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On Varieties of Leibniz-Poisson Algebras with the Identity $\{x, y\} \cdot \{z, t\} = 0$

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Let K be an arbitrary field and let A be a K-algebra. The polynomial identities satisfied by A can be measured through the asymptotic behavior of the sequence of codimensions of A. We study varieties of Leibniz-Poisson algebras, whose ideals of identities contain the identity $\{x, y\} \cdot \{z, t\} = 0$, we study an interrelation between such varieties and varieties of Leibniz algebras. We show that from any Leibniz algebra L one can construct the Leibniz-Poisson algebra A and the properties of L are close to the properties of A. We show that if the ideal of identities of a Leibniz-Poisson variety $\mathcal{V}$ does not contain any Leibniz polynomial identity then $\mathcal{V}$ has overexponential growth of the codimensions. We construct a variety of Leibniz-Poisson algebras with almost exponential growth.

Keywords: Poisson algebra, Leibniz-Poisson algebra, variety of algebras, growth of a variety.

Introduction

Let A be an algebra over an arbitrary field. A natural and well established way of measuring the polynomial identities satisfied by A is through the study of the asymptotic behavior of it's sequence of codimensions $c_n(A)$, $n = 1, 2, \dots$. The first result on the asymptotic behavior of $c_n(A)$ was proved by A.Regev in [1]. He showed that if A is an associative algebra $c_n(A)$ is exponentially bounded. Such result was the starting point for an investigation that has given many useful and interesting results.

For associative algebras A.R.Kemer in [2] proved that the sequence $c_n(A)$ is either polynomially bounded or grows exponentially. Then A.Giambruno and M.V.Zaicev in [3] and [4] showed that the exponential growth of $c_n(A)$ is always an integer called the exponent of the algebra A.

When A is a Lie algebra, the sequence of codimensions has a much more involved behavior. I.B.Volichenko in [5] showed that a Lie algebra can have overexponential growth of the codimensions. Starting from this, V.M.Petrogradsky in [6] exhibited a whole scale of overexponential functions providing the exponential behavior of the identities of polynilpotent Lie algebras.

In this paper we study Leibniz-Poisson algebras satisfying polynomial identities. Remark that if a Leibniz-Poisson algebra A satisfies the identity $\{x, x\} = 0$ then A be a Poisson algebra. Poisson algebras arise naturally in different areas of algebra, topology, theoretical physics. We study varieties of Leibniz-Poisson algebras, whose ideals of identities contain the identity $\{x, y\} \cdot \{z, t\} = 0$. We show that the properties of such Leibniz-Poisson algebras are close to

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the properties of Leibniz algebras. We show that Leibniz-Poisson algebra can have overexponential growth of the codimensions and construct a variety of Leibniz-Poisson algebras with almost exponential growth.

1. Preliminaries

Let $A(+,\cdot,\{,\},K)$ be a K-algebra with two binary multiplications $\cdot$ and $\{,\}$. Let the algebra $A(+,\cdot,K)$ with multiplication $\cdot$ be a commutative associative algebra with unit and let the algebra $A(+,\{,\},K)$ be a Leibniz algebra under the multiplication $\{,\}$. The latter means that $A(+,\{,\},K)$ satisfies the Leibniz identity

$$\{\{x,y\},z\} = \{\{x,z\},y\} + \{x,\{y,z\}\}.$$

Assume that these two operations are connected by the relations $(a,b,c \in A)$

$$\{a\cdot b,c\} = a\cdot\{b,c\} + \{a,c\}\cdot b,$$

$$\{c,a\cdot b\} = a\cdot\{c,b\} + \{c,a\}\cdot b.$$

Then the algebra $A(+,\cdot,\{,\},K)$ is called a Leibniz-Poisson algebra.

We make the convention that brackets in left-normed form arrangements will be omitted:

$$\{\ldots\{\{x_1,x_2\},x_3\},...,x_n\} = \{x_1,x_2,...,x_n\}.$$

Let $L(X)$ be a free Leibniz algebra with multiplication $[,]$ freely generated by the countable set $X=\{x_1,x_2,\ldots\}$. Let also $F(X)$ be a free Leibniz-Poisson algebra. Denote by P_n^L and P_n the vector spaces in $L(X)$ and $F(X)$ accordingly, consisting of the multilinear elements of degree n in the variables $x_1,\ldots,x_n$.

Proposition 1 ([7]). *A basis of the vector space P_n consists of the following elements:*

$$x_{k_1}\cdot\ldots\cdot x_{k_r}\cdot\{x_{i_1},\ldots,x_{i_s}\}\cdot\ldots\cdot\{x_{j_1},\ldots,x_{j_t}\}, \tag{1}$$

where:

(i) $r\geqslant 0$, $k_1<\ldots<k_r$;

(ii) all elements are multilinear in the variebles x_1,...,x_n;

(iii) each factor $\{x_{i_1},\ldots,x_{i_s}\}$,..., $\{x_{j_1},\ldots,x_{j_t}\}$ in (1) is left normed and has lenngth $\geqslant 2$;

(iv) in each product (1) the shorter factor precede the longer: $s\leqslant ...\leqslant t$;

(v) if two consecutive factors in (1) are brackets $\{\ldots\}$ of equal length

$$\ldots\cdot\{x_{p_1},\ldots,x_{p_s}\}\cdot\{x_{q_1},\ldots,x_{q_s}\}\cdot\ldots,$$

then $p_1<q_1$.

Denote by Γ_n the subspace of P_n spanned by the elements (1) with $r=0$.

Denote by $L_{\geqslant 2}(X)$ the subspace of the free Leibniz algebra $L(X)$ spanned by the elements $[x_{i_1},\ldots,x_{i_n}]$ with $n\geqslant 2$. Also denote by $PL_{\geqslant 2}(X)$ the subspace of $F(X)$ spanned by the elements $\{x_{i_1},\ldots,x_{i_n}\}$ with $n\geqslant 2$. Obviously, $L_{\geqslant 2}(X)\cong PL_{\geqslant 2}(X)$ as Leibniz algebras. We will use only the notation $L_{\geqslant 2}(X)$ everywhere as $L_{\geqslant 2}(X)=PL_{\geqslant 2}(X)$ up to isomorphism of Leibniz algebras.

Let $\mathcal{V}$ be a variety of Leibniz-Poisson algebras (pertinent information on varieties of PI-algebras can be found, for instance, in [8], [9]). Let $Id(\mathcal{V})$ be the ideal of identities of $\mathcal{V}$. Denote

$$P_n(\mathcal{V}) = P_n/(P_n \cap Id(\mathcal{V})), \quad c_n(V) = dim\ P_n(\mathcal{V}).$$

For a variety of Leibniz algebras V_L denote

$$P_n^L(\mathcal{V}_L) = P_n^L/(P_n^L \cap Id(\mathcal{V}_L)), \quad c_n^L(\mathcal{V}_L) = dim\ P_n^L(\mathcal{V}_L).$$

Let $Id(A)$ be the ideal of the free algebra $F(X)$ of polynomial identities of A.

The next proposition shows how from every Leibniz algebra one can construct a Leibniz-Poisson algebra with some conditions of the source Lebniz algebra.

Proposition 2 ([7]). *Let A_L be a nonzero Leibniz algebra with multiplication $[,]$ over an infinite field K and let*

$$A = A_L \oplus K$$

be a vector space with multiplications $\cdot$ and $\{,\}$ defined as

$$\begin{gathered}(a+\alpha)\cdot(b+\beta) = (\beta a + \alpha b) + \alpha\beta,\\ \{a+\alpha, b+\beta\} = [a,b], \quad a,b \in A_L, \quad \alpha,\beta \in K.\end{gathered} \tag{2}$$

Then the algebra $(A, +, \cdot, \{\}, K)$ is a Leibniz-Poisson algebra and the following conditions are true:

(i) $Id(A_L) = Id(A) \cap L_{\geqslant 2}(X)$ and the algebra A satisfies the identity $\{x_1, x_2\} \cdot \{x_3, x_4\} = 0$;

(ii) for any $n \geqslant 2$

$$\Gamma_n(A) = P_n^L(A) = P_n^L(A_L)$$

up to isomorphism of vector spaces;

(iii) for any n the following equality holds:

$$c_n(A) = 1 + \sum_{k=2}^{n} \binom{n}{k} \cdot dim\ P_k^L(A_L).$$

2. Leibniz-Poisson Algebras with Identity $\{x_1, x_2\} \cdot \{x_3, x_4\} = 0$

Denote by $Id(\{x_1, x_2\} \cdot \{x_3, x_4\})$ the ideal of identities of the free Leibniz-Poisson algebra $F(X)$ generated by the element $\{x_1, x_2\} \cdot \{x_3, x_4\}$.

Theorem 1. *Let $\mathcal{V}_L$ be a variety of Leibniz algebras over an infinite field K defined by a system of identities*

$$\{f_i = 0 \mid f_i \in L_{\geqslant 2}(X),\ i \in I\} \tag{3}$$

and let $\{g_j \in Id(\{x_1, x_2\} \cdot \{x_3, x_4\}) \mid j \in J\}$, where $|J| > 0$, be a set of elements in the ideal $Id(\{x_1, x_2\} \cdot \{x_3, x_4\})$. Let $\mathcal{V}$ be a variety of Leibniz-Poisson algebras defined by the system of identities

$$\{f_i = 0,\ g_j = 0 \mid i \in I,\ j \in J\}.$$

Then:

(i) $Id(\mathcal{V}_L) = Id(\mathcal{V}) \cap L_{\geqslant 2}(X)$;
(ii) $P_n^L(\mathcal{V}) = P_n^L(\mathcal{V}_L)$;
(iii) $c_n(\mathcal{V}) \geqslant 1 + \sum\limits_{k=2}^{n} \binom{n}{k} \cdot c_k^L(\mathcal{V}_L)$;
(iv) if $|I| = 0$ *then* $c_n(\mathcal{V}) \geqslant [n! \cdot e] - n$, *where* $e = 2.71...$, $[\ ]$ *is an integer part of a number.*

Proof. *(i)* Let $f \in Id(\mathcal{V}_L)$. Then f follows from the system of identities (3). Therefore, $f \in Id(\mathcal{V}) \cap L_{\geqslant 2}(X)$ and $Id(\mathcal{V}_L) \subseteq Id(\mathcal{V}) \cap L_{\geqslant 2}(X)$. We will show that $Id(\mathcal{V}) \cap L_{\geqslant 2}(X) \subseteq Id(\mathcal{V}_L)$.

Let $\mathcal{W}$ be a Leibniz-Poisson variety defined by the system of identities (3) and the identity $\{x_1, x_2\} \cdot \{x_3, x_4\} = 0$. Since the element $\{x_1, x_2\} \cdot \{x_3, x_4\}$ generates the ideal $Id(\{x_1, x_2\} \cdot \{x_3, x_4\})$ and $|J| > 0$ then $\mathcal{W} \subseteq \mathcal{V}$, $Id(\mathcal{V}) \subseteq Id(\mathcal{W})$.

Let $L(X, \mathcal{V}_L)$ be the relatively free algebra of the variety $\mathcal{V}_L$ of countable rank. Theorem of Birkhoff implies that the algebra $L(X, \mathcal{V}_L)$ generates the variety $\mathcal{V}_L$. Hence $Id(\mathcal{V}_L) = Id(L(X, \mathcal{V}_L))$. Let $A = L(X, \mathcal{V}_L) \oplus K$ be a Leibniz-Poisson algebra with the multiplications (2). Proposition 2 implies that $A \in \mathcal{W}$, hence $Id(\mathcal{W}) \subseteq Id(A)$. Proposition 2 also implies the equality

$$Id(\mathcal{V}_L) = Id(L(X, \mathcal{V}_L)) = Id(A) \cap L_{\geqslant 2}(X).$$

Since $Id(\mathcal{V}) \subseteq Id(\mathcal{W}) \subseteq Id(A)$, it follows

$$Id(\mathcal{V}) \cap L_{\geqslant 2}(X) \subseteq Id(\mathcal{W}) \cap L_{\geqslant 2}(X) \subseteq Id(A) \cap L_{\geqslant 2}(X) = Id(\mathcal{V}_L).$$

(ii) Condition *(i)* implies that $Id(\mathcal{V}) \cap P_n^L = Id(\mathcal{V}_L) \cap P_n^L$ for any $n \geqslant 2$. Therefore,

$$P_n^L(\mathcal{V}_L) = P_n^L/(Id(\mathcal{V}_L) \cap P_n^L) = P_n^L/(Id(\mathcal{V}) \cap P_n^L) = P_n^L(\mathcal{V}).$$

(iii) follows from *(ii)* and [7, Proposition 4].

(iv) Applying the formula

$$n! \cdot \sum_{k=0}^{n} \frac{1}{k!} = [n! \cdot e],$$

inequality from *(iii)* and $P_n^L = n!$, we obtain that

$$c_n(\mathcal{V}) \geqslant 1 + \sum_{k=2}^{n} \binom{n}{k} \cdot k! = 1 + \sum_{k=2}^{n} \frac{n!}{(n-k)!} =$$

$$= /t = n - k/ = 1 + \sum_{t=0}^{n-2} \frac{n!}{t!} = n! \cdot \sum_{t=0}^{n} \frac{1}{t!} - n = [n! \cdot e] - n.$$

□

Define the lower and upper exponents for the codimension sequence $\{c_n(\mathcal{V})\}_{n \geqslant 1}$ as follows:

$$\underline{EXP}(\mathcal{V}) = \varliminf_{n \to \infty} \sqrt[n]{c_n(\mathcal{V})}, \qquad \overline{EXP}(\mathcal{V}) = \varlimsup_{n \to \infty} \sqrt[n]{c_n(\mathcal{V})}.$$

If the lower and the upper limits coincide, we use the notation $Exp(\mathcal{V})$.

Theorem 2. *Let $\mathcal{V}_L$ be a variety of Leibniz algebras over an infinite field K defined by the system of identities (3) and let $\mathcal{V}$ be a variety of Leibniz-Poisson algebras defined by the system of identities (3) and the identity $\{x_1, x_2\} \cdot \{x_3, x_4\} = 0$. Then:*

1) For any $n \geqslant 2$

$$\Gamma_n(\mathcal{V}) = P_n^L(\mathcal{V}) = P_n^L(\mathcal{V}_L)$$

up to isomorphism of vector spaces.

2) Let

$$u_s^n(x_1,...,x_n),\ s=1,\ldots,c_n^L(V_L), \tag{4}$$

be a basis of the vector space $P_n^L(\mathcal{V}_L)$, $n\geqslant 2$. Then $P_n(\mathcal{V})$ has a basis

$$\begin{gathered} x_1\cdot\ldots\cdot x_n,\\ x_{i_1}\cdot\ldots\cdot x_{i_{n-k}}\cdot u_s^k(x_{j_1},\ldots,x_{j_k}), \end{gathered} \tag{5}$$

$k=2,\ldots,n,\quad s=1,\ldots,c_k^L(\mathcal{V}_L),\quad i_1<\ldots<i_{n-k},\quad j_1<\ldots<j_k$;

3) For any n

$$c_n(\mathcal{V})=1+\sum_{k=2}^{n}\binom{n}{k}\cdot\dim\ P_k^L(\mathcal{V}_L).$$

4) If exponent $EXP(\mathcal{V}_L)$ exists, then $EXP(\mathcal{V})=EXP(\mathcal{V}_L)+1$, in particular if there exist constants $d\geqslant 0$, α and β such that for all sufficiently large n the double inequality holds

$$n^\alpha d^n\leqslant c_n^L(\mathcal{V}_L)\leqslant n^\beta d^n,$$

then there exist constants γ and δ such that for all sufficient large n the following double inequality holds

$$n^\gamma(d+1)^n\leqslant c_n(\mathcal{V})\leqslant n^\delta(d+1)^n.$$

5) If some Leibniz algebra A_L generate the variety V_L, then the Leibniz-Poisson algebra $A=A_L\oplus K$ with multiplications (2) generates the variety V.

6) If $|I|<+\infty$ and the variety $\mathcal{V}_L$ has the Specht property (i.e. all subvarieties of $\mathcal{V}_L$, including $\mathcal{V}_L$ itself, are finite based), then the variety $\mathcal{V}$ has the Specht property.

7) Let $\mathcal{W}$ be a proper subvariety of $\mathcal{V}$. Then the ideal of identities $Id(\mathcal{W})\cap L_{\geqslant 2}(X)$ determines the proper subvariety of $\mathcal{V}_L$.

8) The variety $\mathcal{V}_L$ is nilpotent if and only if the variety $\mathcal{V}$ has a polynomial growth.

Proof. 1) The equality $P_n^L(\mathcal{V}_L)=P_n^L(\mathcal{V})$ follows from Theorem 1. Since for any n holds equality

$$\Gamma_n=P_n^L\oplus Id(\{x_1,x_2\}\cdot\{x_3,x_4\})\cap\Gamma_n,$$

then

$$\begin{gathered} \Gamma_n(\mathcal{V})=\Gamma_n/(Id(\mathcal{V})\cap\Gamma_n)=\\ =\frac{P_n^L\oplus Id(\{x_1,x_2\}\cdot\{x_3,x_4\})\cap\Gamma_n}{Id(\mathcal{V})\cap(P_n^L\oplus Id(\{x_1,x_2\}\cdot\{x_3,x_4\})\cap\Gamma_n)}=\\ =\frac{P_n^L\oplus Id(\{x_1,x_2\}\cdot\{x_3,x_4\})\cap\Gamma_n}{(Id(\mathcal{V})\cap P_n^L)\oplus(Id(\{x_1,x_2\}\cdot\{x_3,x_4\})\cap\Gamma_n)}\cong\\ \cong P_n^L/(Id(\mathcal{V})\cap P_n^L)=P_n^L(\mathcal{V}). \end{gathered}$$

2) Follows from 1) and [7, Proposition 4].

3) Follows from 2).

4) Follows from 3) and the equality $(t+1)^n=\sum\limits_{k=0}^{n}\binom{n}{k}\cdot t^k$.

5) Let some Leibniz algebra A_L generates the variety $\mathcal{V}_L$. Define the Leibniz-Poisson algebra $A=A_L\oplus K$ with multiplications (2). Then Proposition 2 and Theorem 1 imply such equalities

$$Id(A)\cap L_{\geqslant 2}(X)=Id(A_L)=Id(\mathcal{V}_L)=Id(\mathcal{V})\cap L_{\geqslant 2}(X), \tag{6}$$

with $Id(\mathcal{V}) \subseteq Id(A)$. We will show that $Id(A) \subseteq Id(\mathcal{V})$.

Denote by B the subspace of the free Leibniz-Poisson algebra $F(X)$ spanned by the elements

$$\{x_{i_1}, \ldots, x_{i_s}\} \cdot \ldots \cdot \{x_{j_1}, \ldots, x_{j_t}\}, \quad s \geqslant 2, \ldots, t \geqslant 2.$$

In particular $\Gamma_n = B \cap P_n$, $n = 1, 2, \ldots$ Note that

$$B = L_{\geqslant 2}(X) \oplus B \cap Id(\{x_1, x_2\} \cdot \{x_3, x_4\}). \tag{7}$$

From [7] it follows that the ideal of identities $Id(A)$ is generated by the set of identities $B \cap Id(A)$. Let $f \in B \cap Id(A)$. Since

$$Id(\{x_1, x_2\} \cdot \{x_3, x_4\}) \subseteq Id(A)$$

and (7) then

$$B \cap Id(A) = L_{\geqslant 2}(X) \cap Id(A) \oplus B \cap Id(\{x_1, x_2\} \cdot \{x_3, x_4\}).$$

Hence there exist unique

$$g \in L_{\geqslant 2}(X) \cap Id(A), \quad h \in B \cap Id(\{x_1, x_2\} \cdot \{x_3, x_4\}),$$

such that $f = g + h$. (6) implies that $g \in Id(\mathcal{V})$. Obviously, $h \in Id(\mathcal{V})$, hence $f = g + h \in Id(\mathcal{V})$. Thus $Id(A) = Id(\mathcal{V})$.

6) Let $|I| < +\infty$ and the variety of Leibniz algebras $\mathcal{V}_L$ has the Specht property. Let $\mathcal{W}$ be a subvariety of the variety $\mathcal{V}$. Obviously, $Id(\mathcal{W}) \cap L_{\geqslant 2}(X)$ is an ideal of identities of the free Leibniz algebra $L(X)$. Theorem 1 implies that

$$Id(\mathcal{V}_L) \subseteq Id(\mathcal{W}) \cap L_{\geqslant 2}(X).$$

Hence the ideal of identities $Id(\mathcal{W}) \cap L_{\geqslant 2}(X)$ is generated by a finite number of elements $f_1, \ldots, f_k \in L_{\geqslant 2}(X)$.

Using the notations of 5), we have

$$B \cap Id(\mathcal{W}) = L_{\geqslant 2}(X) \cap Id(\mathcal{W}) \oplus B \cap Id(\{x_1, x_2\} \cdot \{x_3, x_4\}). \tag{8}$$

Since $Id(\mathcal{W})$ is generated by $B \cap Id(\mathcal{W})$ (see [7]) then the variety $\mathcal{W}$ is generated by the elements $f_1, \ldots, f_k$ and $\{x_1, x_2\} \cdot \{x_3, x_4\}$.

7) Let $\mathcal{W}$ be a proper subvariety of $\mathcal{V}$. Then the strict inclusion $Id(\mathcal{V}) \subsetneq Id(\mathcal{W})$ holds. We will show that

$$Id(\mathcal{V}_L) \subsetneq Id(\mathcal{W}) \cap L_{\geqslant 2}(X),$$

where $Id(\mathcal{W}) \cap L_{\geqslant 2}(X)$ is an ideal of identities of $L(X)$.

Since $Id(\mathcal{W})$ is generated by the set $B \cap Id(\mathcal{W})$ (see [7]) and $Id(\mathcal{V}) \subsetneq Id(\mathcal{W})$, there is such element $f \in B \cap Id(\mathcal{W})$ that $f \notin Id(\mathcal{V})$. Equality (8) implies that there exist unique

$$g \in L_{\geqslant 2}(X) \cap Id(\mathcal{W}), \quad h \in B \cap Id(\{x_1, x_2\} \cdot \{x_3, x_4\})$$

such that $f = g + h$. Since $h \in Id(\mathcal{V})$ and $f \notin Id(\mathcal{V})$, we obtain that

$$g \notin L_{\geqslant 2}(X) \cap Id(\mathcal{V}) = Id(\mathcal{V}_L).$$

Therefore, $Id(\mathcal{V}_L) \subsetneq Id(\mathcal{W}) \cap L_{\geqslant 2}(X)$.

8) Follows from 1), 3) and [7, Theorem 1] $\square$

Corollary. *Let $L(X)$ be a free Leibniz algebra over infinite field K and let $L(X) \oplus K$ be a Leibniz-Poisson algebra with multiplications (2). Then:*

(i) $Id(L(X) \oplus K) \cap L(X) = \{0\}$.

(ii) $Id(L(X) \oplus K) = Id(\{x_1, x_2\} \cdot \{x_3, x_4\})$, i.e. the ideal of identities of the algebra $L(X) \oplus K$ is generated by the identity $\{x_1, x_2\} \cdot \{x_3, x_4\} = 0$.

Denote by $\widetilde{\mathcal{V}}_1$ the variety of Leibniz-Poisson algebras defined by the identity $\{x_1, x_2\} \cdot \{x_3, x_4\} = 0$. Theorems 1 and 2 imply that the codimension growth of $\widetilde{\mathcal{V}}_1$ is overexponential.

Proposition 3. *For any $n \geqslant 1$ the codimension of the identities of $\widetilde{\mathcal{V}}_1$ satisfy*

$$c_n(\widetilde{\mathcal{V}}_1) = [n! \cdot e] - n.$$

Proposition 4. *Let ${}_3\widetilde{\mathcal{N}}$ be a Leibniz-Poisson variety, defined by the identity*

$$\{x_1, \{x_2, \{x_3, x_4\}\}\} = 0.$$

Then the variety $\widetilde{\mathcal{V}}_1 \cap {}_3\widetilde{\mathcal{N}}$ over a field K of characteristic 0 has almost exponential growth of the codimension sequence.

Proof. [11] and [10] implies that the variety of Leibniz algebras ${}_3\mathcal{N}$, defined by the identity

$$[x_1, [x_2, [x_3, x_4]]] = 0,$$

has almost exponential codimension growth. Therefore, by Theorem 1, the variety of Leibniz-Poisson algebras $\widetilde{\mathcal{V}}_1 \cap {}_3\widetilde{\mathcal{N}}$ has overexponential codimension growth.

Let $\mathcal{W}$ be a proper subvariety of $\widetilde{\mathcal{V}}_1 \cap {}_3\widetilde{\mathcal{N}}$. Condition 7) of Theorem 2 implies that the ideal of identities $Id(\mathcal{W}) \cap L_{\geqslant 2}(X)$ defines the proper subvariety of ${}_3\mathcal{N}$, which has exponentially bounded codimension growth. By condition 4) of Theorem 2, the sequence of codimensions of $\mathcal{W}$ is exponentially bounded. $\square$

Denote by $\widetilde{\mathcal{N}_s\mathcal{A}}$ the variety of Leibniz-Poisson algebras, defined by the identity

$$\{\{x_1, x_2\}, \dots, \{x_{2s+1}, x_{2s+2}\}\} = 0.$$

Proposition 5. *Variety $\widetilde{\mathcal{V}}_1 \cap \widetilde{\mathcal{N}_s\mathcal{A}}$ over a field K of characteristic 0 has the Specht property.*

Proof. [12] implies that the variety of Leibniz algebras $\widetilde{\mathcal{N}_s\mathcal{A}}$, defined by the identity

$$[[x_1, x_2], ..., [x_{2s+1}, x_{2s+2}]] = 0.$$

has the Specht property. Therefore, by 6) of Theorem 2, $\widetilde{\mathcal{V}}_1 \cap \widetilde{\mathcal{N}_s\mathcal{A}}$ has the Specht property. $\square$

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О многообразиях алгебр Лейбница-Пуассона с тождеством $\{x, y\} \cdot \{z, t\} = 0$

Сергей М. Рацеев

В данной работе исследуются многообразия алгебр Лейбница-Пуассона, идеалы тождеств которых содержат тождество $\{x, y\} \cdot \{z, t\} = 0$, исследуется взаимосвязь таких многообразий с многообразиями алгебр Лейбница. Показано, что из любой алгебры Лейбница можно построить алгебру Лейбница-Пуассона с похожими свойствами исходной алгебры. Показано, что если идеал тождеств многообразия алгебр Лейбница-Пуассона $\mathcal{V}$ не содержит ни одного тождества из свободной алгебры Лейбница, то рост многообразия $\mathcal{V}$ является сверхэкспоненциальным. Приводится многообразие алгебр Лейбница-Пуассона почти экспоненциального роста.

Ключевые слова: алгебра Пуассона, алгебра Лейбница-Пуассона, многообразие алгебр, рост многообразия.