Journal of Siberian Federal University. Mathematics & Physics 2012, 7(1), 124–131

УДК 550.383

Three-dimensional Dynamics of the Earth's Radiation Belt Protons During the Magnetic Storm

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Received 28.10.2013, received in revised form 08.11.2013, accepted 20.12.2013

The non-stationary three-dimensional mathematical model describing evolution of proton distribution function is offered. The distribution of the radiation belt protons on the dayside of the Earth's magnetosphere during the magnetic storm is studied. The temporal and spatial evolution of the proton phase space densities in a dipole field is calculated using a three-dimensional model, considering radial, pitch angle and energy diffusions. The loss terms are described due to charge exchange and wave-particle interactions. The simulation starts with a quiet time distribution. The model is tested by comparing calculated proton fluxes with Polar/MICS measurement during the magnetic storm on 21–22 October 1999 and with AMPTE/CCE measurement during the magnetic storm on 2 May 1986. The good consent of the model pitch angle distributions of protons with the experimental data is received.

Keywords: protons, diffusion, magnetic storm, the Earth's magnetosphere.

Introduction

During the magnetic storm, energetic plasmas are injected on the nightside from a boundary near the geosynchronous orbit. In response to the convection electric field, these particles drift inward and are trapped by the geomagnetic field and form the storm time radiation belts. The purpose of the work is further development of three-dimensional models. Therefore, the distribution of the Earth's radiation belt protons during the main phase of a magnetic storm has been studied, using the offered non-stationary three-dimensional mathematical model.

1. The mathematical model

The 3D Fokker-Planck equation for the phase space density, describing radial, pitch angle and energy diffusions, charge exchange and due to wave-particle interactions losses, can be expressed [1–6] by

$$\begin{gathered}\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left(L^{-2} D_{LL} \frac{\partial f}{\partial L}\right) + \frac{1}{\sin\alpha}\frac{\partial}{\partial \alpha}\left(\sin\alpha D_{\alpha\alpha}\frac{\partial f}{\partial \alpha} - \sin\alpha \frac{d\alpha}{dt} f\right) + \\ + \frac{1}{\sqrt{E}}\frac{\partial}{\partial E}\left(\sqrt{E} D_{EE}\frac{\partial f}{\partial E}\right) - \lambda \cdot f - \frac{f}{T_{wp}} + S_{\perp}\sin^2\alpha \cdot f.\end{gathered} \tag{1}$$

Here, f is the phase space density (referred to as distribution function), t is the time, L is the McIlwain parameter, α is the local pitch angle, E is the kinetic energy, D_{LL} is the radial diffusion

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coefficient, $D_{\alpha\alpha}$ is the pitch angle diffusion coefficient, D_{EE} is the energy diffusion coefficient, $\dfrac{d\alpha}{dt}$ is the pitch angle rate, λ is the loss rate for protons against neutralization by charge exchange, T_{wp} is the lifetime due to wave-particle interactions, $S_{\perp}$ is the perpendicular coefficient of the particle source function.

The full description of the equation (1) is presented in [2–6], excepting the expression with the coefficient of diffusion on energy. Therefore, the energy diffusion coefficient is offered as follows:

$$D_{EE} \approx 4000\sqrt{E}L^{-4.5}, L \leq 4.1 \tag{2}$$

and

$$D_{EE} \approx 287\sqrt{E}L^{-4.2}, L > 4.1, \tag{3}$$

where D_{EE} is in units of keV^2/d [7].

The lifetime due to wave-particle interactions is given by the formula [3, 5, 6]

$$T_{wp} = k_T \frac{2R_E L^4 \sqrt{4L-3}\sqrt{m}}{\sqrt{2EL}} \left(1 - 0.15Kp\right), \tag{4}$$

where k_T is the non-dimensional parameter, R_E is the radius of the Earth, m is the mass of a charged particle and Kp is the index of geomagnetic activity.

In the further we shall plot not the distribution functions of f but the corresponding dependencies of differential fluxes (energy spectra) of protons j. It is such a representation that is more convenient for experimental data. For that reason, we shall be using the following relation between the differential flux of particles j and the phase space density f (or the distribution function) $j = 2mEf$, where m is the mass of a proton in this paper.

Then the equation (1) with (2)–(4) is solved numerically using the finite element projection method.

2. Calculations

The simulation starts with the quiet time conditions [8, 9]. The ion composition compiled by Sheldon and Hamilton [8] during the quietest days in 1985 – 1987, seen by the AMPTE/CCE/charge-energy-mass (CHEM) instrument in near-equatorial orbit at $L = 2-9$ R_E, is used as initial distribution before storm onset. This data set provides the average differential ion fluxes in an energy range 1 – 300 keV.

The distribution of the radiation belt protons during the magnetic storm, similar to that on 21 – 22 October 1999 [10] and on 2 May 1986 [9], is studied. The Kp index is shown in Fig. 1 as a function of the run time RT during the magnetic storm on 21 – 22 October 1999 (0000 RT = 0613 UT on 21 October 1999, 2000 RT = 0213 UT on 22 October 1999 — solid line) and during the magnetic storm on 2 May 1986 (0000 RT = 0200 UT on 2 May, 2000 RT = 2200 UT on 2 May 1986 — dashed line).

For protons with $E = 100$ keV, $L = 5$, $\alpha = 90°$, $Kp = 6$ the coefficient of pitch angle diffusion $D_{\alpha\alpha}$ is approximately equal $5 \cdot 10^{-6}$ 1/s [9]. Then using (4) and [9] it is received $k_T \approx 359$. Assuming the approached validity of the received value k_T and for protons with energy $E = 0$ keV, we use this value in further calculations.

The calculations on offered non-stationary three-dimensional mathematical model have been executed for a full range of pitch angles from 0° up to 180° at distances $L = 2.26-6.6$ in the energy range of protons $E = (85-95)$ keV for a real magnetic storm during its 20 hours 30 minutes. In particular, Fig. 2 compares the model proton pitch angle distributions with the pitch angle distributions observed by Polar/MICS (circles) for $E = (80-100)$ keV, $L = 5$ and by AMPTE/CCE (squares) for $E = (100-154)$ keV, $L = 3.97-4.53$. Circles and squares indicate

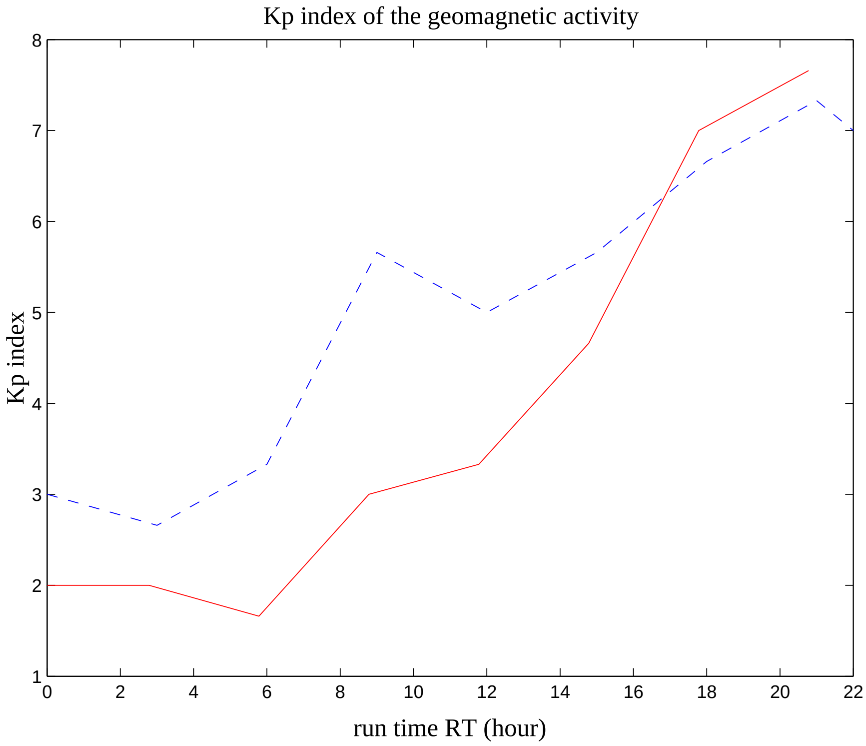

Fig. 1. Kp index as a function of the run time RT during the magnetic storm on 21 – 22 October 1999 (0000 RT = 0613 UT on 21 October 1999, 2000 RT = 0213 UT on 22 October 1999 — solid line) and during the magnetic storm on 2 May 1986 (0000 RT = 0200 UT on 2 May, 2000 RT = 2200 UT on 2 May 1986 — dashed line)

the differential flux at 0000 RT = 0613 UT on 21 October 1999 (prestorm condition) and 1800 RT = 2000 UT on 2 May 1986 (storm main phase), respectively. Dashed and solid lines indicate the model differential flux ($L = 5$, MLT = 0900, $E = 90$ keV) at 0000 RT and ($L = 4$, MLT = 0900, $E = 90$ keV) at 1800 RT for the magnetic storm on 21 – 22 October 1999.

Fig. 3 compares the model pitch angle distributions of protons and pitch angle distributions measured by AMPTE/CCE (squares) for $E = (100 - 154)$ keV, $L = 5.51 - 6.00$, MLT = 0800. Squares indicate the differential flux at 1800 RT = 2000 UT on 2 May 1986 (storm main phase). Dashed and solid lines indicate the model differential flux ($L = 6$, MLT = 0900, $E = 90$ keV) at 0000 RT and 1800 RT for the magnetic storm on 21–22 October 1999, respectively. The pitch angle distribution is pancake-like in the prestorm condition, while it becomes head-and-shoulder (or cap)-like in the storm main phase. The same tendency is seen in, for example, [11].

In total the results of calculations are presented in Fig. 4 and Fig. 5.

Fig. 4 summarizes the model evolution of the proton pitch angle distributions for $E = (85 - 95)$ keV, MLT = 0900, $L = 2.26 - 6.6$ at 0000 RT = 0613 UT on 21 October 1999 (prestorm condition) and 0600 RT = 1213 UT.

Fig. 5 summarizes the model evolution of the proton pitch angle distributions for $E = (85 - 95)$ keV, MLT = 0900, $L = 2.26 - 6.6$ at 1200 RT = 1813 UT on 21 October and 1800 RT = 0013 UT on 22 October 1999 (storm main phase).

Thus, Fig. 4 and Fig. 5 show the temporal variation of the simulated pitch angle distributions of protons for four selected times. At 0000 RT = 0613 UT on 21 October 1999 (prestorm condition), the pitch angle distributions are normal or pancake because the simulation started

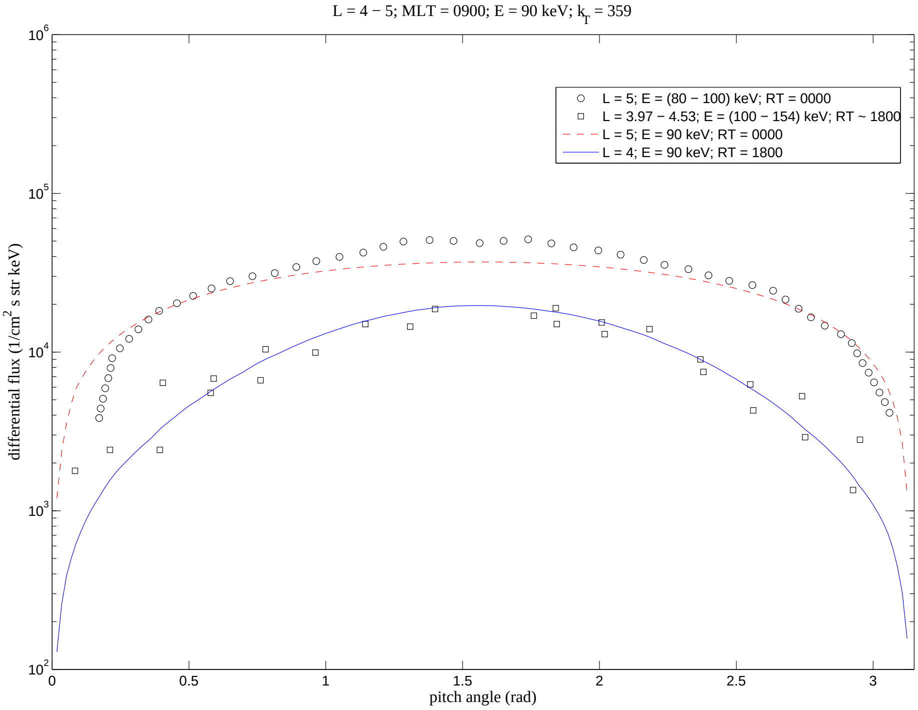

Fig. 2. Model pitch angle distributions of protons and distributions measured by Polar/MICS (circles) for $E = (80 - 100)$ keV, $L = 5$ and by AMPTE/CCE (squares) for $E = (100 - 154)$ keV, $L = 3.97 - 4.53$. Circles and squares indicate the differential flux at 0000 RT = 0613 UT on 21 October 1999 (prestorm condition) and 1800 RT = 2000 UT on 2 May 1986 (storm main phase). Dashed and solid lines indicate the model differential flux ($L = 5$, MLT = 0900, $E = 90$ keV) at 0000 RT and ($L = 4$, MLT = 0900, $E = 90$ keV) at 1800 RT for the magnetic storm on 21 – 22 October 1999

with the normal pitch angle distributions as an initial condition. As time proceeds, the proton flux at pitch angles near 90° increases, while that at pitch angles near 0° and 180° decreases. Finally, the pitch angle distributions become head-and-shoulder (or cap)-like (storm main phase). This tendency is well consistent with the observed head-and-shoulder pitch angle distributions for high-energy protons and electrons on the dayside of the Earth's magnetosphere [e.g., 11–13].

3. Conclusion

1. The non-stationary three-dimensional mathematical model describing the evolution of proton distribution function for a range of pitch angles from 0° up to 180° at distances $L = 2.26 - 6.6$ in the energy range of protons $E = (85 - 95)$ keV is offered.

2. The pitch angle distribution of the Earth's radiation belt protons ($E = (85 - 95)$ keV) in the inner magnetosphere ($L = 2.26 - 6.6$, MLT = 0900) during the magnetic storm on 21 – 22 October 1999 and the magnetic storm on 2 May 1986 is studied.

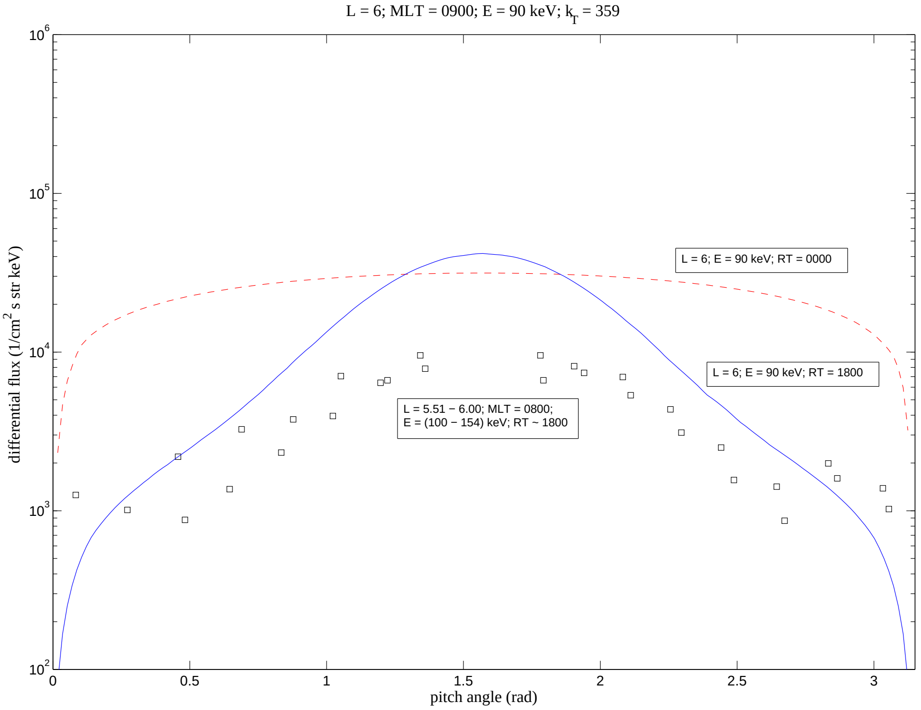

Fig. 3. Model pitch angle distributions of protons and distributions measured by AMPTE/CCE (squares) for $E = (100–154)$ keV, $L = 5.51–6.00$, MLT $= 0800$. Squares indicate the differential flux at 1800 RT $=$ 2000 UT on 2 May 1986 (storm main phase). Dashed and solid lines indicate the model differential flux ($L = 6$, MLT $= 0900$, $E = 90$ keV) at 0000 RT and 1800 RT for the magnetic storm on 21–22 October 1999

3. The pitch angle distribution is pancake-like in the prestorm condition, while it becomes head-and-shoulder (or cap)-like in the storm main phase on the dayside of the Earth's magnetosphere.
4. The good consent of the model pitch angle distributions of protons with the experimental data observed by the Polar/MICS satellite and by the AMPTE/CCE satellite is received.

The author thanks A.S. Smolin for technical help.

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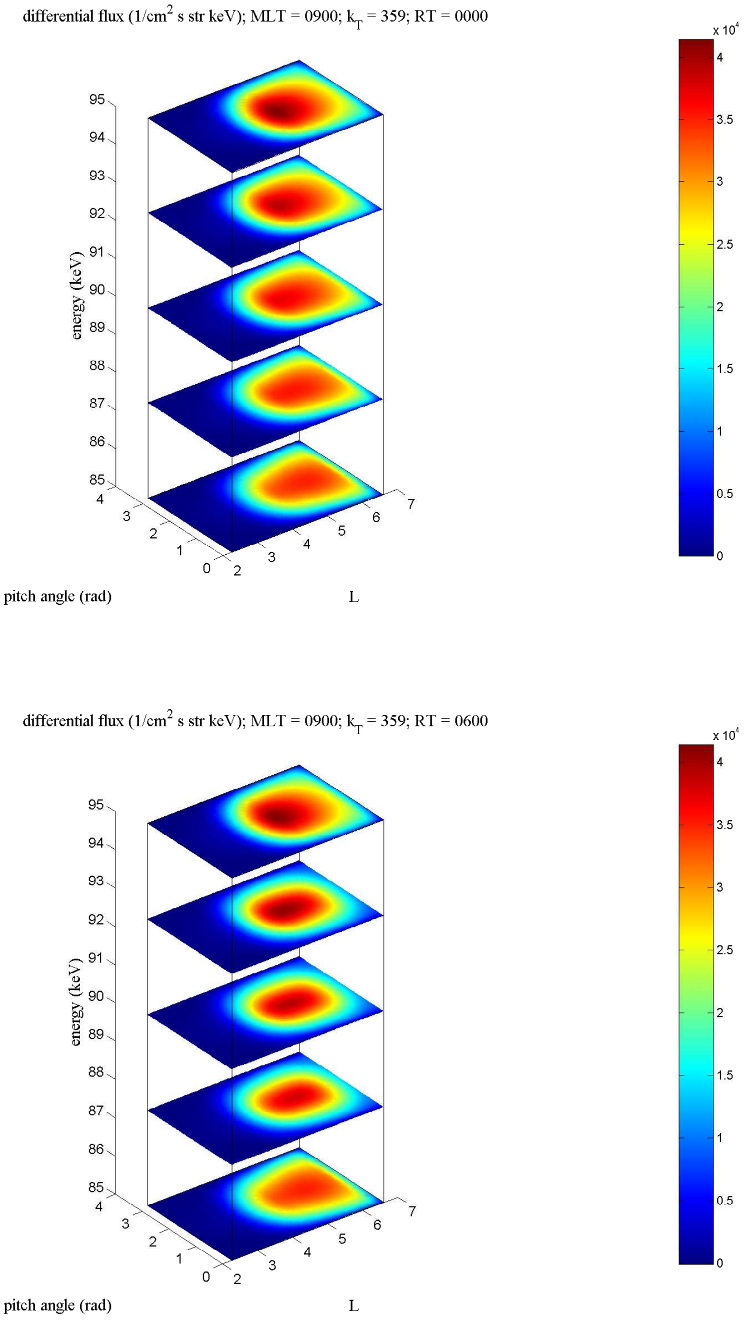
differential flux (1/cm^2 s str keV); MLT = 0900; k$_T$ = 359; RT = 0000

differential flux (1/cm^2 s str keV); MLT = 0900; k$_T$ = 359; RT = 0600

Fig. 4. Model evolution of the proton pitch angle distributions for E = (85–95) keV, MLT = 0900, L = 2.26–6.6 at 0000 RT = 0613 UT on 21 October 1999 (prestorm condition) and 0600 RT = 1213 UT

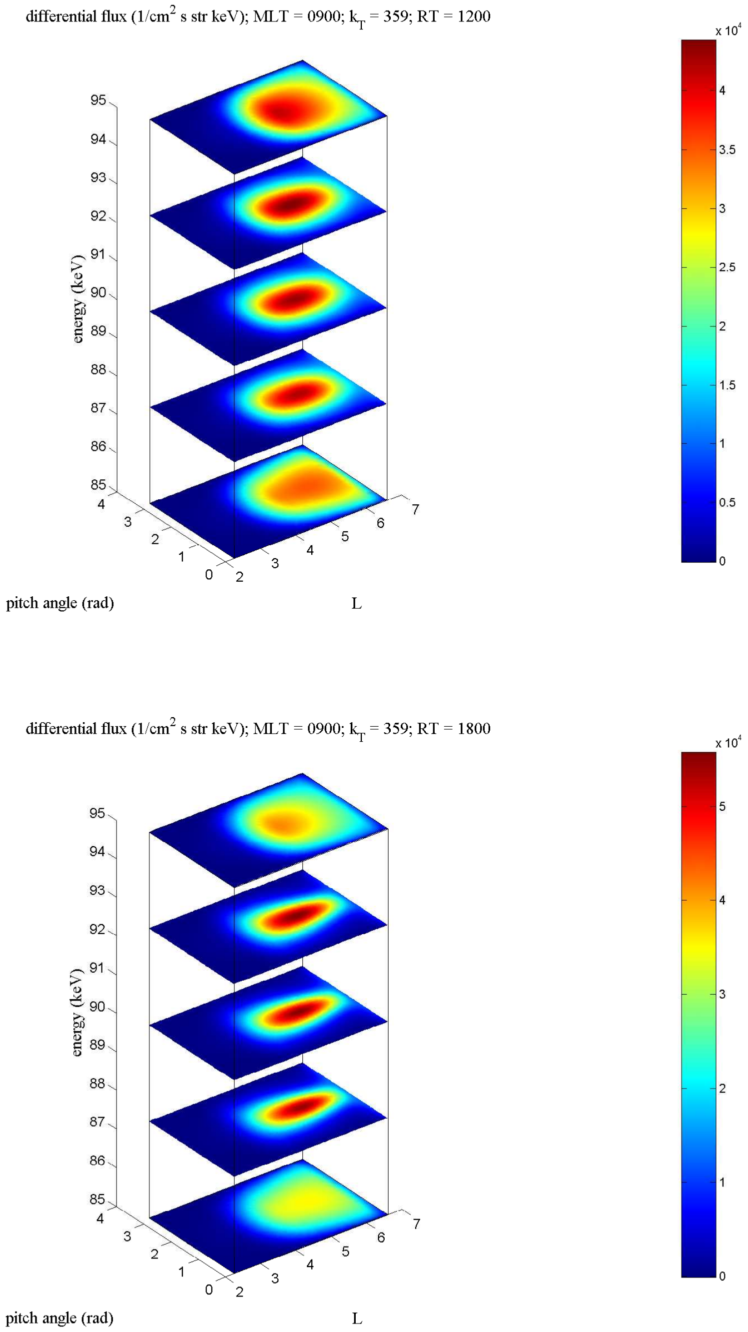

Fig. 5. Model evolution of the proton pitch angle distributions for $E = (85 - 95)$ keV, MLT = 0900, $L = 2.26 - 6.6$ at 1200 RT = 1813 UT on 21 October and 1800 RT = 0013 UT on 22 October 1999 (storm main phase)

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Трехмерная динамика протонов радиационных поясов Земли во время магнитной бури

Сергей В. Смолин

Предложена нестационарная трехмерная математическая модель, описывающая эволюцию функции распределения протонов. Исследовано распределение протонов радиационных поясов на дневной стороне магнитосферы Земли во время магнитной бури. Временная и пространственная эволюция плотностей фазового пространства протонов в дипольном магнитном поле вычисляется, используя трехмерную модель, рассматривающую радиальную, по питч-углам и по энергии диффузии. Выражения потерь описываются вследствие обмена зарядами и взаимодействий волна–частица. Моделирование начинается с распределения магнитоспокойного времени. Модель тестируется сравнением вычисленных потоков протонов с измерением на спутнике Polar/MICS во время магнитной бури 21–22 октября 1999 года и с измерением на спутнике AMPTE/CCE во время магнитной бури 2 мая 1986 года. Получено хорошее согласие модельных питч-угловых распределений протонов с экспериментальными данными.

Ключевые слова: протоны, диффузия, магнитная буря, магнитосфера Земли.