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Local Information Access May Cause a Chaos in Migration

Michael G. Sadovsky[*]
Maria Yu. Senashova[†]
Institute of Computational Modelling, SB RAS,
Akademgorodok, 50/44, Krasnoyarsk, 660036 Russia

A novel model of the targeted migration of a population separated into two subpopulations is proposed. The model is implemented for the local information access. Migration is not random and yields the maximization of a net reproduction, in general; specific version of this general rule is proposed. The local information access means the total lack of knowledge on the environmental conditions and transfer cost at the immigration station, for beings occupying the given station. Various regimes for the models are studied. A reciprocal chaotic-like migrations may take place, for some reasonable parameters figures.

Keywords: recurrent equation, optimality, stability, model.

Introduction

Mathematical modelling is a powerful tool, in a great variety of research areas, including ecology and population biology. Yet, some biologically very important issues fall beyond a comprehensive modelling; modelling of the dynamics of spatially distributed biological communities is number one here. The most popular way to model the dynamics of such communities is based of various "chemical" similarities. Basically, the "reaction–diffusion" systems are used to describe the spatial transfer and its impact on the dynamics.

A reaction–diffusion methodology has a serious intrinsic discrepancy: the organisms must move randomly over a space and have no target in such transfers [1]. Obviously, this assumption is absolutely unfeasible (see, e.g. [2–8]). The assumption towards the idle transfers of organisms in space is obviously less favorable for the species of higher taxonomy rank.

Evolution optimality principle is the way to break through the discrepancy mentioned above. Briefly speaking, it forces a system to evolve to the maximization of net reproduction. This latter is an average number of *per capita* descendants survived at the course of a series of reproductions over an arbitrary long generation line [9–13].

For the spatially distributed communities, the implementation of this principle means that any transfer is to be made to improve a well-being of an organism, not just to roam. Of course, here various spatial sites are to differ in the environmental and reproduction conditions and, besides, the organisms must be able to identify these differences and take them into account when behave. Any specific implementation of this principle requires from a researcher to indicate apparently the knowledge that is available to an organism, for making a decision towards the spatial behaviour.

In general, three typical information patterns could be identified:

✓ **Global information access**. This pattern means that an organism knows everything towards the environmental conditions, including the subpopulation local densities and other key characters both in the residence area, and in the immigration site; it knows transfer cost, costs of emigration and immigration, etc.;

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[*]msad@icm.krasn.ru
[†]msen@icm.krasn.ru

✓ **Total lack of information**. This pattern means that an organism has no information at all, to make a decision towards the transfer of itself in space. One should avoid to miss this pattern with the diffusion approximation, since a reflexivity in behaviour may heavily change the random-like diffusive pattern of the spatial behaviour;

✓ **Local information access**. This pattern means that an organism possesses the information towards the environmental and/or reproductive conditions elsewhere partially. This is the most controversial pattern since an arbitrariness in the choice of the available information and the information content is too great.

Here we present a very simple model of the dynamics of spatially distributed population with local information access pattern. The model is based on the evolution optimality principle, rather than on a chemical kinetics similarity and may make a basis for further development of modelling of spatially distributed populations based on the evolution optimality principle.

1. Basic Model of the Smart Migration with Local Information access

To begin with, we have to introduce, in brief, a simple model of the targeted migration developed for the global information access [14–17]. Consider a population inhabiting two stations (thus, the population consists of two subpopulations). Any movement of individuals in a station are neglected. A migration is the transfer from station to station, only. No effects in the population dynamics from spatial structures are presumed, for each subpopulation, while no migration takes place.

We shall consider the model in discrete time t; the model is also discrete in space, i.e. a population is supposed to inhabit only two stations, and the transfer between these two stations affects the dynamics. The dynamics of each subpopulation is supposed to follow the Verhulst's equation [18–23]. Let N_t be the abundance of the first subpopulation (M_t be that later of the second one, respectively), so that

$$N_{t+1} = a \cdot N_t - b \cdot N_t^2 \qquad (1a)$$

and

$$M_{t+1} = c \cdot M_t - d \cdot M_t^2 \ , \qquad (1b)$$

respectively, in the migration-free case. Here a and c represent fertility of the relevant subpopulation, and b (d) shows the effect of density dependent competition within a subpopulation. The functions

$$k^{\mathrm{I}}(N_t) = a - b \cdot N_t \qquad (2a)$$

and

$$k^{\mathrm{II}}(M_t) = c - d \cdot M_t \qquad (2b)$$

are the net reproduction, in relevant stations, respectively.

Migration is a transfer of a part of subpopulation from one station into the other. Migration itself affects the reproduction, survival and other vital functions of an organism. All these issues will be integrated into the parameter called the *cost of migration* p: no negative impact on the reproduction, survival and other vital functions of an organism is observed, as $p = 1$. Otherwise, a decrease of p yields a decay in survival of organisms. Cost of migration p, $0 < p \leqslant 1$ may be considered as a probability of the successful transfer from station to station, i.e., with no damages for further reproduction [14–17]; see Fig. 1.

Previously (see [14–17]), a global information access model has been presented. The migration was determined from the net reproductions of both stations; besides, the transfer cost was also involved into the migration process determination. Mathematically, it means that the migration
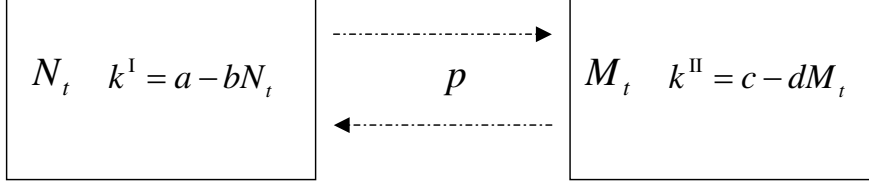
$$\boxed{N_t \quad k^{\mathrm{I}} = a - bN_t} \quad \cdots\!\cdots\!\to \quad p \quad \boxed{M_t \quad k^{\mathrm{II}} = c - dM_t}$$

Fig. 1. The simplest model of optimal migration. k^{I} and k^{II} are the net reproduction in relevant stations, and dotted lines show the migration fluxes

is determined from two equations, simultaneously, from (1). Such interconnection of the equations results from the global information access pattern.

On the contrary, the start of migration, as well as the migration flux is determined independently, for each station, with neither respect to the dynamics of another subpopulation ran at the reciprocal station. Again, there is a great number of ways to define the local information access pattern; here we present the simplest one based on the fertility maximization (so called one step optimization of the net reproduction).

Consider the functions

$$N_{t+1}\left(N_t\right) = a \cdot N_t - b \cdot N_t^2 \tag{3a}$$

$$M_{t+1}\left(M_t\right) = c \cdot M_t - d \cdot M_t^2 \tag{3b}$$

as the function of N_t (M_t, respectively). These are the up-headed parabolas with two zeros: 0 and a/b (0 and c/d, respectively). Both functions reach their maximum at the point

$$N^* = \frac{a}{2b} \qquad \text{and} \qquad M^* = \frac{c}{2d}\,,$$

respectively. Thus, the points N^* and M^* yield the maximal off-spring abundance, at the next generation.

An emigration from a station starts, when

$$N_t > N^*\,, \tag{4a}$$

$$M_t > M^*\,. \tag{4b}$$

It should be stressed, that both inequalities in (4) may take place simultaneously, at the same time moment t. The emigration fluxes are then determined as

$$\Delta_N = N_t - N^* \qquad \text{and} \tag{5a}$$

$$\Delta_M = M_t - M^*\,. \tag{5b}$$

2. Some Properties of the Model

The model (1 – 5) exhibits a number of features peculiar for Verchulst equation, observed for a migration-free case. First of all, a non-zero steady state is possible when $a > 1$ ($c > 1$, respectively), only. There is no steady state but the zero one, when $a \leqslant 1$ ($c \leqslant 1$, respectively). Next, no migration takes place if $1 < a \leqslant 2$ ($1 < c \leqslant 2$, respectively). Indeed, the migration may here take place, if initial abundance of a subpopulation exceeds N^* (M^*, respectively); for $1 < a \leqslant 2$ ($1 < c \leqslant 2$, respectively) steady state is stable, and it is less than N^* (M^*, respectively). Thus, as soon as the system relaxes to the steady state, no migration would be observed. A stable steady state exists for $1 < a \leqslant 3$ ($1 < c \leqslant 3$, respectively). Similar to the simple Verchulst equation [18–20], the model yields an elimination of a subpopulation, as $a \geqslant 4$ ($c \geqslant 4$, respectively).

2.1. Permanent Migration Flux

Consider now the situation where $2 < a \leqslant 3$ but $c \leqslant 2$; thus no migration from the second station into the first one is expected. Then the steady state at the first station

$$N^\circ = \frac{a-1}{b} > N^* \,,$$

and the emigrant flux $\Delta = (a-2)/(2b)$ will appear. This emigration would make the abundance at the station optimal, i.e. equal to N^* (see Eq. (3a)). The emigration will maximize the off-spring at the next generation, and that later would be equal to

$$N_{t+1}^{\max} = a \cdot \frac{a}{2b} - b \cdot \frac{a^2}{4b^2} = \frac{a^2}{4b} \,.$$

Since $a^2/(4b) > N^*$, the excess of a population in the station will again emigrate making the upgraded abundance equal to the optimal one N^*. Here the process ends. So, if no reciprocal migration takes place, then one may observe a limit regime with permanent emigration flux from one station into the other. Thus, for the conditions mentioned above ($2 < a \leqslant 3$ and $c \leqslant 2$) there exists a steady state dynamics, in both stations.

The steady state is provided by the permanent (and constant in value) emigration from the station with greater fertility, to the station with lower fertility.

2.2. Migration Dumps Complexity in Population Dynamics

Consider now the case where $c \leqslant 2$ but $a > 3$. It is a well-known fact, that a subpopulation in a station with a exceeding 3 exhibits a periodical dynamics, in migration free case; the length of a period, as well as the complexity of the limit regime depends on a figure [19–22].

An optimizing migration results in a decomposition of a complex regime into very simple one, with one-way directed migration flux. Indeed, suppose there exists a cycle of the length m: $N_1 \mapsto N_2 \mapsto N_3 \mapsto \cdots \mapsto N_{m-1} \mapsto N_m \mapsto N_1$. Since $a > 3$, then $N_{t+1}^{\max}(N_t) > N^*$, always, and there always exists at least one figure N_j in the cycle so that $N_j > N^\circ > N^*$. As soon as the dynamics reaches this cycle step (N_j, indeed), the migration will cut-off the abundance in the station to the optimal figure N^*. The off-spring number $a^2/(4b)$ exceeds the optimal abundance N^* thus following in a steady state occurrence with permanent emigration flux

$$\Delta_{\text{out}} = \frac{a^2 - 2a}{2b} \,, \tag{6}$$

and stable abundance equal to the optimal value $N^\circ = N^*$. Such behaviour would be observed for any a figure.

2.3. Immigration: Same Type of the Living Cycle

Let now consider the case where the immigration from the reciprocal station takes place. It may take place, since the organisms in various stations emigrate independently. Such immigration shifts the steady state, if the inward immigrant flux Δ_{in} falls within the tolerance range. Still, we consider the situation of $a > 2$.

Indeed, let the situation be similar to that on describe in subsection 2. but a permanent immigration flux Δ_{in} is observed. First of all, the observed dynamics would depend on the Δ_{in} value. When two reciprocal migration fluxes take place, the dynamics would depend on the tolerance capacity of the immigration station.

The living cycle in the model (1–5) consists in the following sequence of events: i) emigration (if necessary); ii) immigration: iii) reproduction. An emigration always maximizes the off-spring number, yielding the current abundance equal to N^*. An immigration flux will break this optimality conditions shifting the abundance upright from the optimal point.

A limit regime of the model $(1-5)$ remains the same, if

$$0 < \Delta_{\text{in}} \leqslant \frac{a^2 - 2a}{2b} \, . \tag{7}$$

In other words, the living cycle of the model is the same, for the case (7). Of course, the immigration flux (7) would shift the observed off-spring number at the next generation, since the number of reproducing organisms would be greater than N^*, while still the tolerant immigration flux will not yield the off-spring number less than N^*. A stable off-spring number, in the case of the tolerant immigration, will be determined by the relation

$$a \left(\frac{a}{2b} + \Delta_{\text{in}} \right) - b \left(\frac{a}{2b} + \Delta_{\text{in}} \right)^2 \, ;$$

again, here $\Delta_{\text{in}} \leqslant (2b)^{-1} \cdot \left(a^2 - 2a \right)$.

Such dynamics would remain for any

$$\Delta_{\text{in}} \leqslant \frac{\sqrt{a^2 - 2a}}{2b} \, ; \tag{8}$$

see Fig. 2, point l in this figure. Here the upgraded abundance still yields the next generation off-spring number $N_{t+1} > N^*$ so that further immigration optimizes the abundance at the station. Obviously, Δ_{in} in (7) is less than Δ_{in} in (8), as $a < 4$.

Fig. 2. Dynamics draft for the case of a reciprocal immigration; the immigration flux is less than Δ_0; $l = (2b)^{-1} \cdot \left(a + \sqrt{a^2 - 2a} \right)$

2.4. Immigration: Cyclic Dynamics

Consider now the case where

$$\Delta_{\text{in}} > \frac{\sqrt{a^2 - 2a}}{2b} \, .$$

In this case, the upgraded abundance $\widetilde{N}_t$ is great enough to give the next generation abundance $N_{t+1} < N^*$. No emigration takes place at the time moment $t + 1$; meanwhile, the immigration with given Δ_{in} (it should be kept in mind that we suppose Δ_{in} to be permanent) shifts the upgraded abundance $\widetilde{N}_{t+1}$ at the time moment $t + 1$ somewhere upright from the point N^*, so that the off-spring number $N_{t+2} > N^*$.

Here the emigration will take place, again, and the upgraded abundance at the station will be equal to $a/(2b)$. For $t + 3$ time moment, the upgraded abundance will again exceed

$$\frac{a}{2b} + \frac{\sqrt{a^2 - 2a}}{2b},$$

and all the dynamics repeats.

Finally, if

$$\Delta_{\text{in}} > \frac{\sqrt{a^2 - 2a}}{2b},$$

then N_{t+1} is less than $a/(2b)$, meanwhile $a/(2b) < N_{t+1} + \Delta_{\text{in}} < l$. This fact follows from the convexity of the function $N_{t+1}(N_t)$. Such type of the dynamics is shown in Fig. 3.

Hence, two types of limit regimes may be observed, for the model $(1 - 5)$, in dependence on the the immigration flux level. The **first type** results in a steady state observed in a station, with permanent emigration flux, with no immigration. Obviously, the reciprocal station also possesses a steady state regime. The **second type** makes a cycle of the length 2; this type results from the **inward migration**, not the dynamically determined cyclic pattern (see [19–22] and Fig. 3). This type of the dynamics requires some peculiar level of the

Fig. 3: Cycle of the length 2, with excessive migration flux

immigration flux, and may not always be observed, for an arbitrary figure of that latter. Finally, quite abundant immigration may stabilize again the dynamics so that a fuzzy-steady state would take place. This latter means that there would be permanent reciprocal migrations, and the abundance at the given stations would persist around some average figure for an arbitrary long time period.
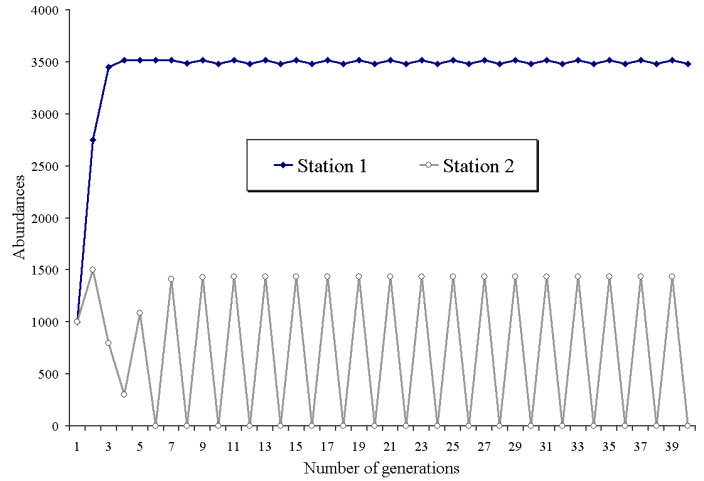
3.   Discussion

Here we present the simplest model of a smart migration addressing the methodology of evolution optimality. The model is quite clear and apparent. In the terms of mathematical technique, the model is a system of two connected (nonlinear) oscillators, discrete both in time, and space. From biological point of view, this model provides the case of the local information access, when a decision towards the migration should be made.

The model (1–5) does not bring serious technical or mathematical problems, at least, for the analysis of limit regimes transformations (bifurcations, etc.). A dynamics of a single population described by Verchulst equation is well known. A model being simple and feasible still yields

the system with locally informed beings. The local information access means that the spatial behaviour of a subpopulation has two (behavioural) options: to remain staying at the station, or to emigrate into the opposite one.

The interaction between the subpopulations is rather weak, and these later "know nothing" about each other, but a place to emigrate, in case of the overpopulation of the station. The situation of the local information access seems to be the most biologically feasible, on contrary to the approximation of the global information access, and complete lack of knowledge of towards the environmental conditions, and the living conditions of a being.

Such behaviour, meanwhile, should be distinguished from the virtual periodicity resulted from the $[0, 1]$ mapping into itself due to the transformation provided by Verchulst equation. Here the basic reason of the cycle of length 2 occurrence is the inlet migration from an outer space, not the intrinsic dynamics of the system.

Also, the evolutionary optimal redistribution of the beings in space described in the model (1–5) looks similar (or pretty close) to the altruistic behaviour observed in numerous biological systems [7,8]. Actually, the mechanism of the space redistribution described above may be applied for the description of the asymmetric strategies of the reproduction; these later are also known as disparity strategies, and they are evolutionary stable, if a significant level of the correlations in the inherited entities takes place [9–11]. Yet, the basic biological issues standing behind the disparity of behavioural strategies implementation, and the emigration strategy provided by the model (1–5) have nothing to do each other. This difference may manifest, within the framework of the model (1–5), in the occurrence of the reciprocal migrations; these later would never take place, in case of the disparity in the reproduction strategies realized in a community.

The model (1–5) considered here still poses a lot of questions. We have not considered the influence of a periodical dynamics (or complex one) resulted from the features of discrete Verchulst equation itself (see [19, 20] for details). There is no evident answer on this question; one may expect the flush-off migration will simplify (complex) limit regimes, and kills a complex behaviour of the system. Probably, this statement holds true for some parameters values, and the area of such simplification of the originally complex behaviour is rather extended (in the space of parameters, of course).

An impact of the transfer cost is another important issue determining both a type, and peculiarities of the dynamics. Everywhere above we supposed the transfer cost be equal to 1. Transfer cost describes the process of a migration, in immediate sense of the word. Probably, it implies three basic issues: a cost of leaving a site, a cost of intrusion a site, and the immediate cost of the transfer between these two sites. All these questions fall beyond the scope of this paper, and would be present separately.

Conclusion

Here we presented the simplest model of evolution optimally migration between two stations, for the case of local information access. That latter means that the beings refer to the local environmental conditions, and local (sub)population density when making a decision towards the transfer from a station (towards an emigration, indeed).

The main purpose of this paper is to show the methodology of the modelling of spatially distributed populations (and other biological communities) with no chemical analogies, rather than to find out some peculiar dynamic regime pretending to match exactly a dynamics of a real biological community.

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Локальная информированность может приводить к хаотической миграции

Михаил Г. Садовский
Мария Ю. Сенашова

Рассмотрена новая модель целенаправленной миграции одного вида, занимающего две стации, в случае локальной информированности. Миграция носит неслучайный характер и максимизирует коэффициент размножения, в статье предложена конкретная реализация этого принципа. Локальная информированность означает полное отсутствие знания об условиях окружающей среды в стации иммиграции и цены перемещения, для особей, населяющих данную стацию. Исследованы различные динамические режимы. Для некоторых биологически осмысленных значений параметров в модели наблюдается хаосо-подобная динамика миграционных потоков.

Ключевые слова: эволюционная оптимальность, модель, устойчивость.