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Solving an Eigen Problem to Create Reliable Tree-Ring Chronologies

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It is very popular in meteorology and climatology to solve the so-called eigen problem for the meteorological field covariation matrix in order to extract "main modes" of the meteorological field variability. In this paper we depict the application of a similar problem for the tree-ring record covariation matrix in order to standardize tree-ring records. For the first time this technique has been applied to a sample set of very long-lived Qilian junipers (Sabina przewalskii Kom.) from the Dulan region in western China and has been earlier published (Yang et al., 2011, 2012a, 2012b). This paper expands our report at the All-Russian conference on dendrochronology (RusDendro-2011) where we have presented the eigen analysis with some recent improvements and additional applications of the technique to a larger sample set of the long-living Chinese junipers, and to another sample set of relative short-living Siberian larches growing in the Yamal Peninsula. We demonstrate that our eigen analysis technique is applicable to heterogeneous sets of tree-ring width records, and of great perspective to create really reliable tree-ring chronologies.

Keywords: the eigen problem, dendrochronology, millennial paleoclimatic reconstruction, solar activity effects on tree growth.

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Introduction

Tree-ring width records are most often used as proxies to reconstruct past climatic variations, and so it is very important to create tree-ring chronologies that would be unaffected by different nonclimatic effects like the biological unique of each individual tree, and micro-environmental conditions of the tree growth, and so reliable indeed. Initially, dendrochronologists mainly tried to reconstruct interannual and interdecadal climatic variations. For this goal it was enough to remove a general trend of the tree growth from each individual tree-ring record to reject age-dependent variations of the growth from further consideration (Fritz, 1976). This simple technique has been called the "classic" standardization (CLS). Unfortunately, the procedure of the trend removing was very subjective in practice of the CLS use. Moreover, it has been proved that CLS is inadequate if the reconstruction of longer (centennial and many-centennial) climatic variations is of interest. It was so because the climate-induced variations periods of which are commensurate to longevities of the trees under consideration turned out to be removed along with the truly age-dependent variations (Cook et al., 1995; Briffa et al., 1996). To overcome this defect, several new kinds of the standardization have been developed such as the age-banding technique and the regional curve standardization (RCS).

RCS is most popular now (Briffa et al., 1992, 1996; Cook et al., 1995; Esper et al., 2002, 2003, 2007; Wilson et al., 2005; Büntgen et al., 2006; D'Arrigo et al., 2006) although there are some limitations on its practical use. In particular, RCS can be only applied to large samples of tree-ring records of any tree species growing in a geographic region that has homogeneous environmental conditions.

Using RCS, all records are aligned with respect to their biological age, and then averaged to create a single time series of tree aging (called the "regional curve" – RC). Certainly, this curve turns out to be truly representative as "the statistical normal" of the tree growth when the first and latest calendar years of the tree records analysed cover the entire time interval under reconstruction (for example, the latest one or two millennia of calendar years) more or less uniformly; moreover, the same is true for the temporal coverage of the records of different lengths. If both of these conditions are fulfilled, one can hope that removals of the contribution of RC from each individual record, either by subtracting RC from that or dividing each record by RC, admit us to create a set of time series of the so-called tree-ring indices that are age-free. It means that tree growth variations represented in these index series do not depend on the tree biological age.

Unfortunately, careful investigations have demonstrated that the above conditions for making RC to be truly representative as the statistical normal of the tree-ring set analysed are never fully met. For example, the first calendar years of individual records might not cover some latest part of the time interval under reconstruction. This circumstance implies a systematic bias in the index time series (Esper et al., 2008; Melvin, Briffa, 2008). A similar bias is inherent in the earliest part of the time interval under reconstruction because the latest years of the tree-ring records can not be incorporated into it.

Then, lengths of the tree-ring records always are very different, and specific peculiarities exist in RCs created by averaging only the records of a particular length (Esper et al., 2007; Esper et al., 2008; Melvin, Briffa, 2008; Datsenko et al., 2010). As a result, the index time series created with use of a single RC contain "fingerprints" of these specific peculiarities. Although certain extensions of

RCS have been recently proposed (Helama et al., 2004, 2005; Nicault et al., 2010, and others), RCs computed in practice are usually flamed by heterogeneous (long- and short-term) climatic and nonclimatic effects. Especially, spurious trends in the scale of the overall length of the tree-ring records analysed are found to be inherent in the tree-ring chronologies being created (Melvin, Briffa, 2008; Yang et al., 2010a).

At last, the RCS technique completely ignores two important circumstances implying trees to grow faster or slower: the heterogeneity of the micro-environmental conditions of the individual tree stands, and the uniqueness of each tree as a living organism. It is a general case that the redistribution of the faster and slower growing trees along the interval of calendar years under reconstruction turns out to be uneven. By this reason, long-term variations of the tree growth represented in any tree-ring chronology have to be essentially effected by the afore-mentioned circumstances.

Thus, because of all above reasons, the tree-ring index time series created by the RCS technique remain to be age- and environment-dependent in practice. The main goal of this paper is to give two examples (for very long-living high-altitude Chinese junipers, and shorter-living conifers growing near the northern timberline in Siberia as well) of this regrettable circumstance by means of demonstration of the "main modes" of the tree-ring index time series variability.

Materials and Methods

Data

The goal of this paper is to testify applicability of the eigen analysis technique to create tree-ring chronologies for both long- and short-living trees. Especially, we try to corroborate that the modes of the tree growth are the same for both long- and short-living trees, and these modes may be depicted well using the basis of the first segments of the Bessel function of the first kind and zero order. It means that these modes are age-dependent. This age-dependence is a very serious obstacle to create any reliable tree-ring chronology. Therefore, we indicate a simpler way, in comparison with the way indicated previously (Yang et al., 2012b), how it is possible to escape the age-dependence of the Bessel basis.

The first application of the eigen analysis of tree-ring records has been previously done (Yang et al., 2012a, 2012b) on an example of a set of 56 junipers aged 603 years or more from the Dulan region in the northeastern part of the Tibetan Plateau in Qinghai Province of western China. This juniper *Sabina przewalskii* Kom. is a unique, very long-lived species (up to 1000 biological years and even longer) which is endemic to China. It grows in open-spaced stands on south-facing slopes near the upper timberline. Researchers started to collect and process Dulan tree cores and discs more than 25 years ago (Wang et al., 1983; Kang et al., 1997). This sample was later supplemented (Kang et al., 2000; Yang et al., 2000; Zhang et al., 2003 ; Sheppard et al., 2004; Liu et al., 2006). Now Chinese scientists continue to add new trees to this sample.

The Dulan region is an arid area with intensive plateau-continental climatic characteristics. Both the East Asian and the Indian monsoons impact the climate of the region. Instead of the usual division of the annual period onto four seasons, it is more appropriate to distinguish only the wet and dry seasons in the Dulan climate. During the wet season this climate is mild, since the East Asian summer monsoons impact the Dulan region. During the dry season, the Westerlies and the Plateau Cold High are important. In general, temperature and precipitation change synchronously in the Dulan region: lower temperature corresponds to lower precipitation, and vice versa. This relationship may indicate that the climatic patterns are combined from warm-wet and cold-dry events not only on a decadal/centennial, but also on a multi-centennial scale. In particular, a multi-proxy analysis (Yang et al., 2000) revealed that the Medieval Warm Epoch was wet and warm, and the Little Ice Age was dry and cold in the Dulan region, although more essential variations were inherent to both of these epochs on centennial and multi-decadal scales in comparison with Europe.

We also used the well-known set of larches sampled by S.G. Shiyatov and his colleagues in the Yamal peninsula (Hantemirov, Shiyatov, 2002), and a subset of the 233 tree-ring width records with longevity no less than 200 years has been chosen from this set.

Besides, in order to corroborate the results published in Yang et al. (2012a, 2012b) and improve the eigen analysis technique we used a much larger sample set (834 records) of the same tree species *S. przewalskii*, both living and archeological trees. This set has been sampled from four mountain valley around the Dulan region. There is full information about pith and bark for each record of this sample set, and careful cross-dating has been done as well. This sample set has been decomposed into six subsets: from 200 to 400 (219 records), from 400 to 600 (220 records), from 600 to 800 (151 records), from 800 to 1000 (122 records), from 1000 to 1500 (91 records), and more than 1500 (31 records) years. Our eigen analysis technique has been applied to each of these subsets. But in this paper we demonstrate the eigen analysis results for the subset 600-800 years only. It is because of two reasons. First, the respective results for all other subset are practically the same as for the subset chosen to demonstrate. Second, the subset chosen is of the same longevity that was inherent to the sample of 56 Dulan's junipers used for our first demonstration of the eigen analysis technique (Yang et al., 2012a, 2012b).

Mention only that we used a slightly improved preprocessing of the long-living Chinese junipers before doing the second step of our eigen analysis. All individual records were scrutinized to recognize the grand maximum of the juvenile tree growth, and all innermost rings before this grand maximum were excluded from further consideration in each individual record. As a rule, the number of the excluded innermost rings was less than 10. But, in some (rather seldom) cases this number was essentially more (up to 60 rings), or, to the contrary, it was equal to zero. Such exclusions of a number of innermost rings was done because these rings are known to be the main source of the tree-ring width record heteroscedasticity. Besides, in order to more diminish heteroscedasticity, we fulfilled the normalization of the remaining parts of the records, i.e. each tree-ring index (computed by means of subtraction of RC from ring width) was divided by the root-mean-square value of the indices of the respective age computed for the studied subset.

Method of the tree-ring record analysis

The starting point of our study is to treat any large set of tree-ring records as a random sample from an unknown probabilistic ensemble. This ensemble is characterized by a probability distribution of the ring width (density, isotope, etc.) values, with its normal (RC) and deviations from RC (index time series) characterized by second-order statistics such as the intra-record covariation matrix, i.e. covariations between all pairs of index values of each tree-ring record averaged over the entire sample set of the records analysed:

$$\mathrm{cov}(\tau_1,\tau_2)=M_i(\tau_1,\tau_2)^{-1}\sum_{i=1}^{M_i(\tau_1,\tau_2)}\left(\Delta_i(\tau_1)\right)\left(\Delta_i(\tau_2)\right). \qquad (1)$$

Here $\Delta_i(\tau_j),\ j=1,2$ is the index value for the ring *j* of the tree-ring record *i*, τ_1 and τ_2 are the biological tree ages being compared, and $M_i(\tau_1,\tau_2)$ is the number of compared pairs of ring widths in the record *i*. The maximal length of the records analysed $K=\max_i\left|\tau_1-\tau_2+1\right|$ determines the order of the intrarecord covariation matrix (usually $K\approx 10^2-10^3$). The computation of the matrix (1) is especially easy if the length is the same for all records analysed.

Note that Equation (1) may be well used to estimate the intra-record covariation matrix in both cases: when either RC is subtracted from individual records ($\Delta_i(\tau)=\delta R_i(\tau)-RC(\tau)$) or these records are divided by RC ($\Delta_i(\tau)=\frac{\delta R_i(\tau)}{RC(\tau)}-1$). But, it is for certain that the second order statistics will be different in these cases, and so that definition of the tree-ring index $\Delta_i(\tau)$ has to be chosen which ensures better properties of the second order statistic in principle. Any way, information about tree growth variability and persistence in the tree-ring ensemble can best be extracted by solving the so-called eigen problem for the intra-record covariation matrix:

$$\mathrm{cov}(\tau_1,\tau_2)\varphi_k(\tau_2)=\lambda_k\varphi_k(\tau_1),\ k=1,2,...,K \qquad (2)$$

where λ_k and $\varphi_k(\tau)$ are the so-called eigen values and vectors (called “empirical orthogonal functions” in traditional meteorological applications) of the covariation matrix, respectively. There are numerous standard routines to solve the eigen problem numerically in FORTRAN and MATHLAB. The number of eigenvectors is equal to *K*. These eigenvectors reveal specific shapes of temporal tree-ring variations (often called “modes”) typical for $\Delta_i(\tau),\ i=1,2,...,N$ where *N* is the number of the tree-ring records analysed. Projections of $\Delta_i(\tau)$ on the eigenvectors given by:

$$PC_i(k)=\sum_{\tau=1}^{K}\Delta_i(\tau)\varphi_k(\tau),\ k=1,2,...,K \qquad (3)$$

are called “principal components” (*PC*s). Each $PC_i(k)$ represents the contribution of the respective mode to the general variability of the index time series, and the eigen values are nothing than less the variances of principal components ($\lambda_k=N^{-1}\sum_{i=1}^{N}\left(PC_i(k)\right)^2$), i.e. measures of the general variability. It should be noted that there are some time series of principal components in applications of the empirical orthogonal functions to meteorology and climatology. However, in the application of the eigen analysis to tree-ring records that we consider, each $PC_i(k)$ is a single number for each individual tree-ring record depending only on the ordinal number of the eigenvector.

Eigenvectors are usually regularized with respect to the eigen value decreasing. The vectors corresponding to the first (larger) eigen values $\lambda_k,\ k=1,2,...$ represent the most influential (essential) modes of the temporal tree growth variations, while those vectors which correspond to smaller eigen values $\lambda_k,\ k=\widehat{K}+1,\widehat{K}+2,...,K-2,K-1,K$ represent the least influential modes, perhaps, induced by different kinds of noise present in tree-ring records. As it is shown in numerous meteorological and climatological applications, one can wait that the number $\widehat{K}$ of the essential modes (eigenvectors) usually is much less than the number *K* of all the modes (eigenvectors). Indeed, the main interest in paleoclimatology is to reconstruct centennial/multi-centennial climatic variations. In the general case (see Sonechkin, 1971 for explanation), the shape of the eigen vector is so more oscillatory than the number of this eigenvector is larger. By this reason, one can usually exclude from further consideration those $PC_i(k)$ for $k > \widehat{K}$, and try to reconstruct age-free variations of tree growth over an interval of calendar years (create a tree-ring chronology) by means of a summation of only the $PC_i(k)$ corresponding to larger eigen values:

$$\Delta_i(\tau)=\sum_{k=1}^{\widehat{K}} PC_i(k)\varphi_k(\tau),\ k=1,2,...,\widehat{K}<K, \tag{4}$$

with consequent aligning of $\widehat{\Delta}_i(\tau)$ with respect to their calendar years. It has been shown that the shapes of the eigenvectors corresponding to the essential principal components can be well approximated by several first segments of the Bessel function of the first kind and zero order (Yang et al., 2011, 2012a). All of these are evidently age-dependent, and thus it is impossible to escape some nonclimatic influences on the tree growth depicted by the respective principal components.

However, the afore-mentioned endogenous micro-environmental variations and the unique biological nature of each tree, which affect growth of individual trees, unfortunately turn out to be characterized as being very low-frequency. Therefore, as it has been shown (Yang et al., 2011), the intra-record covariation matrix does not reveal any essential decrease to zero when the age shift between compared pairs of tree-rings increases. As a result, the largest eigenvalue corresponds to that mode (eigenvector) which components are of the same sign. Moreover, the shape of this eigenvector is very specific and age-dependent. It can be approximated well by the very first segment of the Bessel function of the first kind and zero order (Yang et al., 2011, 2012a). Basing on this mode, it is possible to separate trees with different biological potential of growth, i.e. trees which grow either fast or slow independently from any variation of climate. It means that the contributions of the first principal component for all trees $PC_i(1), i = 1,2,N$ have to be excluded from any consideration if we want to create reliable (induced by climatic variations only) tree-ring chronology.

In sum, when the eigen analysis of tree-ring records is used, the steps of the tree-ring record processing are follow:

- Selection of a subset of records of a certain length from a (rather large) sample set of tree-ring records of interest.
- Alignment of all records of this subset according to their biological ages in order to compute the RC of the subset.
- Transformation of these records to their index time series either by means of subtraction of RC from individual records or by means of division of these records by RC.

Note that dendrochronologists use the record division by RC. But Cook and Peters (1996) have indicated some defects of this defining the index time series. Some other defects have also been indicated (Yang et al., 2012b). Therefore, our recommendation is to use the subtraction of RC.

- Computation of the intra-record covariation matrix for the index time series, and then solution of the eigen problem for this matrix.
- Consideration of the eigenvalue spectrum of the problem in order to chose a number of tree growth modes for further processing.
- Computation of the principal components for some first eigenvectors in order to compute the contributions of the respective modes into the index time series variability.
- Testify of the redistribution of the mode contributions along the time interval of calendar years under reconstruction in respect of the null-hypothesis (the

uniformity of the redistribution). If the null-hypothesis is accepted the contribution of the respective mode should be considered as induced by nonclimatic (micro-environmental and/or biological) effects. Only if the null-hypothesis is rejected the contribution can be considered as climate induced, and so the respective $PC_i(k)$ may b3e used to include its contribution into the tree-ring chronology.

Results

Short-living trees

In our application of the eigen analysis technique to an Yamal subset of relatively short-living larches, we used three different definition of the tree-ring width index: subtraction of RC from original records; division of these records by RC, and the RC subtraction with the preliminary logarithmic transformation of the records. Fig. 1-3 show our numerical estimation of the 1-6 eigenvectors for these cases. Comparing these figures with the respective figures shown in Yang et al. (2012b), one can be convinced that the shapes of the eigenvectors for the Yamal's short-living larches are qualitatively the same that were found for the long-living Chinese junipers.

For example, in the case of the RC subtraction, the eigenvectors look to be rather well approximated by the first segments of the Bessel function of the first kind and zero order. The only one difference exists that the Bessel approximation is not applicable to some longer (than it is inherent to the Chinese junipers) portion of the index time series during the juvenile growth of the larches considered. It is because the longevity of the juvenile growth takes essentially longer part of the entire tree life for the short-living larches in comparison with such longevity for the long-living junipers. Perhaps, the innermost rings corresponding to the juvenile growth of larches have to be excluded from further consideration when the eigen analysis is applied to these larches. But this way, we lose some essential part of the tree-ring information because the longevity of the mature growth of the larches is commensurate to the longevity of their juvenile growth (both are equal to about 100 biological years for the Yamal subsample).

In the case of the record division by RC the first eigenvector looks to be almost constant function of the tree age, the second eigenvector seems to be similar to cosine the period of which is equal to the doubled tree longevity (400 biological years) period. But subsequent eigenvectors remain to be of more complex shapes and (it is essential for the subsequent steps of the eigen analysis) age-dependent. In the case of the preliminary logarithmic transformation of the records the situation is rather similar to the second case of the record division by RC, i.e. the first and second eigenvectors reveal themselves as stationary tree growth variations with respect to the biological age, but all subsequent eigenvectors reveal variations nonstationary with respect to the tree age.

Thus, neither division of original records by RC, nor their preliminary logarithmic transformation can ensure tree growth variations to be homoscedastic. It is just the same conclusion that has been obtained for the long-living Chinese junipers (Yang et al., 2012b). Therefore, the recommendation may be confirmed here to use the subtraction of RC from records instead of the traditional record division by RC.

By the way, recently Briffa and Melvin have announced (*www.cru.uea.ac.*uk) a new Yamal chronology, created by means of the traditional RCS, with current warming trend being very expressed. Our conclusion about an unavoidable heteroscedasticity of the index time series of the Yamal larches, created with use of the record division by RC, allows us to assert that the trend-

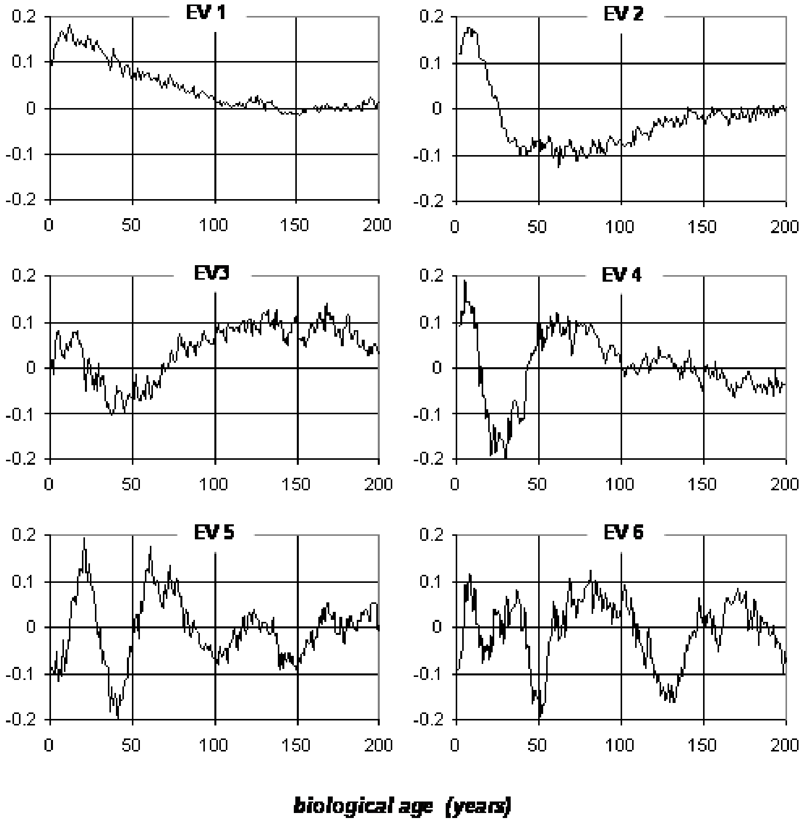

Fig. 1. Six eigenvectors of the intra-record covariation matrix of the Yamal ring width sample set corresponding to 1-6th largest eigenvalues. Tree-ring indices are defined by means of subtractions of RC from individual records

like ingredient of their chronology is very affected by some remaining heteroscedasticity effects (by the afore-mentioned existence of the faster and lower growing trees).

Long-living trees

Processing the new subset of the long-living Chinese junipers, we used the eigen analysis technique with some improvements. First of all, because the character of the juvenile growth of these junipers essentially differs from the character of their mature growth, using full information about the pith offset, we have done careful visual inspection of each tree-ring width record in order to recognize the position of the grand maximum of the juvenile growth. It turned in some cases (but rather seldom) the grand maximum is observed for the innermost ring. But, much more often, the maximum takes place within an interval of biological tree ages from a few years and to about 70 years. We have excluded all inner rings before the grand maxima from each individual ring width record, and then computed RC for the only rings of the mature tree growth. Fig. 4 shows the RC (the graph marked “1”), which we obtained by such manner, together with root-mean-square deviations (the graph marked “2”) computed for each biological age of the mature tree growth for the 151 records as a whole sample set.

One can see that the values of the root-mean-square deviation are practically the same for the 100-600s rings. However, this value is about two times more for a few first rings. It is maximal

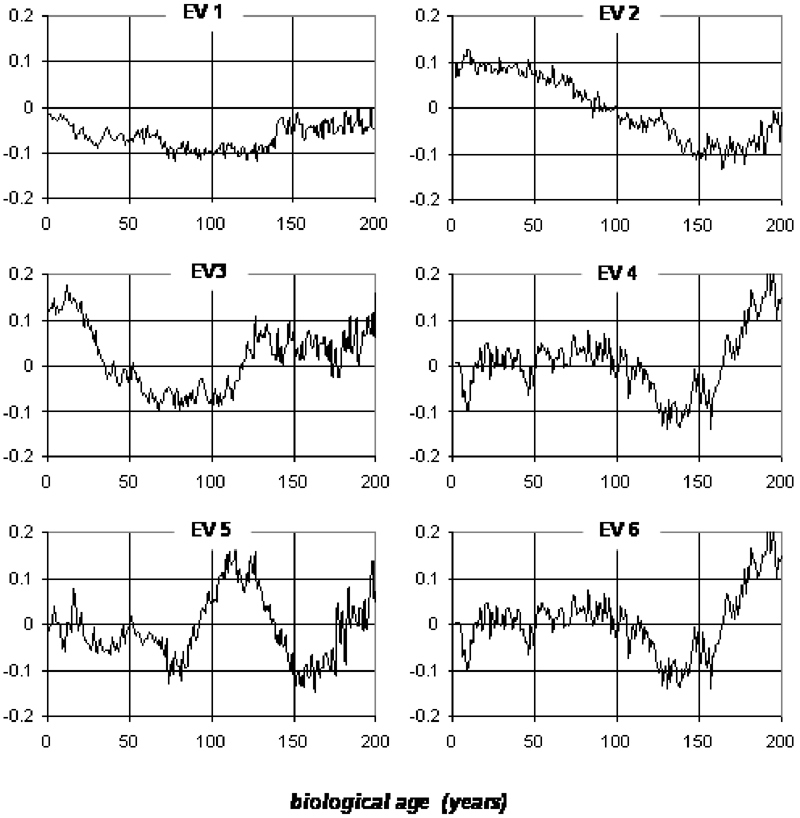

Fig. 2. Six eigenvectors of the matrix of the Yamal sample set (the same as in Fig. 1), where tree-ring indices are defined with use of preliminary logarithmic transformations of individual records

for the ring that corresponds to the grand maximum of the juvenile tree growth. It means that the tree-ring width variability remains to be slightly heteroscedastic even if the only mature tree growth is considered. But the remaining heteroscedasticity is essentially less pronounced than in the case of the consideration of both stages of the juvenile and mature tree growth.

The eigen value spectra for the subset of the 151 long-living Chinese junipers, computed with use different afore-mentioned definitions of the tree-ring width index look to be very similar to the respective spectra of the 56 Chinese junipers of the same longevity (Yang et al., 2011 b, 2012b). Each of these spectra is of a hyperbolic shape, i.e. the first (maximal) eigen value is much more than the second and third eigen values, and all subsequent eigen values are much smaller. Thus, a rather small number of the main tree growth ‘modes” seems to be enough to depict well heterogeneous tree growth variations. These “modes” may be represented by the eigenvectors of the intrarecord covariation matrix of the tree-ring index variations corresponding to the very first (larger) eigen values.

Fig. 5 shows 1-10th eigenvectors of the index time series defined by the RC subtraction. Comparing this figure with the respective figure of the eigen vectors in (Yang et al., 2012a, 2012b), one can see that the shapes of the eigenvectors shown in Fig. 5 are essentially smoother (especially for the 1-5th eigen vectors), and more similar to the very first segments of the Bessel function of the first kind and zero

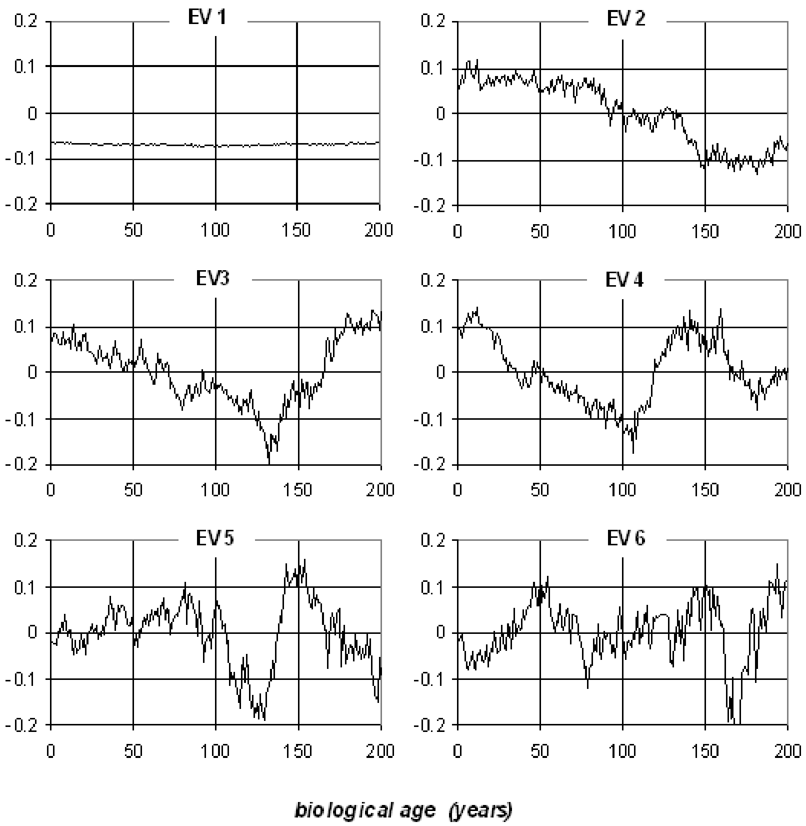

Fig. 3. Six eigenvectors of the matrix of the Yamal sample set (the same as in Fig. 1), where tree-ring indices are defined by means of divisions of individual records by RC

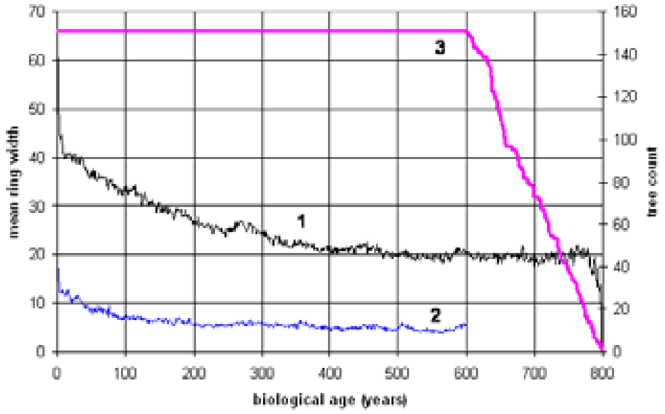

Fig. 4. The regional curve of a sample subset of the Chinese junipers *Sabina przewalskii* with the longevity of 600-800 years (1). The graph of the root-mean-square deviation of individual tree-ring indices from RC as a function of the tree age (2). The graph of the tree count for each biological tree age of the trees (3)

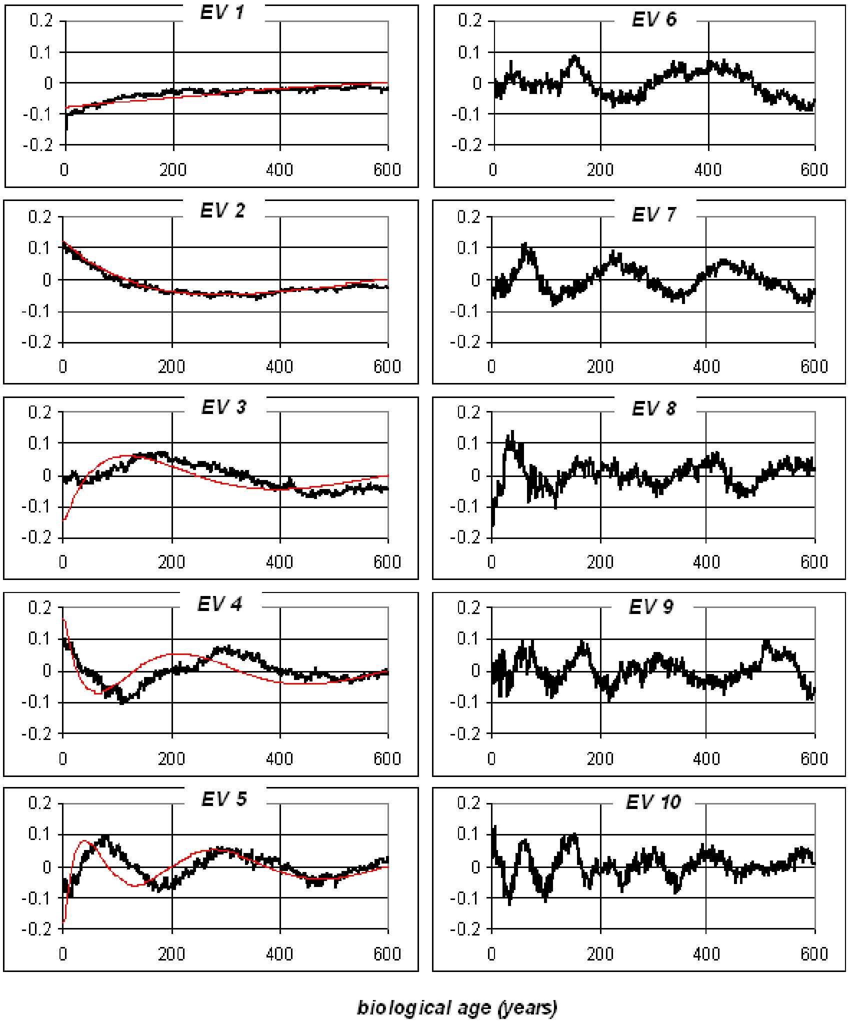

Fig. 5. Ten eigenvectors of the intra-record covariation matrix of the Chinese ring width sample set corresponding to 1-10s largest eigenvalues. Tree-ring indices are defined by means of subtractions of RC from individual records

order (shown by grey lines in Fig. 5 for the only 1-5th eigenvectors). The better smoothness of the eigenvectors is a consequence of two factors: a larger number of the tree-ring width records used (151 versus 56 in the study of Yang et al. (2012a, 2012b)), and the exclusion from consideration some parts of the tree-ring width records before the juvenile grand maxima of their tree growth. The absence of any singularity at the very beginning of the eigenvector curves (corresponding biological years just after the juvenile grand maximum) confirms well that the exclusion of some innermost rings before the juvenile grand maximum of the tree growth is the right way to diminish the heteroscedasticity of the tree-ring width variability.

However, the variability of the mature tree growth become fully homoscedastic only after normalizing the index time series, i.e. dividing each mature index time series by the value of the root-mean-square deviation of the respective age. The curves of the 1-10th eigenvectors of the normalized covariation matrix confirm this circumstance (Fig. 6). Indeed, these new curves

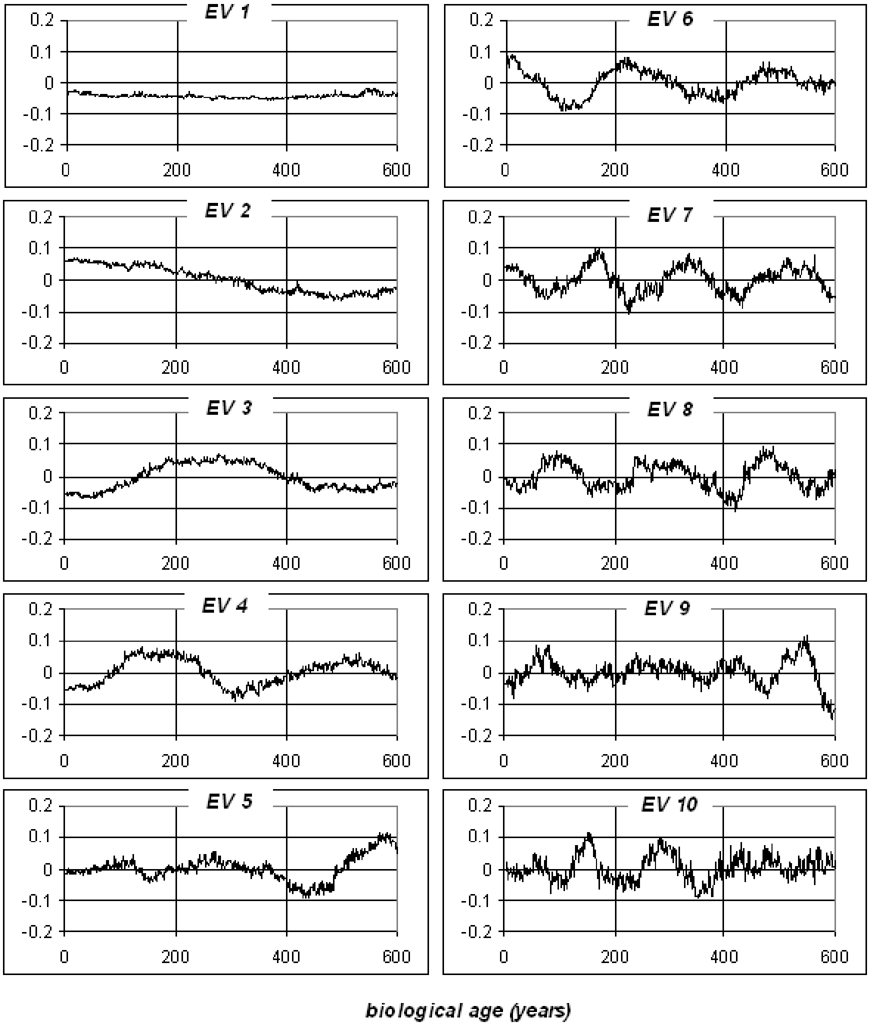

Fig. 6. Ten eigenvectors of the matrix of the Chinese sample set (the same as in Fig. 5), where tree-ring indices are defined after the preliminary normalization of each individual record by means of dividing its ring width by the root-mean-square deviation of the respective tree age

of the eigenvectors look to be rather similar to a family of the trigonometric functions, sine and cosine periods of which are multipliers of the longevity of the tree-ring width records considered. For example, the first eigenvector looks to be of almost constant value. The second eigenvector is similar to the cosine of the period which is equal to the doubled longevity of the records (1200 biological years). The third eigenvector is similar to the sine of the period which is equal to the longevity of the records (600 biological years). The fourth eigenvector is similar to sine of the 300-year long period. The next eigenvectors would be similar to the sines and cosines of the progressively diminishing periods. But, because of the very equality of the eigen values corresponding to these eigenvectors, accurate computation of these vectors is impossible to fulfill. By this computational reason, the 6-10th eigenvectors shown in Fig. 6 are a mixture of the true eigenvectors, and so their shapes look to be more complex and deviate from shapes of the trigonometric functions. Stress, it is inevitable difficulty of any eigen problem numerical solution when the eigen value spectrum is hyperbolic.

In order to overcome this difficulty a recommendation has been given (Yang et al., 2012b) to use the traditional Fourier basis of sines and cosines instead of the "optimal" trigonometric base of the main "modes" of the eigen. Excellent correspondence of the shapes of the 1-5th eigenvectors shown in Fig. 6 confirms that this recommendation can be useful indeed.

Applying all afore-mentioned steps of the eigen analysis technique to a sample set of 56 Dulan junipers of the 600-year life-span we could create a tree-ring chronology of the Chinese junipers that cover the time interval of the last millennium AD 1000-2000 (Fig. 7 upper part). Comparing the 25-year moving average of this tree-ring chronology (shown by light grey line) with the solar forcing of the climate system during the same time interval (Mann et al., 2005), one can see that these time series correspond well to each other in centennial time scale. In particular, main minima of the tree growth with more or less reasonable delays followed by the well-known minima in solar activity: Oort (1040-1080), Wolf (1280-1350), Spoerer (1450-1550), Maunder (1645-1715), and Dalton (1790-1820). It should be emphasized that none tree-ring chronology, earlier built for the Dulan region, did not show such a high degree of compliance with the solar forcing. At the same time, it should be noted that our tree-ring chronology does not reproduce the many-centennial decline in solar forcing from the Medieval Warm Epoch to the Little Ice Age and the subsequent increase in solar forcing to the present. The reason of the last circumstance is clear: the 2-10 modes of the Dulan juniper variability selected by our eigen analysis catch only those climate-induced tree growth variations periods of which are no more than one half of the tree-ring record length (600 years). We believe that similar restriction on the ability of trees to reproduce super long-term climate-induced variations inherent in all tree-ring sample sets.

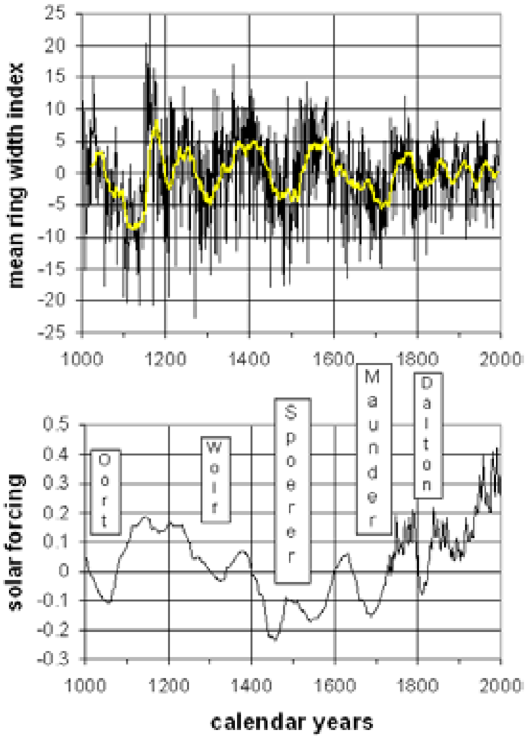

Fig. 7. Chinese's chronology reconstructed with use PC2-10 over the time period AD 1000-2000 (upper part), and solar forcing (W/m^2) computed for tropical Pacific according Mann et al. (2005) (lower part)

Discussion

A few years ago a completely new, mathematically well-grounded technique of the tree-ring record standardization called the eigen analysis of tree-ring records has been developed (Yang et al., 2011, 2012a, 2012b). The aim of this technique consists of an justification of different peculiarities of tree growth that can affect the dendrochronologies being created when the so-called regional curve standardization is used to process raw tree-ring data. It turned out that these peculiarities reveal themselves in specific shapes of the "main modes" tree growth variability. These "modes" can be represented by the eigenvectors of the intrarecord covariation matrix of the tree-ring index records. In turn, these eigenvectors are well approximated on a special basis of the very first segments of the Bessel function of the first kind and zero order. This basis is obviously age-dependent. The existence of this age-dependence proves the poor representativeness of the regional curve as a tree growth "normal".

The hyperbolic shape of the eigen value spectrum of the intrarecord covariation matrix found in all our numerical solutions of the eigen problem evidences that the very first eigen "mode" is prominent. It is indebted to the existence of fast and slow growing trees independently from any variations of climatic conditions, i.e. this "mode" is a characteristic of the biological uniqueness of every tree as a living organism. Thus, this mode has to be excluded from consideration when any tree-ring chronology is created, and so the capability of tree-ring chronologies to reproduce super low-frequency climate-induced variations is hard limited by about one half of the tree-ring record length being analysed. This uniqueness was completely neglected by our predecessors who created millennial-long tree-ring chronologies on the base of the regional curve standardization.

By this reason, one may believe that all already published millennial tree-ring chronologies based on the regional curve standardization are essentially affected by biological and micro-environmental effects. Therefore, it is almost for certain that these chronologies are unreliable as sources of information about many-centennial changes of climate in the past.

Besides, some age-dependent peculiarities of the tree-ring index time series are induced by the well-known property of the tree growth heteroscedasticity. Taking this in mind, we propose two simple improvements of the preliminary pre-processing of the tree-ring width data. First of all, we propose to exclude some initial period of the juvenile tree growth (before the grand maximum of some innermost rings) from further consideration. Secondly, we propose to compute the age-dependent curve of the root-mean-square deviation of the mature parts of the tree-ring index time series, in order to normalize these index time series before the computation of their intrarecord covariation matrix. If this matrix would be computed by this way, its eigenvectors turn out to be of the shapes that are well-approximated by the trigonometric functions, and so the traditional Fourier basis seems to be useful to extract the main "modes" of the tree growth with a certain confidence. We suppose that such improved eigen analysis technique will be capable to reproduce climate-induced tree growth variations commensurate to the overall length of the tree-ring records being used.

Conclusion

Verifying the new technique of the eigen analysis of tree-ring records technique on a sample set of relative short-living larches growing in the Yamal Peninsula we confirm that the eigen analysis can be really applied to relative short-living trees sampled in cold and wet regions near the upper and northern timberline of the North Eurasia.

Considering an the essentially larger sample set of the long-living Chinese junipers, we also confirm all results of the eigen analysis obtained in our pioneering study published earlier (Yang et al., 2011, 2012a, 2012b). Namely, the regional curve does not include within itself all peculiarities of the tree growth depending from the age of trees. Some of these peculiarities remain to be inherent to the tree-index time series independently from the definition used to compute deviations of individual tree-ring width records from this regional curve.

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Решение проблемы собственных значений для построения реалистических дендрохронологий

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В метеорологии и климатологии широко используется решение так называемой проблемы собственных значений для ковариационной матрицы метеорологических полей, для того чтобы выявить «главные моды» изменчивости этих полей. В статье мы описываем приложение сходной проблемы для ковариационной матрицы рядов годичных колец деревьев, чтобы стандартизовать эти ряды. Впервые данная техника была приложена к выборке рядов очень долгоживущих можжевельников (Sabina przewalskii Kom.), произрастающих в районе Дулан Западного Китая, и описана в публикациях (Yang et al., 2011, 2012a, 2012b). Эта статья излагает наш доклад на Всероссийской конференции по дендрохронологии (РусДендро–2011), где мы представили наш анализ с некоторыми недавними усовершенствованиями и дополнительными приложениями этой техники к большей выборке долгоживущих китайских можжевельников, а также к другой выборке относительно недолгоживущих сибирских лиственниц, произрастающих на полуострове Ямал. Мы показываем, что наша техника анализа приложима к разнородным выборкам рядов толщин годичных колец деревьев и перспективна для построения реально значимых дендрохронологий.

Ключевые слова: проблема собственных значений, дендрохронология, тысячелетняя палеоклиматическая реконструкция, эффекты солнечной активности на рост деревьев.