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Chaotic Instantons and Ground Quasienergy Splitting in Kicked Double-well System with Time-reversal Symmetry

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Chaotic instanton approach was used to describe tunneling properties of the particle in the kicked double well system. Effective Hamiltonian for the kicked system was constructed using matrix expansion formula for one period evolution operator exponent. Chaotic instanton approximation was constructed in the framework of the effective model. This approximation was used for estimation of the particle energy range on the chaotic instanton trajectory. Formula for ground quasienergy splitting was obtained averaging nonperturbed trajectory action in the obtained energy range in the framework of chaotic instanton approach. Results of numerical calculations for the ground quasienergy splitting dependence on both the perturbation strength and frequency are in good agreement with the derived analytical formula.

Keywords: double-well potential, chaotic instanton, quasienergy levels.

Introduction

Investigation of the influence of small perturbation at the behavior of the nonlinear dynamical systems attracts permanent interest for the several last decades [1–3]. The connection between the semiclassical properties of perturbed nonlinear systems and purely quantum processes such as tunneling is a reach rapidly developing field of research nowadays [3, 4]. Our insight in some novel phenomena in this field was extended during the last decades. The most intriguing among them are the chaos assisted tunneling (CAT) and the closely related coherent destruction of tunneling (CDT).

The former in particular is an enhancement of tunneling in the perturbed low-dimensional systems at small external field strengths and driving frequencies [5–8]. This phenomenon takes place when levels of the regular doublet undergo an avoided crossing with the chaotic state [9,10]. The later, CDT phenomenon, is a suppression of tunneling which occurs due to the exact crossing of two states with different symmetries from the tunneling doublet [11]. In this case tunneling time diverges which means the total localization of quantum state on the initial torus.

CAT phenomenon as well as CDT were experimentally observed in a number of real physical systems: whispering gallery-type modes of microwave cavity having the form of the annular billiard [12], ultracold atoms [13,14], dielectric microcavities [15], two coupled optical waveguides [16,17]. Recently experimental evidence of coherent control of single particle tunneling in strongly driven double well potential was reported in Ref. [18].

A number of effective approaches [19–21] are used for analysis of the classical and quantum properties of the perturbed systems [22, 23]. The most common methods which are used to

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investigate the interplay between semiclassical properties of perturbed nonlinear systems and quantum processes are numerical methods based on Floquet theory [4, 24, 25]. Among other approaches we would like to mention the scattering approach for billiard systems [26] and approach based upon the presence of a conspicuous tree structure hidden in a complicated set of tunneling branches [27].

In this paper we will consider the original analytical approach based on instanton technique. Enhancement of tunneling in the system with perturbation in framework of this approach occurs due chaotic instantons which appear in perturbed case. This approach was proposed in [28–30] and used in [31]. Chaotic instanton approach will be developed here using effective model and used for description of the enhancement of tunneling in the kicked double well system. The purpose of the present study is to prove the ability of proposed chaotic instanton approach to give quantitative analytical description of tunneling well agreed with independent numerical calculations based on Floquet theory. It will give additional support and pulse for the further development of analytical methods to investigate tunneling phenomenon in quantum systems with mixed classical dynamics. Alternative approach based on quantum instantons which are defined using an introduced notion of quantum action was suggested in [32, 33]. Analytical approach for dynamical tunneling in perturbed systems based on classical resonances consideration was developed in [22, 34].

Double well potential is a special toy model among other nonlinear systems. This system is simple and nonlinear at the same time. It is convenient to use this model for tunneling analysis. This system is well studied in the nonperturbed case, e.g. on the base of instanton technique [35, 36] or WKB method [37]. Double well potential is often used for description of processes which occurred in wide range of real physical systems: such as light waves in two periodically-curved waveguides [17] or flipping of the ammonia molecule [38]. Perturbation in this paper is regarded in the form of the periodic kicks. One of the attractive features of this type of perturbation is the extensively-investigated simple quantum map which stroboscobically evolves the system from kick n to kick $n+1$.

The main aim of this study is a development of the chaotic instanton approach on the base of the construction of effective model. Kicked double well system is investigated using developed approach. Effective model is constructed using matrix expansion formula for one period evolution operator in Sec. 1. Properties of the Euclidean phase space are investigated in Sec. 2. Results which were obtained for effective model are used in Sec. 3 to derive analytical formula for lowest quasienergy doublet splitting dependence on perturbation parameters. Numerical calculations are performed in the same section to check the validity of this formula.

1. Effective Hamiltonian for the Kicked System

Hamiltonian of the particle in the double-well potential can be write down in the following form:

$$H_0 = \frac{p^2}{2m} + a_0\, x^4 - a_2\, x^2, \tag{1}$$

where m is the mass of the particle, a_0, a_2 are parameters of the potential. We consider the perturbation of the kick-type and choose it as follows:

$$V_{per} = \epsilon\, x^2 \sum_{n=-\infty}^{+\infty} \delta(t - nT), \tag{2}$$

where ϵ and T are perturbation strength and period, respectively, t is the time. Hamiltonian of the perturbed system is the following:

$$H = H_0 + V_{per}. \tag{3}$$

Euclidean equations of motion of the particle in the nonperturbed double-well potential ($\epsilon = 0$) have a well known solution — instanton. This solution is used for calculation of the ground energy splitting in the system without perturbation [35,36] and explains the rate of the tunneling process in it. Another solutions of the Euclidean equations of motion besides ordinary instanton are required to explain dynamical tunneling in perturbed system. Perturbation destroys the separatrix and trajectories near it become unbounded. Narrow stochastic layer is formed near nonperturbed separatrix due to perturbation. "Chaotic instanton" is appeared in this layer. It is the closest perturbed trajectory to separatrix. Thus it plays a dominant role in tunneling in perturbed system. This configuration will be investigated both analytically and numerically in Sec. 2.

We use Euclidean time in this section to work with instantons. Now we implement Wick rotation ($t \to -i\tau$) and define Euclidean Hamiltonian in the following form:

$$\mathcal{H}^E = \mathcal{H}_0^E - \epsilon\, x^2 \sum_{n=-\infty}^{+\infty} \delta(\tau - nT), \tag{4}$$

where $\mathcal{H}_0^E$ is the nonperturbed Euclidean Hamiltonian which can be write down as follows:

$$\mathcal{H}_0^E = \frac{p^2}{2m} - a_0\, x^4 + a_2\, x^2. \tag{5}$$

It is seen that all perturbed trajectories are consisted of nonperturbed trajectory parts connected by jumps which caused by kicks. In our approach we use the following estimative relation between the nonperturbed trajectory action and the Euclidean energy difference from the separatrix which can be obtained by expanding of the action variable near separatrix:

$$S[x(\tau,\xi)] = \pi J(E_{inst} - \xi) = S[x_{inst}(\tau,0)] - \alpha \sqrt{\frac{m}{a_2}}\, \xi, \tag{6}$$

where $S[x_{inst}(\tau,0)] = S_{inst} = 2\sqrt{m}\, a_2^{3/2}/(3\, a_0)$ is the nonperturbed instanton action, $\alpha = (1 + 18 \ln 2)/6$ is the numerical coefficient.

Now lets construct the effective Hamiltonian for the double well system with the perturbation of the kick-type. Initially we will calculate this Hamiltonian in the real time. Effective Hamiltonian in the Euclidean time will be obtained in the end of the section and it will be used for phase space analysis in the next section. We will construct an effective Hamiltonian for the system under investigation using the following definition [19]:

$$e^{-i\hat{H}_{eff}T} = e^{-i\hat{H}_0 T/2} e^{-i\epsilon \hat{x}^2} e^{-i\hat{H}_0 T/2}, \tag{7}$$

where RHS is a one-period evolution operator of the kicked system. Evolution operator has such a simple form due to type of the perturbation. Particle behaves like a nonperturbed between kicks. While during kicks the influence of the nonperturbed Hamiltonian is negligible in comparison with perturbation. The moment of kick is chosen in the middle of the period in order to conserve the time-reversal symmetry in the system ($t \to -t$) [3]. We restrict our consideration by sufficiently small values of both the perturbation strength and period. Deriving H_{eff} from definition (7) and using matrix expansion formula for the exponents we can write down effective Hamiltonian in the following way:

$$\hat{H}_{eff} = \frac{i}{T} \ln \left[1 - i\hat{H}_0 T - i\epsilon\hat{x}^2 - \frac{\hat{H}_0^2 T^2}{2} - \frac{\epsilon^2 \hat{x}^4}{2} - \frac{\epsilon T}{2}(\hat{H}_0 \hat{x}^2 + \hat{x}^2 \hat{H}_0) + O(3) \right],$$

where we show terms up to the second order on ϵ and T. Finally we obtain the effective Hamiltonian for the kicked system expanding the logarithm in the series:

$$\hat{H}_{eff} = \hat{H}_0 + \frac{\epsilon\nu}{2\pi}\hat{x}^2 - \frac{\epsilon T\hbar^2}{6m}\frac{p^2}{2m} + \frac{\epsilon T\hbar^2}{3m}a_0\hat{x}^4 - \frac{\epsilon T\hbar^2}{6m}a_2\hat{x}^2 + \frac{\epsilon^2\hbar^2}{3m}\hat{x}^2 + O(3). \qquad (8)$$

Nonperturbed Hamiltonian (1) is the first term in the last formula. The second term has the same order (zero) as nonperturbed part in small parameters (ϵ, T). Terms with first order are absent in the expression (8). Next terms which are shown in (8) have the second order in perturbation parameters. Terms with higher orders are not shown in last expression. We restrict our consideration by two first terms and neglect the others. Thus effective Hamiltonian for the kicked double well system can be write down in the following form:

$$H_{eff} = \frac{p^2}{2m} + a_0 x^4 - \left(a_2 - \frac{\epsilon\,\nu}{2\pi}\right)x^2. \qquad (9)$$

This Hamiltonian is coincided with nonperturbed Hamiltonian (1) with parameter a_2 replaced by $\tilde{a}_2$ which is defined as follows:

$$\tilde{a}_2 = a_2 - \frac{\epsilon\nu}{2\pi}. \qquad (10)$$

In contrast to kicked system (3) effective system with Hamiltonian (9) is autonomous. It should be mentioned that coefficient $\tilde{a}_2$ is negative for large values of perturbation parameters. There are no wells and tunneling in effective system with negative coefficient $\tilde{a}_2$. It restricts the range for the perturbation parameters variation. We use condition $\tilde{a}_2 = 0$ and obtain the following requirement:

$$\epsilon\,\nu < (\epsilon\,\nu)_{max} = 2\pi a_2. \qquad (11)$$

Thus we should use intermediate values of the perturbation frequency. Frequency should be not small to use matrix expansion formula and not larger than $2\pi a_2/\epsilon$. Effective potential for the set of perturbation strength and perturbation frequency $\nu = 4$ is shown in the Fig. 1. The model

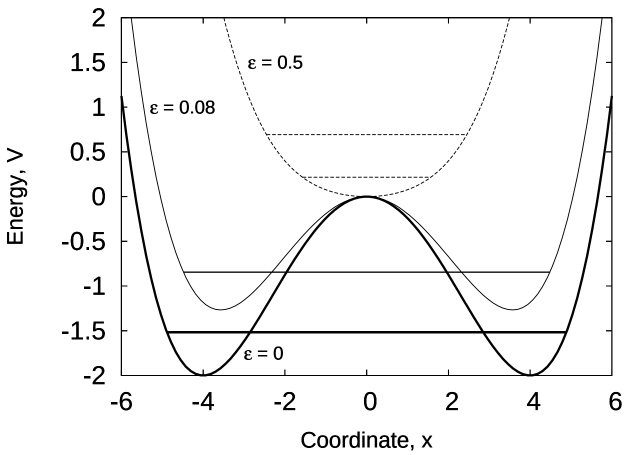

Fig. 1. Potential and two lowest levels for the effective model. Perturbation frequency $\nu = 4$, perturbation strength $\epsilon = 0$ (thick solid line), $\epsilon = 0.08$ (thin solid line) and $\epsilon = 0.5$ (dashed line). The model parameters are $m = 1$, $a_0 = 1/128$, $a_2 = 1/4$

parameters are the following $m = 1$, $a_0 = 1/128$, $a_2 = 1/4$. Two lowest energy levels are shown by horizontal lines in the figure. One can see that for parameters which satisfy to condition (11) two lowest levels are quasidegenerate ($\epsilon = 0$ and $\epsilon = 0.08$). Wells are became closer and barrier height is decreased when we increase perturbation parameters. It should enhance tunneling rate in kicked system. These consequences will be checked in the following sections for quasienergy spectrum and tunneling process in the system. Two lowest levels are shifted one from another for large values of the perturbation strength and frequency ($\epsilon = 0.5$ and $\nu = 4$ in case shown in the Fig. 1).

Now we will perform the transformation for effective Hamiltonian (9) into Euclidean time and obtain the following expression:

$$H^E_{eff} = \frac{p^2}{2m} - a_0 x^4 + \left(a_2 - \frac{\epsilon\,\nu}{2\pi}\right) x^2. \tag{12}$$

This Hamiltonian will be used in the following section to analyze the perturbed system phase space in Euclidean time and to construct a chaotic instanton approximation using effective model.

2. Euclidean Phase Space of the Perturbed and Effective Systems

Let us regard a possibility to describe properties of the classical motion in the kicked double well system in Euclidean time using effective model (12).We will conduct numerical simulations of the particle dynamics in the kicked system and in the framework of effective model for the same initial conditions. The deviation between finishing points of motion in phase space will be a criterion of the applicability of effective model in classical case.

We will consider kicked system (4) with perturbation parameters $\epsilon = 0.02$ and $\nu = 7$ and effective model with parameter $\tilde{a}_2$ which is defined by the expression (10). Separatrix in the effective model is shown in the Fig. 2 by thick solid line while separatrix in nonperturbed system by dashed line. Comparison of the classical motion on one period of the perturbation in the effective (thick solid lines) and kicked (thin solid lines) systems from the set of initial points (thick points) are shown in the inset in the Fig. 2. Trajectory in the effective model is a smooth line in the phase space. The trajectory for the perturbed system is a broken line due to the kick in the middle of the period. Trajectories for two systems lie close to each other as it is shown in the inset in the Fig.2.

In order to check applicability of the effective model for the description of the perturbed system phase space we perform numerical simulations of the particle dynamics using direct simulations for the kicked system and model calculations using effective Hamiltonian (12) from the set of initial points. Energy range for this set was chosen from the bottom of the wells ($E_{bottom} = 0$) to effective model separatrix energy which can be write down in the following way:

$$E_{top} = \tilde{a}_2^2/(4a_0). \tag{13}$$

Fifty energy values were chosen for this energy interval. One hundred of initial points were randomly evaluated for each energy value. Comparison of the energy of finishing points obtained in two models shows that effective model sufficiently strictly describe the classical dynamics of particles in the kicked double well system. Mean arithmetical deviation in energy is less then 0.1% of the barrier height. Thus phase portrait of the effective model coincides with the stroboscopic map for the kicked system besides separatrix region. Narrow stochastic layer should be placed near the separatrix in the kicked system.

Chaotic instanton from dynamical point of view is a set of nonperturbed trajectories connected by jumps. These jumps are induced by kicks with period T. Energy of the nonperturbed trajectories will be changed after each kick. Effective model can be used for analysis of the kicked system phase space. Thus we can use effective model separatrix to describe chaotic instanton in

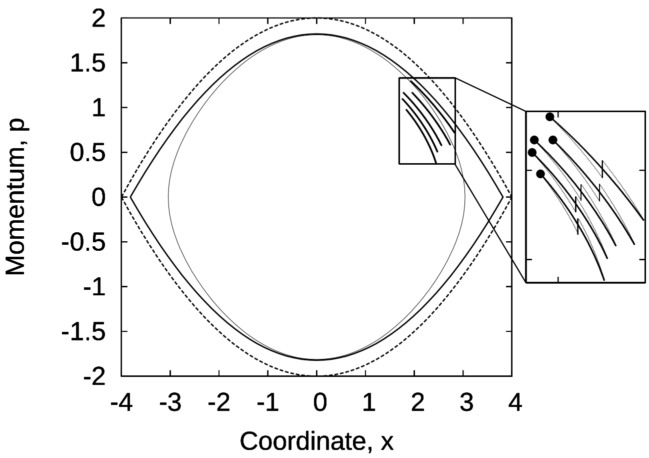

Fig. 2. Separatrix in the effective model for the kicked system with perturbation parameters $\epsilon = 0.02$ and $\nu = 7$ (thick solid line). Comparison of the particle classical motion on one period of the perturbation in effective (thick solid lines) and kicked (thin solid lines) systems from the set of initial points (thick points) are shown in the inset. The model parameters are $m = 1$, $a_0 = 1/128$, $a_2 = 1/4$

kicked system. This separatrix is shown by thick solid line in the Fig. 2. The chaotic instanton can be described in phase space by the following expression:

$$p_s(x) = \sqrt{2m\left(\frac{\tilde{a}_2^2}{4a_0} + a_0x^4 - \tilde{a}_2x^2\right)}. \tag{14}$$

Energy of the particle which is moving over the chaotic instanton trajectory is changing from maximum energy when particle momentum is zero to minimum at point where particle coordinate is zero. Lets write down expressions for these energy borders in explicit form. Chaotic instanton energy is maximum when $p_s(x_\pm) = 0$. This point is a turning point for the effective model ($x_\pm = \pm\sqrt{\tilde{a}_2/(2a_0)}$). Using last condition we obtain the following expression:

$$E_{max} = \mathcal{H}_0^E(p_s = 0, x_\pm) = \frac{a_2^2}{4a_0} - \frac{(\epsilon\nu)^2}{16a_0\pi^2} = E_{inst} - \frac{(\epsilon\nu)^2}{16a_0\pi^2} \approx E_{inst}, \tag{15}$$

where $E_{inst} = a_2^2/(4a_0)$ — energy of nonperturbed instanton. We restrict our consideration by small values of the perturbation strength ϵ and intermediate values of the perturbation frequency ν. Thus we neglect the second term in the expression (15). Chaotic instanton minimal energy is calculated from the condition $x = 0$ for expression (14). This energy is coincided with the energy on the effective model separatrix (13). Minimal energy can be write down in the following way:

$$E_{min} = \mathcal{H}_0^E(p_s, x = 0) = E_{top} = \tilde{a}_2^2/(4a_0) = \frac{a_2^2}{4a_0} - \frac{a_2\epsilon\nu}{4a_0\pi} + \frac{(\epsilon\nu)^2}{16a_0\pi^2} \approx E_{inst} - \frac{a_2\epsilon\nu}{4a_0\pi}, \tag{16}$$

where we neglect terms higher than linear one. Nonperturbed trajectory with energy E_{min} is shown in the Fig. 2 by thin solid line. It is seen that it coincides with the effective separatrix for $x = 0$. Nonperturbed trajectory with energy E_{max} is close to nonperturbed separatrix. Using last two expressions (15) and (16) we obtain the formula for energy range of the chaotic instanton:

$$\Delta\mathcal{H}^E_{ch.inst.} = E_{max} - E_{min} = \frac{a_2\,\epsilon\,\nu}{4\,a_0\,\pi}. \tag{17}$$

Expression (17) will be used in the following section in order to obtain analytical formula for lowest quasienergy doublet splitting dependence on perturbation parameters in the kicked system.

3. Ground Doublet Quasienergy Splitting Formula and Numerical Calculations

Lowest doublet energy splitting in nonperturbed double well potential is the following (see review [36]):

$$\Delta E_0 = 2\omega_0 d_0, \tag{18}$$

where ω_0 — small oscillation frequency near the bottom of the wells , d_0 — nonperturbed instanton density which in two-loop approximation can be write down in the following way [39]:

$$d_0 = \sqrt{\frac{6}{\pi}}\,\sqrt{S_{inst}}\,exp\left(-S_{inst} - \frac{71}{72}\frac{1}{S_{inst}}\right), \tag{19}$$

where S_{inst} — nonperturbed instanton action.

Ground doublet quasienergy splitting ($\Delta\eta$) in kicked system in our approach expresses in terms of chaotic instanton density (d) through formula which is similarly to (18):

$$\Delta\eta = 2\,\omega_0\,d, \tag{20}$$

where chaotic instanton density can be calculated by averaging the nonperturbed instanton density (19) over nonperturbed trajectory action (6) from $S(E_{min})$ to $S(E_{max})$:

$$d = \frac{1}{S(E_{max}) - S(E_{min})}\int_{S(E_{min})}^{S(E_{max})} d\,S\sqrt{\frac{6}{\pi}}\,\sqrt{S[x(\tau,\xi)]}\,exp\left(-S[x(\tau,\xi)] - \frac{71}{72}\frac{1}{S[x(\tau,\xi)]}\right).$$

This integral can be transformed to the integral over the energy difference $\xi = E_{inst} - E$ using formula (6) and we obtain the following expression:

$$\begin{aligned} d &= \frac{1}{\Delta\mathcal{H}^E_{ch.inst.}}\int_0^{\Delta\mathcal{H}^E_{ch.inst.}} d\,\xi\,\sqrt{\frac{6}{\pi}}\,\sqrt{S[x(\tau,\xi)]}\,exp\left(-S[x(\tau,\xi)] - \frac{71}{72}\frac{1}{S[x(\tau,\xi)]}\right) \approx \\ &\approx \frac{1}{\Delta\mathcal{H}^E_{ch.inst.}}\int_0^{\Delta\mathcal{H}^E_{ch.inst.}} d\,\xi\,\sqrt{\frac{6}{\pi}}\,\sqrt{S_{inst}}\,exp\left(-S_{inst} + \alpha\sqrt{\frac{m}{a_2}}\xi - \frac{71}{72}\frac{1}{S_{inst}}\right) = \\ &= \frac{d_0}{\alpha\sqrt{m/a_2}\,\Delta\mathcal{H}^E_{ch.inst.}}\left(e^{\alpha\sqrt{m/a_2}\,\Delta\mathcal{H}^E_{ch.inst.}} - 1\right). \end{aligned}$$

Expanding the exponent in a series and reducing the denominator we obtain the formula for chaotic instanton density in the following way:

$$d \approx d_0\left(1 + \frac{\alpha}{2}\sqrt{\frac{m}{a_2}}\,\Delta\mathcal{H}^E_{ch.inst.}\right) \approx d_0\,exp\left(\frac{\alpha}{2}\sqrt{\frac{m}{a_2}}\,\Delta\mathcal{H}^E_{ch.inst.}\right). \tag{21}$$

Now we can write down analytical formula for ground quasienergy levels splitting using expressions (17), (18), (20) and (21):

$$\Delta\eta(\epsilon,\nu) = \Delta E_0\,e^{k\,\epsilon\,\nu}, \tag{22}$$

where $k = \dfrac{\alpha\sqrt{m\,a_2}}{8\,\pi\,a_0}$. The last exponential factor in the expression (22) is responsible for the tunneling enhancement in the perturbed system. In nonperturbed case formula (22) coincides with the expression (18). Formula (22) will be checked in numerical calculations.

For the computational purposes it is convenient to choose the eigenvectors of harmonic oscillator as basis vectors. In this representation matrices of the Hamiltonian (1) and the perturbation (2) are real and symmetric. They have the following forms ($n \geqslant m$):

$$
\begin{aligned}
H^0_{m\,n} = \delta_{m\,n} &\left[\hbar\omega\left(n+\frac{1}{2}\right)+\frac{g}{2}\left(\frac{3}{2}\,g\,a_0\,(2m^2+2m+1)-a_2'(2m+1)\right)\right]+\\
&+\delta_{m+2\,n}\,\frac{g}{2}\,(g\,a_0(2m+3)-a_2')\,\sqrt{(m+1)(m+2)}+\\
&+\delta_{m+4\,n}\frac{a_0 g^2}{4}\sqrt{(m+1)(m+2)(m+3)(m+4)},\\
V_{m\,n} = \epsilon\,\frac{g}{2}\,&\left(\delta_{m+2\,n}\,\sqrt{(m+1)(m+2)}+\delta_{m\,n}(2m+1)\right),
\end{aligned}
$$

where $g = \hbar/m\omega$ and $a_2' = a_2 + m\,\omega^2/2$, $\hbar$ is Planck constant which we put equal to 1, ω — frequency of the basis harmonic oscillator which is arbitrary, and so may be adjusted to optimize the computation. We use the value $\omega = 0.2$ with parameters $m = 1$, $a_0 = 1/128$, $a_2 = 1/4$ which are chosen in such a way that nonperturbed instanton action is large enough for energy splitting formula for nonperturbed system to be valid and not too big in order to decrease errors of numerical calculations. The matrix size is chosen to be equal to 200×200. Calculations with larger matrices give the same results. System of computer algebra Mathematica was used for numerical calculations.

We calculate eigenvalues of the one-period evolution operator (RHS of the expression (7)) and obtain quasienergy levels (η_k) which are related with the evolution operator eigenvalues (λ_k) through the expression $\eta_k = i\,\ln\lambda_k/T$. Then we get ten levels with the lowest one-period average energy which is calculated using the formula $\langle v_i|\,H_0 + V/T\,|v_i\rangle$ ($|v_i\rangle$ are the eigenvectors of the one-period evolution operator).

Performed numerical calculations give the dependence of the ground quasienergy splitting both on the strength (Fig.3(a)) and the frequency (Fig.3(b)) of the perturbation. Results of numerical calculations are plotted in the Fig. 3 by points. Axis $\Delta\eta$ is shown in logarithmic scale. Obtained dependencies are exponential as it was predicted by chaotic instanton approach and obtained analytical formula (22).

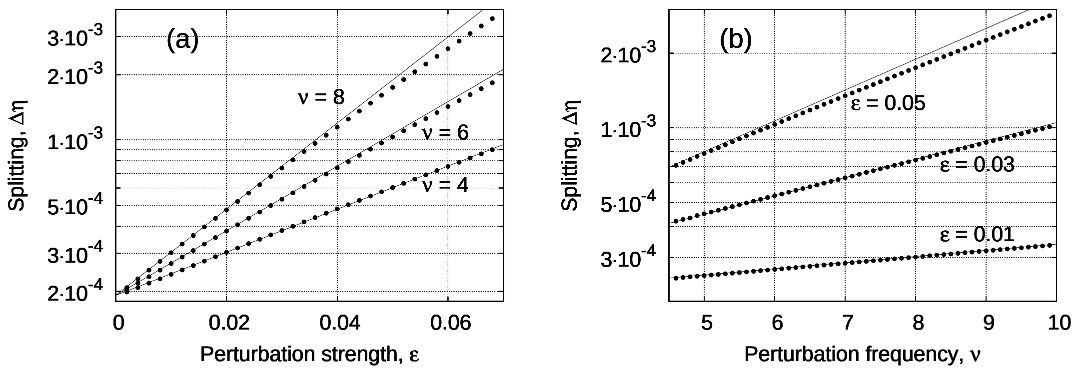

Fig. 3. Quasienergy splitting as a function of the strength (a) and frequency (b) of the perturbation. Lines — analytical formula (22), points — numerical results. The model parameters are $m = 1$, $a_0 = 1/128$, $a_2 = 1/4$

Analytical results are plotted in the Fig. 3 (a) and (b) by straight solid lines. Numerical points lie close to these lines. The agreement between numerical calculations and analytical expression is good in the parametric region considered.

Conclusions

Chaotic instanton approach is used to describe the influence of the perturbation on the quantum properties of the particle in the kicked double well system. Effective model is used for description of the chaotic instanton properties. Effective Hamiltonian is constructed for perturbed system using matrix expansion formula for operator exponent for small strength and period of the perturbation. Dynamical simulations for perturbed system and effective model show that effective Hamiltonian can be used for the description of the phase space of the perturbed system. Approximation for chaotic instanton was constructed in effective model. This approximation is a separatrix in the effective model which can be described analytically. Energy range for the particle which is moved over chaotic instanton trajectory was obtained using mentioned approximation.

Formula for ground quasienergy levels splitting is obtained averaging instanton density in the obtained energy range in the framework of chaotic instanton approach. This formula predicts exponential dependence of the ground doublet splitting on both the perturbation strength and frequency. Numerical calculations for quasienergy levels dependence on the perturbation parameters are performed to check obtained analytical formula. Results of numerical calculations for the quasienergy spectrum confirm the exponential dependence of the ground splitting on both the perturbation strength and frequency. They are in good agreement with the derived analytical formula (22).

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Хаотические инстантоны и расщепление нижних квазиэнергетических уровней в возмущенном двуямном потенциале с симметрией обращения времени

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Для описания характеристик туннелирования частицы в двуямной системе с дельтообразным возмущением был использован метод хаотических инстантонов. Эффективный гамильтониан для данной системы был построен с использованием формулы матричного разложения для оператора эволюции на одном периоде. В рамках эффективной модели была построена аппроксимация хаотического инстантона. Данная аппроксимация была использована для оценки диапазона энергии частицы на траектории хаотического инстантона. В рамках метода хаотических инстантонов выведена формула для расщепления нижних квазиэнергетических уровней путем усреднения действия невозмущенного инстантона в полученном диапазоне. Результаты численных вычислений зависимости расщепления нижних уровней от величины и частоты возмущения находятся в хорошем согласии с полученной аналитической формулой.

Ключевые слова: двуямный потенциал, хаотический инстантон, квазиэнергетические уровни.