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Integral Representations and Volume Forms on Hirzebruch Surfaces

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We construct a class of integral representations for holomorphic functions in a polyhedron in $\mathbb{C}^4$, associated with Hirzebruch surfaces. The kernels of the integral representations are closed differential forms in $\mathbb{C}^4$ associated with volume forms on Hirzebruch surfaces.

Keywords: integral representation, Hirzebruch surface, toric variety.

Introduction

The kernel of the Bochner-Martinelli integral representation in $\mathbb{C}^{n+1}$ is well known to be closely connected with the Fubini-Studi form for the projective space $\mathbb{P}^n = \mathbb{CP}^n$ as follows:

$$\omega(z) = \frac{1}{2\pi i}\frac{d\lambda}{\lambda} \wedge \omega_0([\xi]) \tag{1}$$

(see, for instance, [1, Ch. 3]; [2, Ch. 4]). Here ω is the Bochner–Martinelli form,

$$\omega(z) = \frac{n!}{(2\pi i)^{n+1}} \sum_{k=1}^{n+1} (-1)^{k-1} \frac{\bar{z}_k}{|z|^{2n+2}} d\bar{z}[k] \wedge dz,$$

$dz = dz_1 \wedge \ldots \wedge dz_{n+1}$, and $d\bar{z}[k]$ results from deleting the differential $d\bar{z}$ in $d\bar{z}_k$. The form $\omega_0([\xi])$ is the volume form for the Fubini–Studi metric in $\mathbb{P}^n$ (see [3, p. 21])

$$\omega_0([\xi]) = \frac{n!}{(2\pi i)^n} \frac{E(\xi) \wedge \overline{E(\xi)}}{|\xi|^{2(n+1)}}, \tag{2}$$

where

$$E(\xi) = \sum_{k=1}^{n+1} (-1)^{k-1} \xi_k d\xi[k]$$

is the Euler form and $\xi = (\xi_1, \ldots, \xi_{n+1})$ are the homogeneous coordinates of a point $[\xi] \in \mathbb{P}^n$. Moreover, $\xi, z \in \mathbb{C}^{n+1}$ and $\lambda \in \mathbb{C}$ are connected by the relation $z = \lambda\xi$.

The Bochner–Martinelli form is a “canonical” form of degree $2n + 1$ in $\mathbb{C}^{n+1} \setminus \{0\}$. The latter set is a bundle over $\mathbb{P}^n$ whose fiber is the one-dimensional torus $\mathbb{C}_*$. In other words,

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$\mathbb{P}^n = [\mathbb{C}^{n+1} \setminus \{0\}]/G$, where $G = \{(\lambda, \dots, \lambda) \in \mathbb{C}^{n+1} : \lambda \in \mathbb{C}_*\}$ is the transformation group of diagonal matrices. The projective space is a particular instance of a toric variety. In the general case, each n-dimensional toric variety is some quotient space (see [4, 5, 6])

$$\mathbb{X} = \left[\mathbb{C}^d \setminus Z(\Sigma)\right]/G.$$

Here $Z(\Sigma)$ is the union of some coordinate subspaces in $\mathbb{C}^d$ constructed from a fan $\Sigma \subset \mathbb{R}^n$ with d generators and G is a group isomorphic to the torus $(\mathbb{C}_*)^r,\ r = d - n$, which is also constructed from Σ.

In his report at the "Nordan" conference on complex analysis (Stockholm, April 1999) A. K. Tsikh posed the problem of calculating the volume forms $\omega_0([\xi])$ on toric varieties $\mathbb{X}_k$ (the Fubini–Studi forms) and the canonical forms $\omega(z)$ on $\mathbb{C}^d \setminus Z(\Sigma)$ with the property

$$\omega(z) \sim \frac{1}{(2\pi i)^r}\frac{d\lambda_1}{\lambda_1} \wedge \ldots \wedge \frac{d\lambda_r}{\lambda_r} \wedge \omega_0([\xi]),$$

generalizing (1), where the sign $\sim$ means that the forms have the same residues with respect to $\lambda_1 = \ldots = \lambda_r = 0$. Moreover, he noted that the forms ω may serve as kernels of integral representations in $\mathbb{C}^d$.

In the present work we consider a class of toric varieties of complex dimension 2 called Hirzebruch surfaces. We construct volume forms for this class and canonical forms in $\mathbb{C}^4 \setminus Z'$ where the set Z' is, in general, not the same as the singular set $Z(\Sigma)$. It is shown that the constructed canonical forms define an integral representation in 4-circular polyhedra $G \subset \mathbb{C}^4$. In [7] author considered toric varieties, defined by convex fans. Convexity of a fan provides that the singular set of a canonical form ω coincides with $Z(\Sigma)$. As we will see below in the case of Hirzebruch surfaces fan fails to be convex if $k > 2$.

1. Hirzebruch Surfaces, Moment Maps and Integration Cycles

Hirzebruch surface $\mathbb{X}_k$ is the toric variety defined by the 2-dimensional fan, spanned by the vectors $v_1{=}(1,0)$, $v_2{=}(0,1)$, $v_3{=}(-1,0)$, $v_4{=}(-k,-1)$, where $k \in \mathbb{Z}_+$.

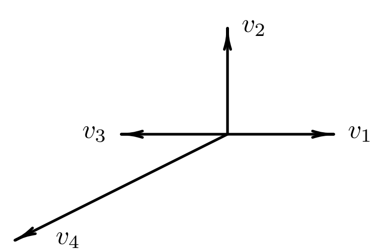

Fig. 1. The fan of $\mathbb{X}_2$.

To each vector v_j we assign a complex variable ζ_j so that $\zeta = (\zeta_1, \zeta_2, \zeta_3, \zeta_4)$ plays role of homogeneous coordinates of Hirzebruch surfaces $\mathbb{X}_k$. Each pair of nonneighboring vectors

v_i, v_j (i.e., those not defining a two-dimensional cone) defines a coordinate plane in $Z(\Sigma)$ (see [7]) so that

$$Z(\Sigma) = \{\zeta_1 = \zeta_3 = 0\} \cup \{\zeta_2 = \zeta_4 = 0\}.$$

The group G is determined by the relations $\sum\limits_j \mu_j v_j = 0$ on the vectors v_j. The following equations

$$\begin{cases} v_1 + v_3 &= 0, \\ kv_1 + v_2 + v_4 &= 0, \end{cases}$$

are all linearly independent relations between the vectors v_k. Consequently, the vectors $\mu_1 = (1, 0, 1, 0)$, $\mu_2 = (k, 1, 0, 1)$ constitute a basis for the lattice of relations. The group G is the 2-parameter surface $\{(\lambda_1\lambda_2^k, \lambda_2, \lambda_1, \lambda_2) : \lambda_j \in \mathbb{C}_*\} \subset (\mathbb{C}_*)^4$, so that

$$\zeta \sim \eta \Leftrightarrow \exists \lambda_1, \lambda_2 : \zeta = (\zeta_1, \zeta_2, \zeta_3, \zeta_4) = (\lambda_1\lambda_2^k\eta_1, \lambda_2\eta_2, \lambda_1\eta_3, \lambda_2\eta_4).$$

The moment map (see, for instance, [5, 8]) $\mu : \mathbb{C}^4 \to \mathbb{R}^4/\mathbb{R}^2 \simeq \mathbb{R}^2$ looks like

$$\mu(\zeta_1, \zeta_2, \zeta_3, \zeta_4) = (\rho_1, \rho_2),$$

where

$$\begin{cases} \rho_1 = |\zeta_1|^2 + |\zeta_3|^2, \\ \rho_2 = k|\zeta_1|^2 + |\zeta_2|^2 + |\zeta_4|^2. \end{cases} \tag{3}$$

For a fixed $\rho = (\rho_1, \rho_2) \in \mathbb{R}^2$, the relations (3) define the set $\Gamma_0^k(\rho) = \mu^{-1}(\rho)$.

The Kähler cone (see, for instance, [5]) for $\mathbb{X}_k$ is defined by the following inequalities:

$$\begin{cases} \rho_1 > 0, \\ \rho_2 > k\rho_1. \end{cases} \tag{4}$$

The fact that the inequalities (4) hold provides that the integration cycle Γ_0^k does not intersect the singular set $Z(\Sigma)$.

2. A Canonical Form and a Volume Form

We write down a form ω in $\mathbb{C}^d \setminus Z(\Sigma)$ that is an analog of the Bochner–Martinelli form and establish its basic properties.

The sought form has bidegree $(4, 2)$ and looks like

$$\omega(\zeta) = \frac{h(\bar\zeta) \wedge d\zeta}{g(\zeta, \bar\zeta)}. \tag{5}$$

The numerator is a form of type $(4, 2)$, where $d\zeta = d\zeta_1 \wedge d\zeta_2 \wedge d\zeta_3 \wedge d\zeta_4$, and

$$h(\zeta) = \zeta_3\zeta_4 d\zeta_1 \wedge d\zeta_2 - \zeta_2\zeta_3 d\zeta_1 \wedge d\zeta_4 + \zeta_1\zeta_4 d\zeta_2 \wedge d\zeta_3 + k\zeta_1\zeta_3 d\zeta_2 \wedge d\zeta_4 + \zeta_1\zeta_2 d\zeta_3 \wedge d\zeta_4 \tag{6}$$

is an analog of the Euler form. The denominator g is the function

$$g(\zeta, \bar\zeta) = |\zeta_1|^4|\zeta_2|^{4-2k} + |\zeta_1|^4|\zeta_4|^{4-2k} + |\zeta_2|^{2k+4}|\zeta_3|^4 + |\zeta_3|^4|\zeta_4|^{2k+4}.$$

Here we have to make one important remark.

Note that g may contain negative powers of ζ. In this case we define the form ω as in (5), whose numerator and denominator are multiplied by the least power of ζ such that the denominator of the resulting form contains no negative powers of ζ. This procedure does not affect the transformation laws of the form ω that we will derive below.

However, the singular set Z_ω of the form ω depends on k. More precisely, we have the following three cases:

1. If $k = 0$ or $k = 1$ then Z_ω coincides with $Z(\Sigma) = \{\zeta_1 = \zeta_3 = 0\} \cup \{\zeta_2 = \zeta_4 = 0\}$;
2. If $k = 2$ then $Z_\omega = Z' := \{\zeta_1 = \zeta_3 = 0\} \cup \{\zeta_1 = \zeta_2 = \zeta_4 = 0\}$;
3. If $k > 2$ then $Z_\omega = Z'' := \{\zeta_1 = \zeta_3 = 0\} \cup \{\zeta_2 = \zeta_4 = 0\} \cup \{\zeta_1 = \zeta_2 = 0\} \cup \{\zeta_1 = \zeta_4 = 0\}$.

Each fixed element $\delta = (\lambda_1\lambda_2^k, \lambda_2, \lambda_1, \lambda_2) \in G$ defines the mapping $\delta : \mathbb{C}^4 \setminus Z(\Sigma) \to \mathbb{C}^4 \setminus Z(\Sigma)$ by the formula $\zeta \to \delta \cdot \zeta$, i.e.,

$$\begin{cases} \zeta_1 \to \lambda_1\lambda_2^k\zeta_1, \\ \zeta_2 \to \lambda_2\zeta_2, \\ \zeta_3 \to \lambda_1\zeta_3, \\ \zeta_4 \to \lambda_2\zeta_4. \end{cases} \tag{7}$$

Proposition 1. *The differential form ω is invariant under the action of δ.*

PROOF. By direct substitution, we obtain the following transformation laws for $h(\bar\zeta)$, $d\zeta$, and $g(\zeta, \bar\zeta)$:

$$h(\bar\zeta) \to \bar\lambda_1^2\bar\lambda_2^{k+2}h(\bar\zeta), \quad d\zeta \to \lambda_1^2\lambda_2^{k+2}d\zeta, \quad g(\zeta,\bar\zeta) \to (\lambda_1\bar\lambda_1)^2(\lambda_2\bar\lambda_2)^{k+2}g(\zeta,\bar\zeta).$$

Inserting them in ω, we arrive at the assertion of the proposition. □

We now describe the behavior of ω under the action of the group $G : \left(\mathbb{C}^4 \setminus Z(\Sigma)\right) \times \mathbb{C}_*^2 \to \mathbb{C}^4 \setminus Z(\Sigma)$, defined by (7).

Lemma 1. *The form $d\zeta$ transforms as follows under the action of (7):*

$$d\zeta \to \lambda_1\lambda_2^{k+1}d\lambda_1 \wedge d\lambda_2 \wedge h(\zeta) + \psi(\lambda, \zeta),$$

where h is determined by (6), and the form ψ has higher degree in ζ than $h(\zeta)$.

Lemma 2. *The form $h(\bar\zeta)$ transforms by the following rule under the action of (7):*

$$h(\bar\zeta) \to \bar\lambda_1^2\bar\lambda_2^{k+2}h(\bar\zeta).$$

It is not hard to prove lemmas 1 and 2 by direct substitution of the action of G into the forms $d\zeta$ and $h(\bar\zeta)$.

Let us note that since the denominator g is a function (not differential form), it transforms by the same rule as in Proposition 1 under the action of (7).

We thus come to the following

Theorem 1. *Under the action of (7) the form ω transforms as follows:*

$$\omega \to \frac{d\lambda_1}{\lambda_1} \wedge \frac{d\lambda_2}{\lambda_2} \wedge \omega_0 + \omega_1 \tag{8}$$

with the positive form

$$\omega_0 = \frac{h(\bar\zeta) \wedge h(\zeta)}{g(\zeta, \bar\zeta)}$$

of homogeneity degree zero under the action of the group G and with some form ω_1, involving no conjugate differentials $d\bar\lambda_i$ and having at most one differential $d\lambda_j$ in each summand.

The form ω_0 is an analog of the Fubini–Studi form (2) for the projective space.

Recall that $\Gamma_0^k = \Gamma_0^k(\rho)$ is the set (3). We now treat it as an integration cycle. The cycle Γ_0^k foliates over $\mathbb{X}_k$ with fibers isomorphic to the real tori $\mathbb{T}^2$ ($\mathbb{T} = \{z \in \mathbb{C} : |z| = 1\}$), i.e.,

$$\Gamma_0^k(\rho)/G_{\mathbb{R}} = \mathbb{X}_k, \tag{9}$$

where $G_{\mathbb{R}} := \{(\lambda_1\lambda_2^k, \lambda_2, \lambda_1, \lambda_2) : |\lambda_j| = 1,\ j = 1,2\}$ (see [5, Theorem 4.1]). From this and Theorem 1 we see that the form ω_0 depends only on the orbits of the group G and consequently is well defined on $\mathbb{X}_k$. Moreover, Γ_0^k is not homologous to zero in $\mathbb{C}^4 \setminus Z(\Sigma)$.

At this point let us note that if $k \geqslant 2$ then the singular set Z_ω does not coincide with $Z(\Sigma)$. (This happens because the fan Σ is not strictly convex.) If $k = 2$ then the singular set Z_ω is a subset of $Z(\Sigma)$, and therefore the cycle Γ_0^k does not intersect Z_ω. If $k > 2$ then the cycle Γ_0^k can intersect the planes $\{\zeta_1 = \zeta_2 = 0\}$ и $\{\zeta_1 = \zeta_4 = 0\}$. In this case we need to prove the following

Proposition 2. *The form ω is bounded in the neighborhood of the planes $\{\zeta_1 = \zeta_2 = 0\}$ and $\{\zeta_1 = \zeta_4 = 0\}$.*

Proof. Let us show that the form ω is bounded in the neighborhood of the plane $\{\zeta_1 = \zeta_2 = 0\}$. Let $|\zeta_1| = \varepsilon_1$, and $|\zeta_2| = \varepsilon_2$. Equalities (3) imply $|\zeta_3|^2 = \rho_1 - \varepsilon_1^2 > \dfrac{\rho_1}{2}$ and $|\zeta_4|^2 = \rho_2 - k\varepsilon_1^2 - \varepsilon_2^2 > \dfrac{\rho_2}{2}$ when ε_1 and ε_2 are sufficiently small. Note that for such $|\zeta_k|$ we have that $g \geqslant \dfrac{\rho_1^2\rho_2^{k+2}}{2^{k+4}}$, and the numerator $h(\bar\zeta) \wedge d\zeta$ is bounded. Therefore, the form ω is bounded in the neighborhood of $\{\zeta_1 = \zeta_2 = 0\}$. Similarly one can show that ω is bounded in the neighborhood of $\{\zeta_1 = \zeta_4 = 0\}$. □

Proposition 2 implies that the form ω is integrable over the cycle Γ_0^k.

Corollary 1. *The equality $\int_{\Gamma_0^k} \omega = C$ holds, where C is some nonzero constant.*

Proof. (8) and (9) imply

$$\int_{\Gamma_0^k} \omega = \int_{|\lambda_1|=1} \frac{d\lambda_1}{\lambda_1} \int_{|\lambda_2|=1} \frac{d\lambda_2}{\lambda_2} \int_{\mathbb{X}_k} \omega_0 = (2\pi i)^2 \int_{\mathbb{X}_k} \omega_0.$$

The last integral is a positive number by positivity of the form ω_0, as required. □

Now, we prove the following

Proposition 3. *The form ω is closed.*

PROOF. In fact we have to demonstrate that $(g/\tilde{g})\bar{\partial}h - \bar{\partial}(g/\tilde{g}) \wedge h = 0$. This would imply that

$$(g/\tilde{g})d(h\wedge d\zeta) - d(g/\tilde{g})\wedge(h\wedge d\zeta) = (g/\tilde{g})dh\wedge d\zeta - d(g/\tilde{g})\wedge h\wedge d\zeta = ((g/\tilde{g})\bar{\partial}h - \bar{\partial}(g/\tilde{g})\wedge h)\wedge d\zeta = 0,$$

i.e., the form ω is closed. By direct calculation of $\bar{\partial}h$ and $\bar{\partial}(g/\tilde{g})$ we get the statement of the proposition. □

Proposition 4. *Let $f(\zeta)$ be a holomorphic function in a neighborhood U about the origin and let ρ_1, ρ_2 be small enough to guarantee $\Gamma_0^k \subset U$. Then the following integral representation is valid:*

$$f(0) = \frac{1}{C}\int_{\Gamma_0^k} f(\zeta)\omega(\zeta), \tag{10}$$

where C is the normalization constant: $\int_{\Gamma_0^k}\omega = C \neq 0$.

PROOF. Since the form $f\omega$ is $\bar{\partial}$-closed, the integral in (10) is independent of $\rho_1, \ldots, \rho_r$. We rewrite it as

$$\int_{\Gamma_0^k} f(\zeta)\omega(\zeta) = \int_{\Gamma_0^k} f(0)\omega(\zeta) + \int_{\Gamma_0^k}(f(\zeta) - f(0))\omega(\zeta) =$$

$$= Cf(0) + \int_{\Gamma_0^k}(f(\zeta) - f(0))\omega(\zeta).$$

Let us show that the last integral vanishes. By substituting $\zeta \to \tau\zeta$, we obtain:

$$\begin{cases} \zeta_1 \to \tau^{k+1}\zeta_1, \\ \zeta_2 \to \tau\zeta_2, \\ \zeta_3 \to \tau\zeta_3, \\ \zeta_4 \to \tau\zeta_4. \end{cases}$$

Then the cycle Γ_0^k goes into the cycle Γ_τ^k:

$$\begin{cases} |\tau^{k+1}\zeta_1|^2 + |\tau\zeta_3|^2 = \rho_1, \\ k|\tau^{k+1}\zeta_1|^2 + |\tau\zeta_2|^2 + |\tau\zeta_4|^2 = \rho_2. \end{cases}$$

The integral goes to

$$\int_{\Gamma_0^k}(f(\zeta) - f(0))\omega(\zeta) = \lim_{\tau\to 0}\int_{\Gamma_\tau^k}(f(\zeta) - f(0))\omega(\zeta) = \lim_{\tau\to 0}\int_{\Gamma_0^k}(f(\zeta\tau) - f(0))\omega(\zeta\tau).$$

By Proposition 1 the form ω is invariant under the substitution $\omega(\zeta\tau) = \omega(\zeta)$. Since all s_k are positive, we have $\lim\limits_{\tau\to 0} f(\zeta\tau) = f(0)$. Thus

$$\lim_{\tau\to 0}\int_{\Gamma_0^k}(f(\zeta\tau) - f(0))\omega(\zeta\tau) = \lim_{\tau\to 0}\int_{\Gamma_0^k}(f(\zeta\tau) - f(0))\omega(\zeta) = 0.$$

The proof of the proposition is now completed. □

3. Integral Representation

We now consider the question of finding a domain D, such that the following integral representation is valid for every point $z \in D$

$$f(z) = \frac{1}{C} \int_{\mu^{-1}(\rho)} f(\zeta)\omega(\zeta - z). \tag{11}$$

Consider the domain $D = D_\rho$:

$$\left\{ \begin{array}{l} |\zeta_1|^2 + |\zeta_3|^2 < \rho_1, \\ |\zeta_2|^2 + |\zeta_4|^2 < \rho_2 - k\rho_1. \end{array} \right. \tag{12}$$

We will show that it is the required domain. Note that D is nonempty if the Kähler conditions (4) are satisfied.

Denote by $Z_z(\Sigma)$ the translate $z + Z(\Sigma)$:

$$Z_z(\Sigma) = \{\zeta_1 - z_1 = \zeta_3 - z_3 = 0\} \cup \{\zeta_2 - z_2 = \zeta_4 - z_4 = 0\},$$

and let Γ_z^k be the translate $z + \Gamma_0^k$:

$$\Gamma_z^k : \left\{ \begin{array}{l} |\zeta_1 - z_1|^2 + |\zeta_3 - z_3|^2 = \rho_1, \\ k|\zeta_1 - z_1|^2 + |\zeta_2 - z_2|^2 + |\zeta_4 - z_4|^2 = \rho_2. \end{array} \right.$$

Denote by $W = W_\rho$ 2-circular polyhedron defined by the system

$$\left\{ \begin{array}{l} |\zeta_1|^2 + |\zeta_3|^2 < \rho_1, \\ k|\zeta_1|^2 + |\zeta_2|^2 + |\zeta_4|^2 < \rho_2. \end{array} \right. \tag{13}$$

By $W_{2\rho}$ we denote the domain like (13), where the right-hand sides of the inequalities are $2\rho_1, 2\rho_2$.

Lemma 3. *For each $z \in D$ the cycle Γ_z^k lies in $W_{2\rho}$. Moreover, if the Kähler conditions (4) are satisfied then the homology $\Gamma_z \sim \Gamma_0^k$ holds in the domain $W_{2\rho} \setminus Z_z(\Sigma)$.*

Proof. Consider the following homotopy of the cycles Γ_0^k and Γ_z^k:

$$\left\{ \begin{array}{l} |\zeta_1 - tz_1|^2 + |\zeta_3 - tz_3|^2 = \rho_1, \\ k|\zeta_1 - tz_1|^2 + |\zeta_2 - tz_2|^2 + |\zeta_4 - tz_4|^2 = \rho_2, \end{array} \right. \tag{14}$$

where $0 \leqslant t \leqslant 1$. We will prove that the cycle (14) is disjoint from $Z_z(\Sigma)$ for any t in the interval $[0, 1]$.

Let us show that the cycle (14) is disjoint from the plane $\{\zeta_1 - z_1 = \zeta_3 - z_3 = 0\}$ in $Z_z(\Sigma)$. Substituting it to (14) we get $(1-t)^2(|\zeta_1|^2 + |\zeta_3|^2) = \rho_1$. The last equality is false since $(1-t)^2 \leqslant 1$ and $|\zeta_1|^2 + |\zeta_3|^2 < \rho_1$.

Similarly we show that the cycle (14) is disjoint from the plane $\{\zeta_2 - z_2 = \zeta_4 - z_4 = 0\}$ in $Z_z(\Sigma)$. Substituting it to (14) we get $k|\zeta_3 - tz_3|^2 = -(\rho_2 - k\rho_1) + (1-t)^2(|\zeta_2|^2 + |\zeta_4|^2)$ that never holds since $(1-t)^2 \leqslant 1$ and $|\zeta_2|^2 + |\zeta_4|^2 < \rho_2 - k\rho_1$. This completes the proof of the lemma. $\square$

We have thus proven the integral representation (11) for functions holomorphic in $W_{2\rho}$. Note that it suffices to take the holomorphy domain of the function $f(z)$ in (11) to be $W = W_\rho$, since the latter is a convex domain whose boundary contains the cycle Γ_0^k. It follows from convexity of W that a function holomorphic in W and continuous in the closure of W can be approximated by polynomials in the closure of W for which the integral representation (11) is proven. Thus, we arrive at the following

Theorem 2. *Suppose that $f(\zeta)$ is a holomorphic function in the domain W defined by (13) and f is continuous in the closure of W. Then the integral representation (11), with the cycle Γ_0^k defined by (3) is valid in the domain D defined by (12).*

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