)}{\partial\mu}\,b_j(y)\right)dyds, \tag{17}$$

where

$$\begin{aligned}W_{1i}(t,\nu,\mu)=&-\big(\omega_{1i}(\nu,\mu)\big)'_\mu\,\frac{t}{2}\exp\Big\{-\omega_{1i}(\nu,\mu)\,\frac{t}{2}\Big\}\times\\ &\times\left[\varphi_{1i}\cos\omega_{2i}(\nu,\mu)\,\frac{t}{2}+\frac{2}{\omega_{2i}(\nu,\mu)}\Big(\varphi_{2i}+\frac{\varphi_{1i}}{2}\omega_{1i}(\nu,\mu)\Big)\,\sin\omega_{2i}(\nu,\mu)\,\frac{t}{2}\right]+\\ &+\exp\Big\{-\omega_{1i}(\nu,\mu)\,\frac{t}{2}\Big\}\Big\{-\frac{\varphi_{1i}t}{2}\,\big(\omega_{2i}(\nu,\mu)\big)'_\mu\,\sin\omega_{2i}(\nu,\mu)\,\frac{t}{2}-\\ &-2\frac{\big(\omega_{2i}(\nu,\mu)\big)'_\mu}{\big(\omega_{2i}(\nu,\mu)\big)^2}\,\Big(\varphi_{2i}+\frac{\varphi_{1i}}{2}\omega_{1i}(\nu,\mu)\Big)\,\sin\omega_{2i}(\nu,\mu)\,\frac{t}{2}+\\ &+\big(\omega_{1i}(\nu,\mu)\big)'_\mu\left[\varphi_{1i}\,\frac{\sin\omega_{2i}(\nu,\mu)\,\frac{t}{2}}{\omega_{2i}(\nu,\mu)}+\frac{\cos\omega_{2i}(\nu,\mu)\,\frac{t}{2}}{\omega_{2i}(\nu,\mu)}\,\Big(\varphi_{2i}+\frac{\varphi_{1i}}{2}\omega_{1i}(\nu,\mu)\Big)\,t\right]\Big\}+\\ &+\int_0^t\int_0^l\big(G_i(t,s,\nu,\mu)\big)'_\mu\,f\left(s,y,\sum_{j=1}^{\infty}a_j(s,\,\nu,\,\mu)\,b_j(y)\right)\,b_i(y)\,dyds,\end{aligned} \tag{18}$$

$$\begin{aligned}\big(G_i(t,s,\nu,\mu)\big)'_\mu=&-G_i(t,s,\nu,\mu)\left[\frac{t-s}{2}\,\big(\omega_{1i}(\nu,\mu)\big)'_\mu+\right.\\ &\left.+\frac{\big(\omega_{2i}(\nu,\mu)+\omega_{1i}(\nu,\mu)\sin\omega_{2i}(\nu,\mu)s\big)'_\mu}{\omega_{2i}(\nu,\mu)+\omega_{1i}(\nu,\mu)\sin\omega_{2i}(\nu,\mu)s}\right]+\\ &+\frac{t-s}{2}\,\big(\omega_{2i}(\nu,\mu)\big)'_\mu\,\frac{2\exp\Big\{-\omega_{1i}(\nu,\mu)\frac{t-s}{2}\Big\}\cos\omega_{2i}(\nu,\mu)\frac{t-s}{2}}{\omega_{0i}(\nu)\,\big(\omega_{2i}(\nu,\mu)+\omega_{1i}(\nu,\mu)\sin\omega_{2i}(\nu,\mu)s\big)}.\end{aligned} \tag{19}$$

By virtue of the conditions of the theorem from (18) and (19) we obtain

$$\|W_1(t,\nu,\,\mu)\|_{B_2(T)} \leqslant \sum_{i=1}^{\infty} \|W_{1i}(t,\nu,\,\mu)\|_{C(D_T)} \leqslant$$
$$\leqslant \sum_{i=1}^{\infty}\left\{|\varphi_{1i}|\cdot\left[\frac{\lambda_i^4 T}{2}\left(1+\frac{\lambda_i^2\omega_{0i}(\nu)}{\sqrt{\rho}}\right)+\frac{\lambda_i^6\mu}{\sqrt{\rho}}\left(T+\frac{2\mu\,\omega_{0i}^2(\nu)}{\sqrt{\rho}}+\frac{T^2\omega_{0i}(\nu)}{2}\right)+\frac{\lambda_i^2\omega_{0i}}{\sqrt{\rho}}\right]+\right.$$
$$\left.+|\varphi_{2i}|\left[\frac{\lambda_i^2\omega_{0i}(\nu)}{\sqrt{\rho}}\left(2T+\frac{4\mu\omega_{0i}(\nu)}{\rho}\right)\right]\right\}+\sum_{i=1}^{\infty}\sqrt{\frac{2}{l}}\lambda_i^4\left[\frac{T}{2}+\frac{1}{\overline{\tau}_0}+\frac{2\lambda_i^2\mu}{\sqrt{\rho}}\,\left(1+T+\lambda_i^4\right)\right]\times$$
$$\times M_1\int_0^T\max_{t\in D_T}\int_0^l\left|f\left(t,y,\sum_{j=1}^{\infty}a_j(t,\,\nu,\,\mu)\,b_j(y)\right)\right|\cdot|b_i(y)|\,dydt\leqslant$$
$$\leqslant B_0(\nu,\mu)+M_1\sum_{i=1}^{\infty}\beta_i\int_0^T\int_0^l\max_{t\in D_T}\left|f\left(t,y,\sum_{j=1}^{\infty}a_j(t,\,\nu,\,\mu)\,b_j(y)\right)\right|\cdot|b_i(y)|\,dydt<\infty. \tag{20}$$

Then from (17) it follows that

$$\left\|\frac{\partial\,a(t,\,\nu,\,\mu)}{\partial\,\mu}\right\|_{B_2(t)}\leqslant\|W_1(t,\nu,\mu)\|_{B_2(T)}+$$
$$+M_1M_2\int_0^t\left(\int_0^l\left|\frac{\partial\,f(s,\,y,\,u)}{\partial\,u}\right|^2\left|\sum_{j=1}^{\infty}\frac{\partial\,a_j(s,\,\nu,\,\mu)}{\partial\,\mu}\,b_j(y)\right|^2dy\right)^{\frac{1}{2}}ds\leqslant \tag{21}$$
$$\leqslant\|W_1(t,\nu,\mu)\|_{B_2(T)}+M_1M_2^2\int_0^t\left\|\frac{\partial\,f(s,\,y,\,u)}{\partial\,u}\right\|_{L_2(D_l)}\left\|\frac{\partial\,a(s,\,\nu,\,\mu)}{\partial\,\mu}\right\|_{B_2(s)}ds.$$

After applying Gronwall-Bellman inequality to (21) and taking into account (20) we derive

$$\frac{\partial\,a(t,\,\nu,\,\mu)}{\partial\,\mu}\in B_2(T).$$

Let us consider the following iteration process

$$\frac{\partial\,a_i^0(t,\,\nu,\,\mu)}{\partial\,\mu}=W_{1i}(t,\nu,\mu),\ \frac{\partial\,a_i^{k+1}(t,\,\nu,\,\mu)}{\partial\,\mu}=W_{1i}(t,\nu,\mu)+$$
$$+\int_0^t\int_0^l G_i(t,s,\nu,\mu)\,\frac{\partial\,f(s,\,y,\,u)}{\partial\,u}\,\left(\sum_{j=1}^{\infty}\frac{\partial\,a_j^k(s,\,\nu,\,\mu)}{\partial\,\mu}\,b_j(y)\right)b_i(y)dyds,\ k=0,1,2,\ldots.$$

It is easy to verify that the following estimates hold

$$\left\|\frac{\partial\,a^1(t,\,\nu,\,\mu)}{\partial\,\mu}-\frac{\partial\,a^0(t,\,\nu,\,\mu)}{\partial\,\mu}\right\|_{B_2(t)}\leqslant$$
$$\leqslant M_1M_2^2\int_0^t\left\|\frac{\partial\,f(s,\,y,\,u)}{\partial\,u}\right\|_{L_2(D_l)}\left\|\frac{\partial\,a^0(s,\,\nu,\,\mu)}{\partial\,\mu}\right\|_{B_2(s)}ds\leqslant$$
$$\leqslant M_1M_2^2\,\|W_1(t,\nu,\mu)\|_{B_2(T)}\,\gamma t\,,$$

$$\left\|\frac{\partial\,a^{k+1}(t,\,\nu,\,\mu)}{\partial\,\mu}-\frac{\partial\,a^k(t,\,\nu,\,\mu)}{\partial\,\mu}\right\|_{B_2(t)}\leqslant$$

$$\leqslant M_1 M_2^2 \int_0^t \left\| \frac{\partial f(s,\, y,\, u)}{\partial u} \right\|_{L_2(D_l)} \left\| \frac{\partial a^k(s,\, \nu,\, \mu)}{\partial \mu} - \frac{\partial a^{k-1}(s,\, \nu,\, \mu)}{\partial \mu} \right\|_{B_2(s)} ds \leqslant$$

$$\leqslant M_1^{k+1} M_2^{2k+2} \left\| W_1(t,\nu,\mu) \right\|_{B_2(T)} \frac{(\gamma t)^{k+1}}{(k+1)!}\,.$$

The existence of solution of countable system (17) in the space $B_2(T)$ follows from the last two estimates. The uniqueness of this solution is proved by the following estimate

$$\left\| \frac{\partial a(t,\, \nu,\, \mu)}{\partial \mu} - \frac{\partial v(t,\, \nu,\, \mu)}{\partial \mu} \right\|_{B_2(t)} \leqslant$$
$$\leqslant M_1 M_2^2 \int_0^t \left\| \frac{\partial f(s,\, y,\, u)}{\partial u} \right\|_{L_2(D_l)} \left\| \frac{\partial a(s,\, \nu,\, \mu)}{\partial \mu} - \frac{\partial v(s,\, \nu,\, \mu)}{\partial \mu} \right\|_{B_2(s)} ds, \tag{22}$$

if we apply to (22) the Gronwall-Bellman inequality.

It follows that

$$\left| \frac{\partial u(t,x,\, \nu,\, \mu)}{\partial \mu} \right| \leqslant \sum_{i=1}^{\infty} \left| \frac{\partial a_i(t,\, \nu,\, \mu)}{\partial \mu} \right| \cdot |b_i(x)| \leqslant$$

$$\leqslant \left(\sum_{i=1}^{\infty} \left| \frac{\partial a_i(t,\, \nu,\, \mu)}{\partial \mu} \right|^2 \right)^{\frac{1}{2}} \cdot \left(\sum_{i=1}^{\infty} |b_i(x)|^2 \right)^{\frac{1}{2}} = M_2 \left\| \frac{\partial a(t,\, \nu,\, \mu)}{\partial \mu} \right\|_{B_2(T)} < \infty.$$

Let us consider the following relation

$$\frac{a_i(t,\, \nu,\, \mu+h) - a_i(t,\, \nu,\, \mu)}{h} = \frac{\exp\Big\{-\omega_{1i}(\nu,\, \mu+h)\, t\Big\} - \exp\Big\{-\omega_{1i}(\nu,\, \mu)\, t\Big\}}{h} \times$$
$$\times \left[\varphi_{1i} \cos \omega_{2i}(\nu,\, \mu+h) \frac{t}{2} + \frac{2}{\omega_{2i}(\nu,\, \mu+h)} \Big(\varphi_{2i} + \frac{\varphi_{1i}}{2}\, \omega_{1i}(\nu,\, \mu+h) \Big) \sin \omega_{2i}(\nu,\, \mu+h) \frac{t}{2} \right] +$$
$$+ \exp\Big\{-\omega_{1i}(\nu,\, \mu)\, \frac{t}{2}\Big\} \cdot \left\{ \frac{\varphi_{1i}}{h} \Big(\cos \omega_{2i}(\nu,\, \mu+h)\, \frac{t}{2} - \cos \omega_{2i}(\nu,\, \mu)\, \frac{t}{2} \Big) + \right.$$
$$+ \frac{1}{h} \Big(\frac{2}{\omega_{2i}(\nu,\, \mu+h)} - \frac{2}{\omega_{2i}(\nu,\, \mu)} \Big) \cdot \Big(\varphi_{2i} + \frac{\varphi_{1i}}{2}\, \omega_{1i}(\nu,\, \mu+h) \Big) \sin \omega_{2i}(\nu,\, \mu+h) \frac{t}{2} +$$
$$+ \frac{2}{\omega_{2i}(\nu,\, \mu)} \left[\frac{\varphi_{1i}}{2} \cdot \frac{\omega_{1i}(\nu,\, \mu+h) - \omega_{1i}(\nu,\, \mu)}{h} \sin \omega_{2i}(\nu,\, \mu+h)\, \frac{t}{2} + \right.$$
$$\left.\left. + \frac{\sin \omega_{2i}(\nu,\, \mu+h)\, \frac{t}{2} - \sin \omega_{2i}(\nu,\, \mu)\, \frac{t}{2}}{h} \Big(\varphi_{2i} + \frac{\varphi_{1i}}{2}\, \omega_{1i}(\nu,\, \mu) \Big) \right] \right\} + \tag{23}$$
$$+ \int_0^t \int_0^l \frac{G_i(t,s,\nu,\mu+h) - G_i(t,s,\nu,\mu)}{h} f\left(s, y, \sum_{j=1}^{\infty} a_j(s,\, \nu,\, \mu+h)\, b_j(y) \right) b_i(y) dy ds +$$
$$+ \int_0^t \int_0^l G_i(t,s,\nu,\mu+h)\, \frac{1}{h} \left[f\Big(s, y, \sum_{j=1}^{\infty} a_j(s,\, \nu,\, \mu+h)\, b_j(y) \Big) - \right.$$
$$\left. - f\Big(s, y, \sum_{j=1}^{\infty} a_j(s,\, \nu,\, \mu)\, b_j(y) \Big) \right] b_i(y) dy ds,$$

where

$$\frac{G_i(t,s,\nu,\mu+h) - G_i(t,s,\nu,\mu)}{h} = 2 \frac{\exp\Big\{-\omega_{1i}(\nu,\, \mu+h)\, \frac{t-s}{2}\Big\} - \exp\Big\{-\omega_{1i}(\nu,\, \mu)\, \frac{t-s}{2}\Big\}}{h} \times$$

$$\times \frac{\sin \omega_{2i}(\nu,\, \mu+h)\, \frac{t-s}{2}}{\omega_{0i}(\nu)\, \Big[\omega_{2i}(\nu,\, \mu+h) + \omega_{1i}(\nu,\, \mu+h)\, \sin \omega_{2i}(\nu,\, \mu+h)\, s\Big]} +$$

$$+2 \exp\Big\{-\omega_{1i}(\nu,\, \mu)\, \frac{t-s}{2}\Big\} \Big\{ \frac{\sin \omega_{2i}(\nu,\, \mu+h)\, \frac{t-s}{2} - \sin \omega_{2i}(\nu,\, \mu)\, \frac{t-s}{2}}{h} \times$$

$$\times \frac{1}{\omega_{0i}(\nu)\, \Big[\omega_{2i}(\nu,\, \mu+h) + \omega_{1i}(\nu,\, \mu+h)\, \sin \omega_{2i}(\nu,\, \mu+h)\, s\Big]} + \frac{\sin \omega_{2i}(\nu,\, \mu)\, \frac{t-s}{2}}{\omega_{0i}(\nu)}\, \frac{1}{h} \times$$

$$\times \Big(\frac{1}{\omega_{2i}(\nu,\mu+h) + \omega_{1i}(\nu,\mu+h)\, \sin \omega_{2i}(\nu,\mu+h)\, s} - \frac{1}{\omega_{2i}(\nu,\mu) + \omega_{1i}(\nu,\mu)\, \sin \omega_{2i}(\nu,\mu)\, s} \Big) \Big\}.$$

Taking the limit $h \to 0$ in (23), we obtain (17). So we have

$$\left| \frac{u(t,x,\nu,\mu+h) - u(t,x,\nu,\mu)}{h} - \frac{\partial\, u(t,x,\nu,\mu)}{\partial\, \mu} \right| \leqslant$$

$$\leqslant M_2 \left\| \frac{a(t,\, \nu,\, \mu+h) - a(t,\, \nu,\, \mu)}{h} - \frac{\partial\, a(t,\, \nu,\, \mu)}{\partial\, \mu} \right\|_{B_2(T)}.$$

The theorem is proved. □

Conclusions

In conclusion, we note that under the conditions of the theorems proved in this paper the solution of mixed value problem (1)–(3) is differentiable with respect to small parameters ν and μ. In particular, it follows that the solution of mixed value problem (1)–(3) depends continuously on small parameters ν and μ. In addition, as $\nu \to 0$ and $\mu \to 0$ the solution of the mixed value problem converges uniformly to the solution of equation (4) with initial condition (2) and boundary conditions (3).

References

[1] V.M.Alexandrov, E.V.Kovalenko, Continuum mechanics problems with mixed boundary conditions, Nauka, Moscow, 1986 (in Russian).

[2] S.D.Algazin, I.A.Kiyko, Flutter of plates and shells, Nauka, Moscow, 2006 (in Russian).

[3] P.P.Kiryakov, S.I.Senashov, A.N.Yakhno, Application of symmetries and conservation laws to solving differential equations, Izdatel'stvo SO RAN, Novosibirsk, 2001 (in Russian).

[4] S.I.Senashov, Conservation laws of plasticity equations, *Dokl. AN SSSR*, **320**(1991), no. 3, 606–608 (in Russian).

[5] O.A.Ladyzhenskaya, V.A.Solonnikov, N.N.Ural'ceva, Linear and Quasilinear Equations of Parabolic Type, Nauka, Moscow, 1967 (in Russian).

[6] S.L.Sobolev, Some applications of functional analysis in mathematical physics, Nauka, Moscow, 1988, (in Russian).

[7] S.I.Pohozhaev, On Prior estimates and gradient catastrophes smooth solutions of hyperbolic systems of conservation laws, *Trudy MI RAN*, **243**(2003), 257–288 (in Russian).

[8] A.A.Samarskiy, On some problems of contemporary theory of differential equations, *Diifferentsial'nye uravneniya*, **16**(1980), no. 11, 1221–1228 (in Russian).

[9] A.M.Nakhushev, An approximation method for solving boundary value problems for differential equations with applications to the dynamics of soil moisture and groundwater, *Diifferentsial'nye uravneniya*, **18**(1982), no. 1, 72–81 (in Russian).

[10] D.G.Gordesiani, G.A.Avalishvili, Solutions of nonlocal problems for one-dimensional vibration environment, *Matem. Model.*, **12**(2000), no. 1, 94–103 (in Russian).

[11] V.B.Dmitriev, Nonlocal problem with integral conditions for free equation, *Vestnik Samarskogo gosuniversiteta, Seriay estestvennyh nauk*, **42**(2006), no. 2, 15–27 (in Russian).

[12] L.S.Pul'kina, A mixed problem with an integral condition for the hyperbolic equation, *Mathem. Notes*, **74**(2003), no. 3, 435–445.

[13] J.W.Strutt Lord Rayleigh, Theory of Sound, 2-nd ed., MacMillan, London, II, 1896.

[14] T.K.Yuldashev, Mixed value problem for nonlinear integro-differential equation with parabolic operator of higher power, *Computational Mathem. and Mathem. Phyzics*, **52**(2012), no. 1, 112–123.

[15] T.K.Yuldashev, On weak solvability of a mixed problem for a nonlinear pseudohyperbolic equation, *Zh. Mat. Obcshestva Srednei Volgi*, **14**(2012), no. 4, 91–94 (in Russian).

[16] T.K.Yuldashev, On solvability of a mixed value problem for a nonlinear pseudohyperbolic equation of fifth order, Abstract book of IV Russian-Armyan Conference on Mathematical Physics, Complex Analysis and Related Topics, Siberian Federal Univers., Krasnoyarsk, 2012, 88–91.

О дифференцируемости по малым параметрам решения смешанной задачи для нелинейного псевдогиперболического уравнения

Турсун К. Юлдашев

В статье доказываются теоремы о дифференцируемости по малым параметрам обобщенного решения смешанной задачи для нелинейного псевдогиперболического уравнения пятого порядка.

Ключевые слова: нелинейное уравнение, дифференцируемость решения, малые параметры, счетная система нелинейных интегральных уравнений.