Journal of Siberian Federal University. Humanities & Social Sciences 1 (2014 7) 4-19
~ ~ ~

УДК 113:530.145

Quantum Measure from a Philosophical Viewpoint

Vasil Penchev*
Bulgarian Academy of Sciences:
Institute for the Study of Societies and Knowledge
31 Neofit Rilski, 1000 Sofia, Bulgaria

Received 06.11.2013, received in revised form 29.11.2013, accepted 26.12.2013

The paper discusses the philosophical conclusions, which the interrelation between quantum mechanics and general relativity implies by quantum measure.
Quantum measure is three-dimensional, both universal as the Borel measure and complete as the Lebesgue one. Its unit is a quantum bit (qubit) and can be considered as a generalization of the unit of classical information, a bit. It allows quantum mechanics to be interpreted in terms of quantum information, and all physical processes to be seen as informational in a generalized sense. This implies a fundamental connection between the physical and material, on the one hand, and the mathematical and ideal, on the other hand. Quantum measure unifies them by a common and joint informational unit.
Quantum mechanics and general relativity can be understood correspondingly as the holistic and temporal aspect of one and the same, the state of a quantum system, e.g. that of the universe as a whole.

Keywords: measurement, quantum mechanics, general relativity, quantum information, entanglement.

The main question is how nothing (pure probability) can turn into something (physical quantity). The best idea is that both have a common measure. This problem is resolved implicitly in quantum mechanics introducing the formalism of wave functions, which are points in Hilbert space. Its approach can be equivalently represented explicitly in terms of quantum measure, i.e. by the notion of "qubit" defined and utilized in the theory of quantum information. The paper addresses the corresponding philosophical interpretation focused on the interrelation of quantum mechanics and general relativity, and thus on the problem of quantum gravity from a methodological viewpoint.

1. The Lebesgue (LM) and Borel (BM) measure

They will be discussed in relation to the axiom of choice ("AC", "NAC" = no axiom of choice) and the continuum hypothesis ("CH", "NCH" = no continuum hypothesis):

LM and BM coincide (AC, CH) Carathéodory's extension theorem as to Borel sets (BS) according to Carathéodory's extension

* Corresponding author E-mail address: decanka@mail.ru

theorem[1], and either can be distinguished only unconstructively (AC; CH or NCH) as to non-Borel sets (NBS) or cannot be juxtaposed at all (NAC; NCH). Then one can ascribe whatever difference including no difference. That incomparability is a typical situation in quantum mechanics and it represents the proper content of "complementarity". Here the Lebesgue measure is implied to be one-dimensional since the real line is such.

Dimensionality, LM, and BM: One must distinguish the dimensionality of the space being measured from the dimensionality of measure, by which the space is measured. The idea of probability as well as that of number in general is to be introduced a universal measure (quantity), by which all (pears, apples, distances, volumes and all the rest) can be measured as separately, item (quality) by item, as together. BM uses n-dimensional spheres, which it compares in radius independently of the number of dimensions. That radius is the Borel measure, and if it is finite, admits the Kolmogorov probability.

One can suppose, though counterintuitively, the case, where the dimensionality of the space being measured is lesser than that of the measure, and that such a case may have a nonempty intersection with NCH. The conjecture would not make much sense while one does not point out a universal measure of the dimensionality greater than one.

2. Quantum measure (QM) and its construction

QM is a three-dimensional universal one. A motivation may be for it to be introduced an as complete (like LM) as universal (like BM) measure. It should resolve the problem for completing BM in general (both AC & NAC):

An alternative, but equivalent approach is to be measured empty intervals (without any points in them), i.e. discrete or quantum leaps, in the same way as complete intervals of continuum. In fact quantum mechanics is what forced the rising of such a measure (& probability).

Moreover, quantum measure is more complete than LM in a sense or even is the most complete measure known to mankind since it can measure not only infinitely small empty, but any finite and even infinite leaps. However it postpones the question to complete them as no need to do it initially, on the one hand, and the general AC & NAC invariance even requires for the complete and incomplete case to be equated therefore rejecting the need of completion, on the other. That rather strange state of affairs is discussed in details below.

Given BM, the construction of QM is the following:

The objective is to be measured all the NBS as being reduced into some combination of the following three types partially complete:

NBS complete in relative complement;

NBS complete in countable union;

NBS complete in countable intersection.

A partial measure (or a partial probability as a finite measure) corresponds in each of the three cases above.

If a NBS is incomplete in one or more, or even in all of the three relation above, its corresponding measure(s) [probability (-es)] is (are) accepted as zero.

BM is the particular case where the three measures (probabilities) coincide. If a NBS is incomplete in any relation, it has a zero BM anyway. That backdoor is substantive for reconciling quantum theory based on QM and general relativity grounded on LM or BM in fact.

That kind of construction will be called tricolor hereinafter. The tricolor has exact correspondences in set theory and logic.

Let us now consider as an example of the case of tricolor or quantum probability compared with the classical one. One substitutes the unit ball for the interval of [0, 1]:

The unit ball can be decomposed in a "spin" way into two orthogonal circles.

The point of the unit ball generalizes that of [0, 1].

The point of the unit ball can be represented equivalently both as the two correlating complex numbers (the two projections on the orthogonal circles) and as three independent numbers (those of the tricolor above).

As the interval of [0, 1] allows of introducing the unit of classical information, a bit, as the unit ball does the same for quantum information, a qubit:

Since a bit can be thought as the alternative choice between two points: 0 or 1, a qubit might be thought as the choice of a point of the ball, i.e. as a choice among a continuum of alternatives in final analysis.

The [0, 1] is the universal measuring unit of all what can be classically measured. It can be illustrated as a "tape measure" for anything which is something, but not nothing. However the unit ball is a more universal measuring unit since it can measure as anything which is something as nothing in a uniform way. In other words, it can measure as the continuous as the discrete without completing the latter with the continuum of a continuous medium of points, i.e. without transforming nothing in something. Consequently, the unit ball is the perfect measure for quantum mechanics since aids it in resolving its main question, namely: How can nothing (pure probability) become something (physical quantity)?

Many philosophers reckon that the same kind of question, why there is something rather than nothing, is the beginning of philosophy, too. Quantum mechanics gives an answer, which is the single one that mankind has managed to reach and which, fortunately or unfortunately, is constructive besides.

Given LM, the construction of QM is the following:

One builds a tricolor measure as the BM for any dimension.

One might consider a "vector" measure, which components are 3D balls. In fact, it is equivalent both to Minkowski and to Hilbert space. That unit-ball vector represents a unit covariant vector, i.e. just a measure.

Any measure of the ball vector would be QM on LM. If the measure is the usual one of the vector length, the measured result would be a 3D ball rather than a 1D length. The axiom of choice does not use in that QM-on-LM construction.

Using the axiom of choice, a ball is equivalent to any set of balls, which is known as the Banach and Tarski paradox[2]. So, one need not construct a ball-vector measure as above since it is directly equal to a ball (i.e. QM) according to the axiom of choice.

The last two paragraphs show the original invariance of QM to the axiom of choice unlike LM and BM. As to BM that invariance is an undecidable statement. One might say that BM even possesses anyway a specific invariance or universality to the axiom of choice: the invariance of incompleteness: BM is incomplete as with the axiom of choice as without it. As to LM, it is complete without AC, but incomplete with AC: Indeed the construction of a Vitali set[3], which is immeasurable by LM, requires necessarily AC. At the same time, the way of its construction shows that any Vitali set is a subset of a null set such as that of all the rational numbers within the interval [0, 1] since there is a one-to-one constructing mapping between the Vitali set and that set of the rational numbers. Consequently LM under the condition of AC is incomplete since there is a subset of a null set, which is immeasurable: the Vitali set.

The consideration shows that LM occupies an intermediate position between the complete

QM and the incomplete BM being partly complete (without AC) and partly incomplete (with AC). Thus LM can also demonstrate AC as the boundary between potential and actually infinity. LM under condition of AC can measure anything which is finite, but nothing which is infinite. QM unlike it can measure both even under AC.

Thus the invariance of QM to the axiom of choice can be added to the motivation of QM since quantum mechanics needs such invariance: Really, quantum measuring requires the axiom of choice, and any quantum state by itself rejects it being due the "no hidden variables" theorems[4,5]. Consequently, the epistemological "equation", which equates any state "by itself" and the result of its measuring, needs that invariance in the case of quantum mechanics.

A problem remains to be solved (as if): Is there a BM or LM, with which no QM corresponds after utilizing the aforesaid procedure? The finite or infinite discrete leaps are described by QM unlike LM and BM: Consequently QM can be accepted as more general. However, are there cases, too, which admit BM or LM, but not QM?

Unfortunately that question is not one of abstract, purely mathematical interest since it is an interpretation of the quantum-gravity problem into the measure-theory language. General relativity uses LM, while quantum mechanics QM. If general relativity is true (as seems) and there is a LM (BM) which is not QM (LM-no-QM), then quantum gravity is an undecidable problem. Vice versa: Quantum gravity is resolvable if and only if QM is more general than (since it cannot be equivalent with) LM (BM).

A try for a short answer might be as follows:

The QM-on-LM construction excludes the LM-no-QM conjecture. However it cannot serve for refusing a nonconstructive proof of LM-no-QM existence in general.

Any pure proof of that kind, which requires necessarily the axiom of choice, can be neglected because of the QM invariance to AC/ NAC.

No other proof of pure LM-no-QM existence can be omitted, but whether there are such ones, no one knows. That pure existence is not only a question of abstract and theoretical interest. It suggests that a more general measure than QM can be ever found on the base of LM-no-QM.

One can suppose a new invariance to CH/ NCN similar to the QM invariance to AC/ NAC. In fact, it would be equivalent to the existence of a countable model for any mathematical structure of first order: This is a well-known direct corollary of the Löwenheim-Skolem theorem[6]. Thus that alleged as a new invariance would not expand out of QM, though. The reason is that CH implies AC.

However one can continue the implication of AC from CH in the following way: AC implies Skolem's "paradox"[7]: The latter implies the impossibility to be compared infinite powers and that CH/ NCH is undecidable for the sake of that. That is: CH implies the undecidability of CH/ NCH, but NCH does not imply that undecidability since cannot imply AC. All this is another argument in favor of QM and against LM-no-QM.

Anyway "QM & an undecidability of QM/ LM" satisfies almost all combinations of AC, CH, and their negations. Moreover it does not require LM-no-QM since LM and QM are complementary to each other where both AC & CH hold.

As to the problem of "quantum gravity", this means the following: Quantum gravity as supposing QM is consistent as with NCH and the AC/ NAC invariance as with CH & AC. However it is not consistent with CH & NAC, in the domain of which general relativity is built, unfortunately.

What about LM-no-QM in "CN & NAC"? Of course, one can construct QM on any LM there, too. That construction implies AC, and since NAC is valid there, the construction is forbidden, though. This is a very amazing state of affairs resembling the human rather than nature laws: QM is possible, but forbidden where general relativity is valid. After daring to construct QM in its territory, anyone turns out to be expulsed automatically in CH & AC where QM is admitted since it is complementary to LM and does not force LM to vanish.

What implies all that? Quantum gravity is a question of choice. One can create the theory as of quantum gravity as of general relativity, however ought to choose preliminary which of them. They should be equivalent to each other in a sense and can be thought as one and the same. Consequently general relativity can be reckoned as the cherished quantum gravity.

That is the case though it is very strange, even ridiculous. If and only if another and more general than QM measure be discovered so that the LM-no-QM be built constructively, then and only then general relativity and quantum gravity will be able to be distinguished effectively, i.e. experimentally. Vice versa: if an experimental refutation of general relativity be observed, a generalization of QM (GQM) will be implied: RIP both for Albert Einstein and for Niels Bohr since general relativity (LM) and quantum mechanics (QM) can be universal only together and reconciled. GQM will be able to resolve the dispute between them or will remove both when it comes. However we have not got any idea about GQM.

Finally, the example of BS can be used to illustrate how the strange kind of as if undecidability of CH to AC, and hence the relation of general relativity and quantum mechanics in terms of measure:

BS implies CH according to the Alexandroff – Hausdorff theorem[8]: Any uncountable BS has a perfect subset (and any perfect set has the power of continuum). However, CH implies AC in turn, and the latter does Skolem's "paradox", i.e. the incomparability (or more exactly, unorderability) of any two infinite powers. Consequently, BS can be consistent as with CH as with NCH since BS and CH are complementary in a sense. If the case is NCH, then AC is not implied and BS remains consistent as with CH as with NCH.

Of course, this should be so as BM is a particular case of QM, and the latter is consistent with NCH (as well as CH & AC).

All illustrate how it is possible for BS and BM to be consistent as with LM as with QM even where CH & NAC hold. That is the domain of general relativity, which should not exist if CH implies AC. Really CH implies AC only that AC implies the undecidabilty of CH or NCH, which allows of existing the area of general relativity.

One can abstract the logical relation of general relativity and quantum mechanics by means of the same one of LM and QM. Roughly speaking, they are complementary because of a similar complementarity of CH/ NCH and AC/ NAC rooted in the amazing or even paradoxical properties of infinity: AC supposes a single infinity which ought to be countable. However, both CH and NCH suggest an infinite set of sets which can be countable (CH) in turn, too.

That unordinary logical relation does not generate any contradictions. In fact, it contravenes only the prejudices. Anyway, we can attempt to explain and elucidate the reason of our confusion and misunderstanding:

Anything in our experience can be either an indivisible whole (a much) or divided in parts (a many): No "much" can be a "many" in the same moment and vice versa.

The above postulate is not valid as to infinity: It can be defined as that "much" which is a "many" or as that "many" which is "much".

Consequently it can be equally seen as a single "much" consisting of a "many" of parts (after AC) or as a many of indivisible wholes ("much"-s) (after CH / NCH).

To reconcile the two viewpoints onto infinity in a single illustration, one can utilize the image of cyclicality.

While anything else consists of something else and is not self-referential or cyclic, infinity is what consists just of it self-referentially or cyclically: Its "much" is forced to return back into it as many units.

AC suggests that cycle while CH or NCH unfold this cycle in a line. Consequently AC sees infinity as a well-ordering (line) bounded as a cycle while CH (or NCH) as many cycles well-ordered in a line.

Any contradictions between them do not arise since both are the same seen from opposite perspectives.

3. QM compared with LM and with BM

QM can be compared as with BM as with the LM to stand out its essence and features:

QM vs. BM:

Similarities:

– Both are supposed to be universal.

– Both generate probabilities where they are bounded.

– Both can be generated by BS.

– There is a common viewpoint, according to which QM can be considered as a three-dimensional or "tricolor" generalization of BM.

Differences:

– QM is complete, BM is not.

– QM is three-dimensional, BM is one-dimensional.

– BM can be considered as the particular case where the three dimensions of QM coincides.

***QM vs. LM*:**

Similarities:

– Both are complete under NAC.

– QM and LM correspond to each other "two-to-two", i.e. "± to ±" or in other symbols "square-to-square".

– No one of QM and LM can be deduced from the other or represented as a particular case of the other.

The differences from each other (see above) focus on a common 3D space where they vanish. One can utilize the metaphor of the two eyes or binocular sight for QM and LM.

Differences:

– QM is three-dimensional, while LM is of an arbitrary even infinite dimensionality.

– The dimensionality of QM does not correspond to that of the space measured, in general. They can be interpreted differently even in the particular case, where they coincide (three dimensions). The dimensionality of LM always coincides with that.

– QM is universal: It does not depend on the dimensionality of the space measured. LM is not universal: It does strictly correspond to the dimensionality of the space measured.

If one uses the metaphor of binocular sight for QM and LM, then their "global focus" is always in the "plane" of QM, while LM can represent the "local development or change" dimension by dimension.

4. The origin of QM

QM arose for quantum mechanics when Heisenberg's (1925) matrix mechanics[9] and Schrödinger's (1926a)[10] wave mechanics were united by the latter one[11].

Though Hilbert space guaranteed as a mathematical enough formalism, as John von Neumann showed[12], for the quantum mechanics, the sense of that guarantee as well as its attitude toward the two initial components, matrix and wave mechanics accordingly, remained misunderstood:

Heisenberg's matrix mechanics represented all quantum motions only as discrete rather than continuous or smooth.

Schrödinger's wave mechanics represented all quantum motions only as smooth rather than discrete, though.

Consequently the sense of quantum mechanics, which unites both by means of Hilbert space, is that, in fact, all quantum motions are invariant to the transition between the discrete and smooth.

However wave mechanics had advantage that it could represent that invariance in terms of the continuous and smooth, which terms were dominating for classical mechanics, though they were only prejudices, a legacy of the past, needless or even harmful:

The determent consisted in that the invariance of the discrete and smooth as to quantum motion remained tightly hidden in the mathematical apparatus of Hilbert space and accordingly misunderstood in physical interpretation.

The real sense of QM is to suggest a common measure both for the discrete and for the continuous and smooth so that to offer a suitable language for their invariance required by quantum mechanics.

The case of a (discrete) quantum leap measured by QM:

Any quantum leap can be decomposed in harmonics by Fourier transform:

Then any of those harmonics can be enumerated and considered as a QM for the n^{th} dimension of Hilbert space.

The n^{th} dimension of Hilbert space can be interpreted as a frequency or consequently, as an energy corresponding one-to-one to it.

The above construction shows the transition from real to complex Hilbert space and the transition from LM to QM as well. By the way the universality of QM is similar to that of complex numbers.

The case of a continuous or smooth physical motion measured by QM:

Since the continuous or smooth physical movement means a motion in Euclidean space, which is the usual three-dimensional one, it can be decomposed in successive 3D spheres or balls corresponding one-to-one as to all points of the trajectory in time as to the successive spheres or balls of the light cone in Minkowski space as well as to the successive dimensions of Hilbert space.

Consequently those points of the trajectory can be enumerated and considered as a QM for the n^{th} dimension of Hilbert space in an analogical way.

Now the n^{th} dimension of Hilbert space can be interpreted as a moment of time corresponding one-to-one to it.

The two above constructions show why QM is universal as well as the sense of that universality. Since frequency (energy) and time are reciprocal (or complementary in terms of quantum mechanics), then they can be juxtaposed as the two dual spaces of Hilbert space connected and mapped one-to-one by Fourier transform.

Max Born's probabilistic mechanics:

Max Born suggested in 1926[13] that the square of the modulus of wave function represents a probability, namely that of the state corresponding to that wave function. However somehow it was called the "statistical interpretation" of quantum mechanics. The term of "interpretation" used by Max Born himself as an expression of scientific modesty and politeness should not mislead. Its utilizing shows a complete misunderstanding of Max Born's conjecture and a yearning for its understatement. In fact it was not and is not an interpretation, but another, third form of quantum mechanics among and with matrix and wave mechanics. This is the cause for one to call it probabilistic mechanics (after the expressions of "wave mechanics" and "matrix mechanics" are common) rather than an interpretation.

Probabilistic mechanics shares Hilbert space with matrix and wave mechanics. However wave function (i.e. a point in Hilbert space) does not mean here a quantum leap decomposed in energies, neither a trajectory decomposed in time moments, but the characteristic function of a complex random quantity (or of two conjugate real quantity).

One should say a few words on the Fourier transform of a complex random quantity and on its characteristic function:

In fact their interrelation is quite symmetric and simple: The Fourier transform and the replacing of a complex random quantity by its conjugate swap the two dual Hilbert space.

Consequently the characteristic function of the conjugate of the complex random quantity is just the complex random quantity itself.

The interpretation of a complex random quantity and its conjugate is simple, too: Since a complex random quantity can be interpreted as two real conjugate (reciprocal) physical quantities such as e.g. time and frequency (energy), then the conjugate of the same random quantity must represent merely swapping between the corresponding physical quantities or the axes of the complex plane, or its rotation of $\pi/2$.

Probabilistic vs. matrix mechanics: If one compares them, the differences would be only two: in interpretation and in choice between NAC and AC.

However the wave function in both cases and despite the differences would be the same and the same point in Hilbert space. That sameness inspires invariance as to probabilistic vs. matrix "interpretation" as to NAC vs. AC.

Since the wave function is a sum of the measured by QM, one can reduce completely that invariance in terms of QM:

QM as quantum probability guarantees the former members, and it decomposed in dimensions (which are harmonics or energetic levels in the case) supplies the latter ones.

A philosopher would emphasize the extraordinary universality both of Hilbert space and of QM contradicting common sense:

Why and where exactly? QM is so universal that can measure both the unordered (and even unorderable in principle) and the well-ordered and thereof ordering it (them):

In our case it can measure and order quantum probabilities (for the unorderable in principle) and quantum leaps (for the well-ordered in harmonics or energies), and therefore QM establishes a one-to-one mapping between quantum probabilities and quantum leaps:

That one-to-one mapping is too shocking to the prejudices. *It shows that a level of energy corresponds exactly to a quantum probability*: That is a physical quantity (what is the former) can be equated with a real number being without any physical dimensionality (what is the latter):

However this is what has been necessary for the objectives declared in the beginning of the paper: to demonstrate how QM allows of becoming nothing to something or vice versa and *eo ipso creatio ex nihilo* or *reductio ad nihilum* (i.e. true creation or true annihilation).

Probabilistic vs. wave mechanics: All what has been said above about the links between probabilistic and matrix mechanics can be almost literally repeated again in that case. The immaterial differences are as follows:

– The dual Hilbert space replaces its dual counterpart.

– The well-ordering in time replaces that in frequency (energy).

The one-to-one mapping based on QM establishes now a correspondence of a wave function as quantum probability with a continuous or smooth trajectory in time.

A threefold (even fourfold) one-to-one mapping arises thereof: It states invariance or equivalence in a sense between the quantum leaps (for the discrete), the smooth trajectories in time (for the continuous) and the quantum probabilities (for the unorderable in principle).

That threefold mapping shows how pure numbers even only the positive integers (for "nothing") can generate physical quantities in pairs of ("reciprocal") conjugates such as frequency (energy) and time. The stages of that generation are as follows:

– Nothing.

– The positive integers are given somehow, maybe by God as Leopold Kronecker reckoned: "Die ganzen Zahlen hat der liebe Gott gemacht, alles andere ist Menschenwerk"[14]:

– Creation: Qubits (or QM) replaces each of the positive integers generating Hilbert space.

– The Hilbert space generates that threefold mapping between quantum probability, energy and time and thereof the physical world arises already, too.

Quantum mechanics seen as the unification of all three kinds of mechanics: probabilistic & matrix & wave mechanics:

Quantum mechanics is better to be understood as the unification of all the three types of mechanics listed above instead only of the last two.

The sense of that unification is the extraordinary invariance (or equivalence in a sense) of the discrete, continuous (smooth) and the probabilistic in the common form of quantum motion:

Quantum motion can be already thought as a relation between two or more states despite whether each of them is considered as a discrete, continuous (smooth) or probabilistic one since it is always represented by one the same wave function in all the three cases:

This calls for far-reaching philosophical conclusions, though:

The difference not only between the discrete and continuous (smooth), but also that between both and the probabilistic is only seeming and accidental or even anthropomorphic in a sense.

Quantum motion breaks down their barriers and allows any transitions between them.

The probabilistic can be located "between" the discrete and continuous (smooth) and can be considered as something like a substance of that kind of transition. Accordingly the discrete and continuous can be supposed as the two extreme or particular cases of the probabilistic, which are opposite to each other, and that is not all:

What is the physically existing according to common sense can be linked only to those two extremes. Physical reality ostensibly consists just of (and in) both since they are all the actual.

According to the same common sense the probabilistic cannot be physically real since is not actual: However quantum mechanics shows that is the case, the probabilistic is physically real: "So much the worse for quantum mechanics because this means that it is incorrect or at least incomplete", declared the common sense then. Quantum mechanics rather than that "common sense" turns out to be the right again, though, experimentally verified[15].

If quantum mechanics is the right, what does it mean about the philosophical interrelation between reality and "virtuality"?

"Virtuality" is a term coined here to denote just that new class required by quantum mechanics and involving both reality, i.e. the discrete and continuous (smooth), and "only" (ostensibly) the possible so that to allow of any transfer between them.

Consequently virtuality is a term for the new constitution of being, according to which the barriers between the actual and the possible are broke down and all the kind of transitions between them are unrestrained.

Thus virtuiality established by quantum mechanics can resolve our properly philosophical (and even theological) problem about *creatio ex nihilo* or *reductio ad nihilum*: The area of probability can describe very well both those *creatio* and *reductio* as states and processes: One can see the actual in creating or annihilating rigorously mathematically, i.e. in the process of creation or annihilation, as the change of probability.

Not less striking is that the new "constitution" of virtuality suggests for mathematics to be more general than physics if the latter is defined and restricted only to the actual; or in other words, mathematics and a new and more general physics can and even should coincide.

How to interpret the fermion and boson kind of spin statistics in the light of that unification?

According to the so-called spin-statistics theorem[16] all the quantum particles can be divided into two disjunctive classes after the second quantization: fermions and bosons:

Since the second quantization maps the wave functions of the quantum particles "two-to-two", it admits two kinds of solving as to a swap of the space-time positions of two quantum particles: symmetric (++, −−) and antisymmetric (+−, −+).

The bosons are supposed to be those of symmetric swap, and the fermions are those of the antisymmetric swap. Turns out the any number of bosons can share one the same state and wave function while if they are fermions, only two.

The following can be easily spotted: Quantum probabilistic mechanics explains very well that property as to the bosons, and quantum matrix-wave mechanics explains it not less successfully as to the fermions:

Indeed the opportunity of sharing a common state or wave function by the bosons is due to the sharing of a common probability by an arbitrary ensemble of quantum particles. That ensemble, which possibly consists of an infinite number of elements, is supposed ***not*** *to be well-ordered*.

The same ensemble already well-ordered can be distinguished in two kinds of well-ordering corresponding to the two fermions admitted in one the same state or wave function. The one is well-ordered ***to***, and the other ***from*** infinity. If the ordering is in time and energy, the one fermion as if corresponds to the discrete "half" of wave-particle duality, and the other accordingly to its continuous (smooth) "half".

Hence one can clearly see that the second quantization giving rise to spin statistics either is equivalent to, or is a particular case of that quantum mechanics, which includes as probabilistic as matrix and wave mechanics. Indeed, the sense of the second quantization is to be defined "quantum field". In fact, this is done by ascribing a wave function (i.e. a quantum state) to each space-time point. That quantum mechanics, which includes as probabilistic as matrix and wave mechanics, ascribes a space-time point to each wave function. Then:

If the quantum field is well-ordered, then the mapping between all the wave functions (Hilbert space) and all space-time points (Minkowski space) is one-to-one, and the second quantization is equivalent to that quantum mechanics, which involves probabilistic mechanics. Then any quantum particle must necessarily be either a boson or a fermion.

If the quantum field is not well-ordered, it admits two opposite options as well as both together:

Two or more space-time points to share one the same wave function, and thus the reverse mapping not to be well-defined: It will not be a standard function.

Two or more wave functions to share one the same space-time point, and thus the straight mapping not to be well-defined: It will not be a standard function.

However the most general case is two or more space-time points to share two or more wave functions (i.e. together both above). If that is the case, it can be equally described as some space-time points, which share a part of some wave functions (entanglement) or as wave functions, which share a "part" of some space-time points ("quantum" gravity). The interrelation or equivalence of entanglement and gravity is being studied.

If the quantum field is not well-ordered as above, it can be represented in a few ways (as well as in their combinations or mappings):

– As a curving of Hilbert to Banach space.

– As a curving of Minkowski to pseudo-Riemainian space.

– As quantum particles with an arbitrary spin: such which can be any real number.

The transitions between the "probabilistic" wave function and "well-ordered" wave function in any of the above ways describe in essence the arising of "something from nothing and from time" ("time" is for the axiom of choice) as a continuous process as a quantum leap as well as a purely informational event.

One can give examples of that arising in terms of classical (gravity) or quantum (entanglement) physics as a continuous process.

Quantum mechanics, QM and AC:

A question may have remained obscure: Does quantum mechanics need AC?

Quantum mechanics is actually the only experimental science, which requires necessarily AC: Roughly speaking, the state before measuring has not to be well-ordered, but after that it has to. This means that measurement supposes the well-ordering theorem, which is equivalent to AC:

The "no hidden variables" theorems do not allow of well-ordering before measuring.

However even only the record of the measured results (which is after measuring, of course) forces they to be well-ordered.

The basic epistemological postulates equate the states before and after measurement and thus imply AC.

Though measuring requires the AC, it remains inapplicable before measuring. Consequently quantum mechanics is ought to be consistent both with AC and NAC in addition.

The only possible conclusion is too extraordinary: Quantum mechanics is consistent as with AC as with NAC. However, quantum mechanics is not consistent with the absence as of AC as of NAC.

We could see above that QM is linked to AC in the same extraordinary way. This means that quantum mechanics is consistent with QM as to AC, which should expect.

Quantum mechanics, QM and the CH:

That extraordinary interrelation between quantum mechanics, QM and AC goes on with CH:

NCH is consistent as with AC as with NAC, thus quantum mechanics is consistent with NCH.

Reversely, CH should (ostensibly) imply AC. However AC implies the undecidability of CH and NCH, then CH implies by means of AC the own undecidability. The only way out is then to admit the complementarity of CH and AC.

Both quantum mechanics and QM share that extraordinary relation to CH by means of AC.

Though the state of affairs is strange, it is not logically contradictory. It messes up only common sense. The cause of that ostensible muddle is the intervention of infinity, of which we try to think as of a finite entity.

5. Physical quantity measured by QM or by LM, or by BM

The definition of physical quantity in quantum mechanics involves measuring by QM. It is a generalization of the corresponding notion in classical physics and exact science.

One can see the quantum at all as that generalization from LM and BM to QM. The correspondence is the following:

A few conclusions can be drawn from that correspondence:

The sense of a point of dual Hilbert space is to be a "unit", or something like a reference frame, which can measure a point of Hilbert space.

The measured value represents the distance between the "unit" point (or its conjugate point, too) and another point, which is for the measured quantity. This distance can be thought of also as a distance in the "reference frame" of that point.

Both cases are "flat": They conserve the measure under translation and rotation. If the translation and rotation are understood as usual as translation and rotation in space-time, then that "flatness" implies the classical laws of conservation complemented by Lorentz invariance. One can especially emphasize the time translation and energy conservation.

The "flatness" in general can be equated with the axiom of choice. Indeed the well-ordering requires that flatness since otherwise a second dimension for ordering appears questioning the well-ordering made only in the first dimension.

The above table (1) can be paraphrased in terms of the "crooked" as the following table 2 asking how both tables can refer to each other:

The "crooked" case is that of general relativity and gravity. The question for the connection between the two tables addresses the problems of quantum gravity in terms of general relativity and measure theory.

To be together in front of the eyes, one can combine the two tables as a new one (3):Table 3:

The above table (3) shows that the problem of quantum gravity is a problem of measure: It concerns the ostensibly contradictory properties of infinity focused on how AC and CH should refer to each other.

The gravity being as "crooked" as smooth in general relativity supposes the "classical" case of NAC and CH. However as CH implies AC, it should not exist. After all it arises anyway since AC in turn implies the undecidability between HC and NHC. In last analysis it concerns the property of infinity to be both cyclic and linear unlike anything in our usual experience.

Table 1. From LM and BM to QM

	Quantity	**Unit**	**Value**
The classical case	Real quantity	Unit	Real number
The quantum case	Wave function	Conjugate wave function	Self-adjoint operator

Table 2. Table 1 paraphrased

	Quantity	Unit	Value
The classical case	Real quantity	Unit	Real number
The "crooked" case	Contravariant vector	Covariant vector	Metric tensor

Table 1 and Table 2 unified

	Quantity	**Unit**	**Value**
The classical case	Real quantity	Unit	Real number
The quantum case	Wave function	Conjugate wave function	Self-adjoint operator
The gravity case	Contravariant vector	Covariant vector	Metric tensor
Quantum gravity	???	???	???

One can think of the quantum and gravity case as complementary. In particular this means that the values of a quantity as a self-adjoint operator or as a metric tensor are complementary, too, as well as QM and the "crooked" LM therefore that QM and LM are even equivalent in the distinctive way of quantum mechanics

Just this complementarity of QM and LM is taken in account as to NCH, which is consistent as with AC as with NAC, and for this, with the curious invariance of AC and NAC.

For the above three as to the CH case, one can be free to suggest that it exactly repeats the NCH case in relation to AC and NAC because of the undecidability between CH and NCH after AC. This would means that any theory of quantum gravity is not necessary since the pair of quantum mechanics and general relativity can represent whatever the case is.

The following should be highlighted in background of the just said: In the same extent, one can be free to admit the opposite: That is the CH case does not repeat the NCH one just because of the used undecidability between CH and NCH after AC. This will say that a theory of quantum gravity is possible though it will be never necessary.

One can compare with the real state of affairs: Indeed many theories of quantum gravity appear constantly and supposedly some of them do not contradict the experiments just because they are possible. However they are not necessary in principle since general relativity does not contradict the experiments, too, and "Occam's razor" removes all of them remaining in hand only general relativity.

The perspective

Is there any measure more general and universal than QM? If there is, it could be called "generalized quantum measure" (GKM). According to current knowledge, we cannot even figure what might cause such a measure to be introduced or what would constitute.

One can postulate the absolute universality of QM. This implies a series of philosophical conclusions and new interpretations of well-known facts. The most of them have been already mentioned in a slightly different context above. What is worth to emphasize here is the following:

A universal measure as QM suggests that all entities are not more than different forms of a substance shared by all of them as their fundament: It is quantum information and represents a general quantity, which is both mathematical and physical in its essence. The longstanding philosophical idea of a single and general substance can be already discussed in terms of exact science.

1 Carathéodory, 1956, 149.
2 Banach and Tarski, 1924.
3 Vitali, 1905.
4 Neumann, 1932, 167-173.

[5] Kochen and Specker 1968, 70.
[6] Löwenheim, 1915; Skolem, 1919a; Skolem, 1919b.
[7] Skolem, 1923.
[8] Alexandroff, 1916; Hausdorff 1916; cf. Sierpiński 1924.
[9] Heisenberg, 1925.
[10] Schrödinger, 1926a.
[11] Schrödinger, 1926b.
[12] Neumann, 1932, 18-100.
[13] Born 1926a; Born, 1926b; Born 1927a; Born 1927b; Born and Fock 1928; Born 1954.
[14] Cited in: Weber, 1893, 19.
[15] Bell 1964; Clauser and Horne 1974; Aspect et al, 1981; Aspect et al, 1982.
[16] Fiertz, 1939; Pauli 1940.

References

1. Alexandroff, P. (1916). On the power of measurable (B) sets [Sur la puissance des ensembles measurable (B)]. *Comptes rendus hebdomadaires des séances de l'Académie des sciences*, 162 (28.02.1916), 323-325.

2. Aspect, A., Grangier, P., Roger, G. (1981). Experimental tests of realistic local theories via Bell's theorem. *Physical Review Letters*, 47 (7), 460-463. doi:10.1103/PhysRevLett.47.460

3. Aspect, A., Grangier, P., Roger, G. (1982). Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedanken Experiment: A New Violation of Bell's Inequalities. *Physical Review Letters*, 49 (2), 91-94. doi:10.1103/PhysRevLett.49.91

4. Banach, S. and Tarski, A. (1924). On the decomposition of sets of points into parts respectively congruent [Sur la decomposition des ensembles de points en parties respectivement congruentes]. *Fundamenta Mathematicae,* 6 (1), 244-277.

5. Bell, J. (1964). On the Einstein – Podolsky – Rosen paradox. *Physics* (New York), 1 (3), 195-200.

6. Born M. (1926a). On the quantum mechanics of shock events [Zur Quantenmechanik der Stoβvorgänge]. *Zeitschrift für Physik*, 37,(12/ December), 863-867.

7. Born, M. (1926b). The quantum mechanics of shock events [Quantenmechanik der Stoβvorgänge]. *Zeitschrift für Physik*, 38 (11-12/ November), 803-827.

8. Born, M. (1927a). The adiabatic principle in quantum mechanics [Das Adiabatenprinzip in der Quantenmechanik]. *Zeitschrift für Physik*, 40, (3-4/ März), 167-192. doi: 10.1007/BF01400360

9. Born, M. (1927b). Physical aspects of quantum mechanics. *Nature*, 119 (05.03.1927), 354-357. doi:10.1038/119354a0

10. Born, M. (1954). *The statistical interpretation of quantum mechanics* (Nobel Lecture, December 11, 1954). Available at: http://nobelprize.org/nobel_prizes/physics/laureates/1954/born-lecture.pdf (accessed 23 November 2013).

11. Born, M. and Fock, V. (1928). Proof of the adiabatic theorems [Beweis der Adiabatensatzes] . *Zeitschrift für Physik A: Hadrons and Nuclei*, 51 (3-4), 165-180. doi: 10.1007/BF01343193

12. Carathéodori, C. Measure and integral and their algebraization [*Mass und Integral und ihre Algebraisierung*]. Basel, Birkhäuser, 1956. 337 p.

13. Clauser, J. and Horne, M. (1974). Experimental consequences of objective local theories. *Physical Review D*, 10 (2), 526-535. doi:10.1103/PhysRevD.10.526

14. Fiertz, M. (1939). On the relativistic theory of force-free particles with arbitrary spin [Über die relativistische Theorie kräftefreier Teilchen mit beliebigem Spin]. *Helvetica Physica Acta*, 12 (1), 3-17. doi:10.5169/seals-110930

15. Hausdorf, F. (1916). The power of Borel sets [Die Mächtigkeit der Borelschen Mengen]. *Mathematische Annalen*, 77 (3), 430-437. doi:10.1007/BF01475871

16. Heisenberg, W. (1925). On the quantum-theoretical interpretation of kinematic and dynamic relations [Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen]. *Zeitschrift für Physik*, 33 (1), 879-893. doi: 10.1007/BF01328377

17. Löwehheim, L. (1915). On possibilities in the calculus of relatives [Über Möglichkeiten im Relativkalkül]. *Mathematische Annalen*, 76 (4), 447-470. doi: 10.1007/BF01458217

18. Kochen, S. and Specker, E. (1968). The problem of hidden variables in quantum mechanics. *Journal of Mathematics and Mechanics*, 17 (1), 59-87. doi:10.1007/978-94-010-1795-4_17

19. Neumann, J. Mathematical foundation of quantum mechanics [*Mathematische Grundlagen der Quantenmechanik*]. Berlin, Verlag von Julius Springer, 1932. 262 p.

20. Pauli, W. (1940). The Connection Between Spin and Statistics. *Physical Review*, 58 (8/October), 716-722. doi: 10.1103/PhysRev.58.716

21. Schrödinger, E. (1926a). An Undulatory Theory of the Mechanics of Atoms and Molecules. *Physical Review*, 28 (6), 1049-1070. doi: 10.1103/PhysRev.28.1049

22. Schrödinger, E. (1926b). On the relation the Heisenber-Bohr-Jordan quantum mechanics to mine [Über das Verhältniss der Heisenberg-Born-Jordanischen Quantenmechanik zu der meinem]. *Annalen der Physik*, 384, (8), 734-756.

23. Sierpiński, W. (1924). On the power of the measurable (B) sets [Sur la puissance des ensembles measurable (B)]. *Fundamenta Mathematicae*, 5 (1), 166-171.

24. Skolem, T. (1919a). Investigation on the axiom of the calculus of classes and on the problem about the products and sums, to which some classes of propositions refer [Untersuchung über die Axiome des Klassenkalküls und über Produktations- und Summationsprobleme, welche gewisse Klassen von Aussagen betreffen]. *Skrifter utgit av Videnskabsakademiet i Kristiania*, I (3), 37-74.

25. Skolem, T. (1919b). Logical and combinatoric investigations on the completeness and provability of mathematical statements in relation to a theorem about dense sets [Logisch-kombinatorische Untersuchungen über die Erfüllbarkeit und Beweisbarkeit mathematischen Sätze nebst einem Theoreme über dichte Mengen]. *Skrifter utgit av Videnskabsakademiet i Kristiania*, I, (3), 1-36.

26. Skolem, T. A few notes to the axiomatic foundation of set theory [Einige Bemerkungen zur axiomatischen Begründung der Mengenlehre]. *Matematikerkongressen i Helsingrofs den 4-7 Juli 1922, Den femte skandinaviska matematikerkongressen, Redogörelse*, Helsinki: Akademiska-Bokhandeln, 1923, pp. 217-232.

27. Weber, *H.* (1893). Leopold Kronecker [Leopold Kronecker]. *Jahresbericht der Deutschen Mathematiker-Vereinigung*, 2 (1891 – 1892), 5-31.

28. Vitali, G. On the problem of the measure of the set of points of a straight line [*Sul problema della misura dei gruppi di punti di una retta*]. Bologna, Gamberini and Parmeggiani , 1905, pp. 231-235.

Квантовое измерение с философской точки зрения

Васил Пенчев
Болгарская академия наук
Институт исследований обществ и знаний
Болгария, 1000 София, Неофит Рильски, 31

В данной статье представлены философские выводы о том, что квантовое измерение предполагает взаимосвязь между квантовой механикой и общей теорией относительности.
Квантовое измерение является трехмерным, таким же универсальным, как мера Бореля, и таким же полным, как мера Лебега. Единица квантового измерения – бит (кубит) – может рассматриваться как генерализация единицы классической информации, бита. Квантовое измерение позволяет интерпретировать квантовую механику в рамках квантовой информации, и все физические процессы рассматриваются как информационные в обобщенном смысле, что предполагает фундаментальную взаимосвязь между физическим и материальным, с одной стороны, и математическим и идеальным – с другой. Квантовое измерение объединяет их посредством одной объединенной информационной единицы.
Квантовая механика и теория общей относительности могут пониматься совместно как целостный и временный аспект одного и того же, состояние квантовой системы, т.е. состояние Вселенной в целом.

Ключевые слова: измерение, квантовая механика, теория общей относительности, квантовая информация, перепутывание.