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# Modelling of a Sandwich Plate Cross-section with Different Moduli of the Material under Cylindrical Loads

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**Abstract.** A model of three-layer sandwich plate consisting of two layers of composite material connected by an elastic isotropic layer is considered in the paper. Composite layers with different tensile and compression moduli of elasticity are described as an orthotropic material reinforced with parallel carbon fibres. Constitutive equations of the model are based on the generalized rheological method. The energy functional is constructed with the use of the Lagrange variational method which is minimized using the initial stress method and the finite element method. The results of a series of computational experiments are presented wherein the stress-strain state of a vertical section of a plate under the action of cylindrical load is calculated.

**Keywords:** composite material, multi-modular theory of elasticity, generalized rheological method, composite plate, finite element method.

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Composite materials are materials consisting of two or more distinct components and they have properties different from the properties of the original materials. In addition, the composition and distribution of individual components are known in advance, the proportion of each component is not lower than a certain value, and there are clear boundaries separating the starting materials [1,2]. Despite the heterogeneity of composites on micro-scale they can be considered as homogeneous materials on macro-scale. The components of a composite material are divided into a continuous phase, which is called the matrix, and a reinforcing phase. Moreover, the same material can play the role of a matrix or be a reinforcing material in various composites [3].

Various industries such as automotive industry, mechanical engineering, aircraft manufacturing and space industry widely use composite materials. The use of composite materials is growing in aerospace industry. The share of composites ranges from 15% to 30% of the total weight in modern aircraft, and in rocket engines reaches 90% [4, 5].

One of the composite materials is sandwich structures consisting of a filler and a shell. Polymers reinforced with glass fibre, carbon fibre or biofilter [6] can be used as shell material. Sandwich structures are increasingly used in industry, building structures and transportation due to their light weight and strength under heavy loads. A sandwich structure in which the shell is a fibrous composite reinforced with long parallel fibres, and the filler is an elastic isotropic material is considered in this paper. Since such shell material has different moduli of elasticity and different strengths it is necessary to take this into account when calculating structures made of such material [7].

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One of the approaches that allows one to construct model that takes into account different moduli of elasticity of a composite under tension and compression is the generalized rheological method [8]. The method provides thermodynamically correct governing equations for fibre composites. The method is based on the construction of rheological schemes using basic elements (elastic spring, viscous damper and plastic hinge) and a new element — hard contact that simulates the behaviour of ideal granular medium with absolutely solid particles. The rheological method has proven itself well in modelling the dynamics and statics of granular and porous materials with a threshold change in rigidity during the collapse of pores. A similar change in stiffness occurs in fibre composite when the sign of deformation changes.

# 1. Generalized rheological method

Let us construct the scheme describing a three-layer structure that consists of two layers of a multi-modulus composite and one layer of isotropic filler. Fig. 1 shows rheological scheme consisting of five elastic elements and two rigid contact elements, where $\sigma$ is the actual stress tensor, $a_i$ is the tensor of elastic moduli in compression for $i$-th layer, $b_i$ is the tensor of additional moduli under tension for the $i$-th layer.

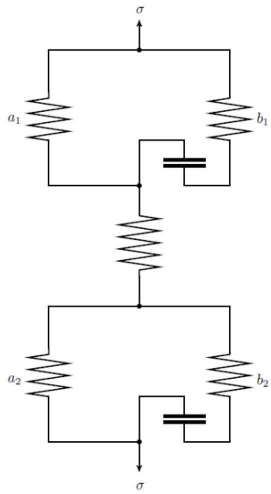


Fig. 1. Scheme of three-layer sandwich plate

Derivation of the rigid contact equations used to describe heteromodularity is presented in [9, 10]. Two equivalent variational inequalities for the reverse rigid contact are

$$\sigma(\varepsilon - \tilde{\varepsilon}) \geqslant 0, \quad \varepsilon, \tilde{\varepsilon} \leqslant 0; \qquad (\sigma - \tilde{\sigma})\varepsilon \geqslant 0, \quad \sigma, \tilde{\sigma} \geqslant 0. \tag{1}$$

Let us consider layer of composite material separatly.

Layer of a material with different moduli is described by the diagram shown in Fig. 2, where $\varepsilon$ is the strain tensor, $\sigma$ is the actual stress tensor, $\sigma'$ is the additional stress tensor, $a$ is the tensor of elastic moduli in compression, $b$ is the tensor of additional moduli in tension. The governing equations of the stress-strain state of elastic composites for finite linear or non-linear relations between stress tensors $\sigma$ and strain tensors $\varepsilon$ admit the potential representation

$$\sigma = \frac{\partial \Phi(\varepsilon)}{\partial \varepsilon}, \quad \varepsilon = \frac{\partial \Psi(\sigma)}{\partial \sigma}. \tag{2}$$

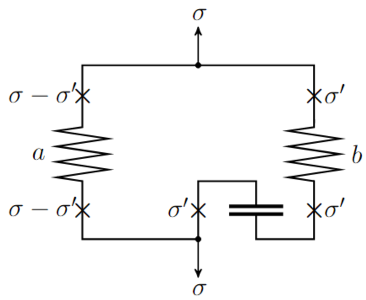


Fig. 2. Scheme of layer of composite material

Here $\Phi$ and $\Psi$ are elastic potentials of stress and strain related to each other through the Young transformation:

$$\Psi(\sigma) = \sup_{\varepsilon}(\sigma : \varepsilon - \Phi(\varepsilon)), \tag{3}$$

$$\Phi(\varepsilon) = \sup_{\sigma}(\sigma : \varepsilon - \Psi(\sigma)), \tag{4}$$

where the colon denotes double convolution of tensors.

According to [11, 12] such scheme corresponds to the following governing equation

$$\sigma = a : \varepsilon + b : (\varepsilon - \Pi(\varepsilon)), \tag{5}$$

where $\Pi(\varepsilon)$ is the projection of tensor $\varepsilon$ onto the cone $C$ according to the norm $|\varepsilon| = \sqrt{\varepsilon : b : \varepsilon}$ and stress and strain potentials

$$\Phi(\varepsilon) = \frac{1}{2}\varepsilon : a : \varepsilon + \frac{1}{2}(\varepsilon - \Pi(\varepsilon))^2, \tag{6}$$

$$\Psi(\sigma) = \frac{1}{2}\,\sigma : a^{-1} : \sigma - \frac{1}{2}\|\pi(\bar{\sigma})\|^2, \tag{7}$$

where $\pi(\bar{\sigma})$ is the projection of stress tensor $\bar{\sigma}$ onto the cone that is conjugate to the cone $C$ according to the norm $\|\sigma\|^2 = \sigma : (a^{-1} + b^{-1}) : \sigma$. The equivalent form of equation (9) are two equations for the actual stress tensor $\sigma$, the additional stress tensor $\sigma'$ and the intrinsic strain tensor of the rigid contact $\varepsilon'$:

$$\sigma - \sigma' = a : \varepsilon, \quad \sigma' = b : (\varepsilon - \varepsilon'). \tag{8}$$

Let us assume that governing equation at each point of composite layers of the plate has the form

$$\sigma = a(x_1, x_2, x_3) : \varepsilon + b(x_1, x_2, x_3) : (\varepsilon - \Pi(\varepsilon)), \tag{9}$$

where $a(x_1, x_2, x_3)$ and $b(x_1, x_2, x_3)$ take constant values $a_1, b_1$ and $a_2, b_2$ for each layer, respectively, cones $C_i$ are half-spaces associated with direction of reinforcement. In the filler layer, potentials and the governing equation take the form

$$\Phi(\varepsilon) = \frac{1}{2}\varepsilon : a_m : \varepsilon, \quad \Psi(\sigma) = \frac{1}{2}\,\sigma : a_m^{-1} : \sigma, \quad \sigma = a_m : \varepsilon. \tag{10}$$

## 2. Sandwich plate section

Let us consider the stress-strain state of the sandwich plate section. Let the $x_1$ axis of the Cartesian coordinate system $Ox_1x_2$ be located in the direction of fibre. Let us assume that during compression the plate material is described by the Hooke law for transversally isotropic body. Then the first equation of system (8) can be written in the following matrix form

$$\begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{pmatrix} = \begin{pmatrix} \frac{1}{E_1} & -\frac{\nu_2}{E_2} & 0 \\ -\frac{\nu_1}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{2G} \end{pmatrix} \begin{pmatrix} \sigma_{11} - \sigma'_{11} \\ \sigma_{22} - \sigma'_{22} \\ \sigma_{12} - \sigma'_{12} \end{pmatrix}, \tag{11}$$

where $E_1$ and $E_2$ are the Young moduli along the fibre and perpendicular to the fibre, respectively, $\nu_1$ and $\nu_2$ are the corresponding Poisson's ratios, and $G$ is the shear modulus. When the strain of fibres is positive additional stress is

$$\sigma'_{11} = b_{11}\varepsilon_{11},$$

which is introduced using tensor $b$. In the case under consideration it has only one non-zero component $b_{11}$. To ensure that introduced tensor is non-degenerate and positive definite small positive components $\beta$ and $\gamma$ are introduced, and they subsequently tend to zero. Let us write the second equation of system (8)

$$\begin{pmatrix} \varepsilon_{11} - \varepsilon'_{11} \\ \varepsilon_{22} - \varepsilon'_{22} \\ \varepsilon_{12} - \varepsilon'_{12} \end{pmatrix} = \begin{pmatrix} \frac{1}{b_{11}} & 0 & 0 \\ 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{2\gamma} \end{pmatrix} \begin{pmatrix} \sigma'_{11} \\ \sigma'_{22} \\ \sigma'_{12} \end{pmatrix}. \tag{12}$$

Substituting the resulting expressions into (9), governing equations for the plane stress state are obtained:

$$\begin{cases} \sigma_{11} = \dfrac{E_1(\varepsilon_{11} + \nu_2\varepsilon_{22})}{1 - \nu_1\nu_2} + b_{11}(\varepsilon_{11} - \Pi_{11}), \\ \sigma_{22} = \dfrac{E_2(\varepsilon_{22} + \nu_1\varepsilon_{11})}{1 - \nu_1\nu_2}, \\ \sigma_{12} = 2G\varepsilon_{12}. \end{cases} \tag{13}$$

Let us write out tensor $a^{-1} + b^{-1}$ and represent it in matrix form

$$\begin{pmatrix} \frac{1}{E_1} + \frac{1}{b_{11}} & -\frac{\nu_2}{E_2} & 0 \\ -\frac{\nu_1}{E_1} & \frac{1}{E_2} + \frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{2G} + \frac{1}{2\gamma} \end{pmatrix}. \tag{14}$$

Consider the minor of size $2 \times 2$ of the matrix $(a^{-1} + b^{-1})^{-1}$:

$$\frac{1}{(b_{11} + E_1)(\beta + E_2) - \nu_1\nu_2 b\beta} \begin{pmatrix} b_{11}(\beta + E_2)E_1 & \nu_1 b\beta E_2 \\ \nu_1 b_{11}\beta E_2 & \beta(b_{11} + E_1)E_2 \end{pmatrix}. \tag{15}$$

Taking the limit $\beta, \gamma \to 0$, matrix with single non-zero element $bE_1/(b + E_1)$ is obtained. Thus, components of the conditional stress tensor $\bar{\sigma}$ are

$$\bar{\sigma}_{11} = \frac{E_1 b_{11}}{b_{11} + E_1} \left( \frac{\sigma_{11}}{E_1} - \nu_2 \frac{\sigma_{22}}{E_2} \right), \quad \bar{\sigma}_{22} = \bar{\sigma}_{12} = 0. \tag{16}$$

Let us assume that each layer of the shell is reinforced with parallel fibres in the plane of the plate at an angle $R_i$ to the $x$ axis. Let us write down the governing equations for this case. Since the rotation occurs in the plane of the plate, components $\sigma_{22}$ and $\sigma_{12}$ remain unchanged:

$$\begin{cases} \sigma_{11} = \left( \dfrac{(E_1 \cos R_i + E_2 \sin R_i)(\varepsilon_{11} + \nu_2 \varepsilon_{22})}{1 - \nu_1 \nu_2} + b_{11} \cos R_i (\varepsilon_{11} - \Pi_{11}) \right), \\ \sigma_{22} = \dfrac{E_2(\varepsilon_{22} + \nu_1 \varepsilon_{11})}{1 - \nu_1 \nu_2}, \\ \sigma_{12} = 2G\varepsilon_{12}. \end{cases} \tag{17}$$

Let us consider the following problem. Region $\Omega$ with boundary $\Gamma$ coincide with the vertical section of the sandwich plate. Boundary $\Gamma$ consists of a part $\Gamma_u$ on which there are no movements and part $\Gamma_\sigma$ that does not intersect with it, and distributed load is specified on part $\Gamma_\sigma$:

$$\begin{cases} u = 0 & \text{на } \Gamma_u, \\ \sigma n = q & \text{на } \Gamma_\sigma. \end{cases} \tag{18}$$

It is required to determine the vector displacement field $u$ and the tensor field $\sigma$ that satisfy the differential equations

$$\nabla \cdot \sigma = 0, \qquad 2\varepsilon(u) = \nabla u + (\nabla u)^*,$$

and boundary conditions (18), and for which the following variational equations are satisfied in $\Omega$

$$\sigma_i = a_i : \varepsilon_i + b_i : (\varepsilon_i - \Pi_i(\varepsilon_i)), \quad \sigma_m = a_m : \varepsilon_m. \tag{19}$$

Components of the small strain tensor are related to displacements as follows

$$\varepsilon_{11} = \frac{\partial u_x}{\partial x}, \quad \varepsilon_{22} = \frac{\partial u_y}{\partial y}, \quad \varepsilon_{12} = \frac{1}{2}\left( \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right).$$

Let us formulate variational principles that are equivalent to the differential formulation of the problem under consideration. The required displacement field minimizes the integral

$$J(u) = \int_\Omega (\Phi(\varepsilon(u)))\, d\Omega - \int_{\Gamma_\sigma} qu\, d\Gamma \tag{20}$$

on the linear space $U$ of generalized functions $u \in H^1(\Omega)$.

To obtain the equation of the stress-strain state, the Lagrange variational principle is used. The actual distribution of plate displacements is minimized on a set of variations consistent with the main boundary conditions by the elastic energy functional:

$$\sum_{i=1,2} \left( \int_{\Omega_i} \left( \frac{1}{2} \nabla u : (a_i + b_i) : \nabla u - b_i : \Pi_i(\varepsilon)\sigma : \nabla u \right) d\Omega_i - \int_{\Gamma_i} \vec{q} \cdot u\, d\Gamma_i \right) + \\ + \int_{\Omega_m} \left( \frac{1}{2} \nabla u : a_m : \nabla u \right) d\Omega_m - \int_{\Gamma_m} \vec{q} \cdot u\, d\Gamma_m = 0,$$

where index $i$ denotes the layer number, $u$ is the vector field of displacements in $\Omega$, $\nabla$ is the Hamilton operator, $\vec{q}$ is the stress vector at the boundary of the plate $\Gamma_\sigma$, $a_i$ is the tensor of elastic moduli under compression, $b_i$ is the tensor of additional moduli under tension, $a_m$ is the tensor of elastic moduli of the interlayer.

Let us apply the obtained constitutive equations to the analysis of the plane stress state of the section of a sandwich plate loaded along the edge with static self-balanced stress system, using the initial stress method. To do this, defining equation of the fibre composite in form (19) is replaced the with the following iterative formula

$$\sigma^k = \sum_{i=1,2} \left( (a_i + b_i) : \varepsilon^k - \Delta\sigma_i^{k-1} \right) + a_m : \varepsilon^k, \quad \Delta\sigma_i^{k-1} = b_i : \Pi_i(\varepsilon^{k-1}).$$

At the first step, the problem for unstressed plate is solved when initial stress tensor $\Delta\sigma^0$ is identically equal to zero. In this case, the elastic modulus tensor is $a + b$. For the next steps, tensor $\Delta\sigma^{k-1}$ is determined, using the projection of tensor $\varepsilon^{k-1}$. Taking into account the iterative formula, the elastic energy functional takes the following form

$$\sum_{i=1,2} \left( \int_{\Omega_i} \left( \frac{1}{2} \nabla u : (a_i + b_i) : \nabla u - \Delta\sigma^{k-1} : \nabla u \right) d\Omega_i - \int_{\Gamma_i} \vec{q} \cdot u \, d\Gamma_i \right) + \\ + \int_{\Omega_m} \left( \frac{1}{2} \nabla u : a_m : \nabla u \right) d\Omega_m - \int_{\Gamma_m} \vec{q} \cdot u \, d\Gamma_m = 0. \quad (21)$$

To ensure the uniqueness of the solution any point on the plate is fixed, and rotation around this point is excluded. By minimizing functional (21) at each step of the algorithm, the required displacement vector $u$ is obtained.

# 3. Numerical results

The finite element method is used for the numerical solution. The triangular Lagrange element with three nodes is used, and displacements $u_x, u_y$ are specified at the nodes. An irregular triangular mesh is constructed in domain $\Omega$. Vector of generalized coordinates $U$ of dimension $2n$ is introduced, where $n$ is the number of grid nodes. The functional is represented as a sum of integrals over all triangles of the mesh

$$J(U) = \sum_{l=1}^{m} \iint_{\Omega_l} \left( (U_l)^T S^T K(x_1, x_2) S U_l - b\Pi(S U_l^{k-1}) S U_l - q_l U_l \right) dx_1 \, dx_2, \quad (22)$$

where $\Omega_l$ is the domain of the $l$th finite element, $U_l$ is the local vector of generalized coordinates, $S_l$ is the local matrix of displacements and deformations, $K$ is the matrix of elastic constants, $q$ is a global vector of generalized forces, the superscript $T$ means transpose. When conducting computational experiments, sandwich plates with different layer thicknesses were considered. Loading schemes are presented in Fig. 3. The shell parameters corresponded to carbon fibre plastic are $E_1^+ = 114$, $E_1^- = 57$, $E_2 = 14$, $G = 3.5$ GPa, $\nu_1 = 0.19$. The filler is isotropic epoxy resin with parameters $E = 4$, $G = 1.54$GPa, $\nu = 0.3$. In the first series of computational experiments, tension-compression along fibres is considered. The figures show axial displacements for the plate with shell layer thickness of 1 mm and filler of 3 mm. The force of 50 kN (tension, Fig. 5) and −50 kN (compression, Fig. 6) is applied to the right side of the plate.

Similar calculations are carried out for the transverse direction. The deformation is calculated under the action of distributed load applied to the upper boundary of the plate. Displacements for tension and compression are shown in Fig. 7.

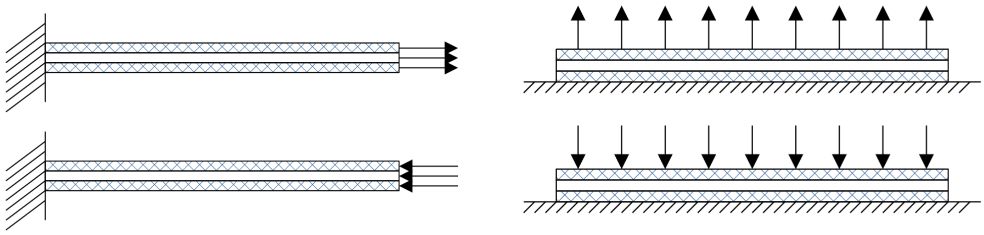

Fig. 3. Schemes of sandwich plate loading under tension-compression along and across fibres

As a result of calculations for the described material, the following values of effective elastic moduli for the sandwich structure are obtained

$$E_1^+ = 41\text{ГПа}, \quad E_1^- = 23\text{ГПа}, \quad E_2^+ = 5.88\text{ГПа}, \quad E_2^- = 5.93\text{ГПа}.$$

A series of computational experiments on bending of the sandwich plate under the action of concentrated force (diagram is shown in Fig. 4) is carried out.

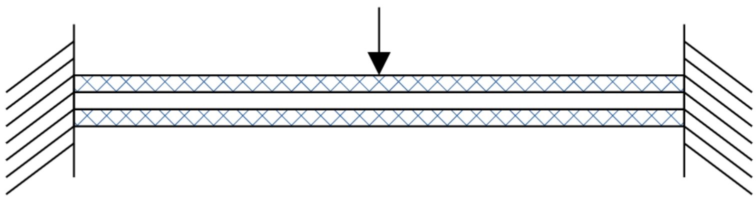

Fig. 4. Loading diagram for cylindrical bending

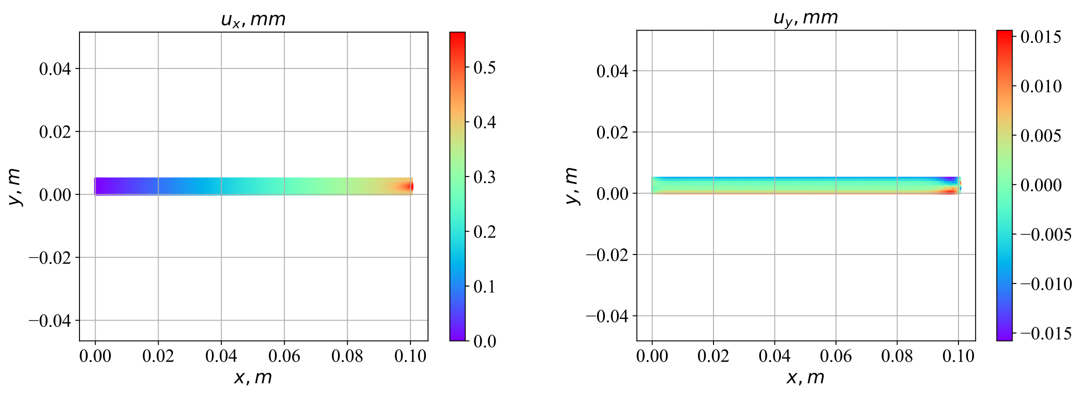


Fig. 5. Displacement of sandwich plate under tension with force applied along the shell reinforcement

Displacements and strains $\varepsilon_{11}$ during bending for sandwich plate with shell layer thickness of 4 mm and filler layer of 4 mm are shown in Fig. 8. One can observe the distribution of tension and compression zones near edges and the centre of the plate.

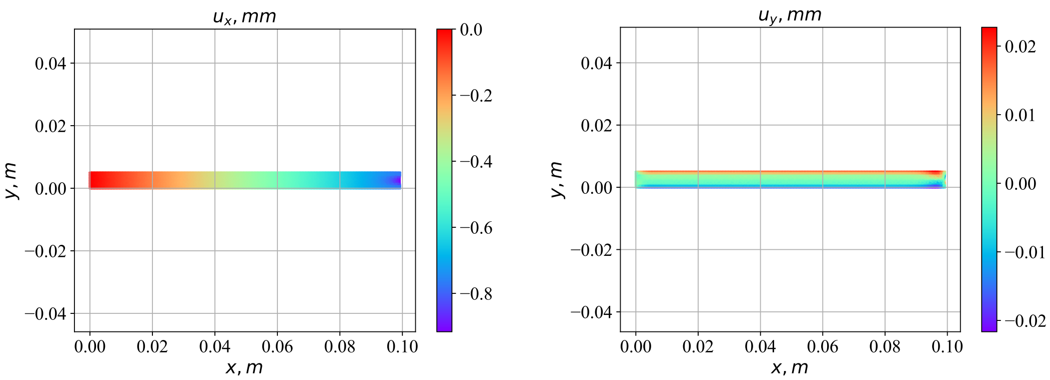


Fig. 6. Displacement of sandwich plate under compression with force applied along the shell reinforcement

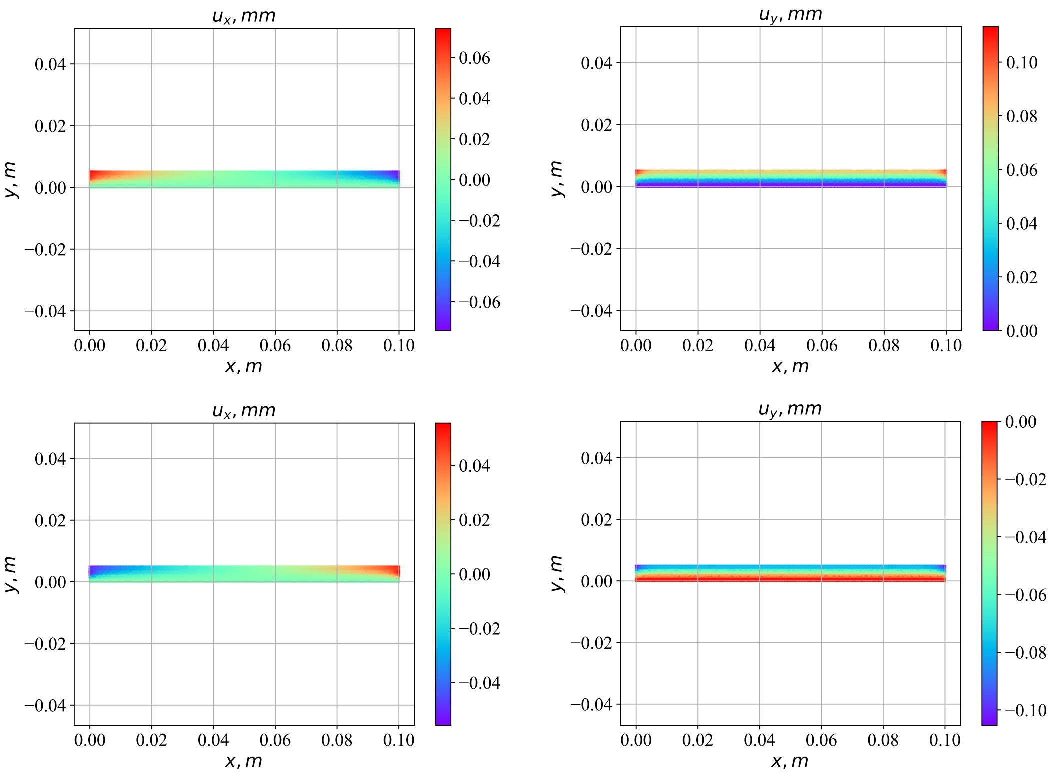


Fig. 7. Displacement of sandwich plate under tensile (top pictures) and compressive (bottom pictures) with force applied across the shell reinforcement

Tab. 1 shows values of deflections for various ratios of shell and filler thicknesses, where $T_i$ is the thickness of the interlayer, $T_a$ is the thickness of the reinforcement, $w_d$ is the deflection when difference in modularity is taken into account, $w$ is the deflection when difference in modularity is not taken into account. As thickness of the shell layers increases the influence of different moduli on the value of deflection is also increased. When difference in moduli is not taken into account different moduli the error of calculation of deflection can reach 10%.

Table 1. Deflection of sandwich plate under the action of concentrated force

| $T_i$, mm | $T_a$, mm | $w_d$, mm | $w$, mm | $\delta w$, % |
|---|---|---|---|---|
| 0.24 | 0.96 | 1.88 | 1.66 | 11% |
| 0.48 | 0.72 | 1.99 | 1.78 | 10% |
| 0.72 | 0.48 | 2.11 | 1.9 | 10% |
| 0.96 | 0.24 | 2.24 | 2.08 | 7% |

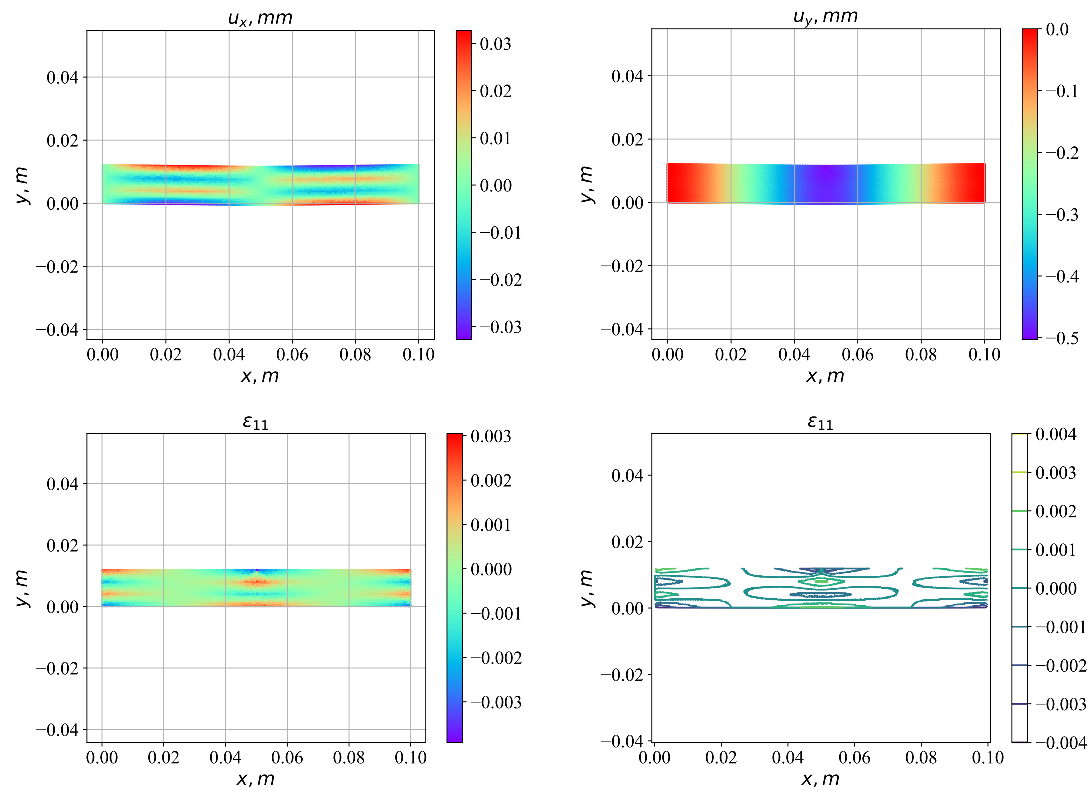


Fig. 8. Strain and displacement of sandwich plate with equal thickness of layers under the action of concentrated force

# Conclusion

Model of sandwich plate that takes into account the different resistance of the material to tension and compression was considered. Computational algorithm for solving the problem of calculating the stress-strain state of sandwich plate section under the influence of cylindrical load has been developed. The developed model allows one to determine tension-compression zones of the sandwich plate section. Analysis of the results of numerical calculations showed the influence of different moduli on the deformed state of sandwich plate under cylindrical bending.

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# Моделирование сечения сэндвич-пластины при цилиндрических нагрузках с учетом разномодульности материала

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**Аннотация.** В работе рассматривается модель трехслойной сэндвич-пластины, состоящей из двух слоев композитного материала, связанного упругой изотропной прослойкой. Слои композитного материала моделируются с учетом различных модулей упругости при растяжении и сжатии и представляют собой ортотропный материал, армированный параллельными углеродными волокнами. Представлена модель на основе обобщенного реологического метода, с помощью которого получены определяющие уравнения. С помощью вариационного метода Лагранжа построен функционал энергии, минимизация которого проведена с использованием метода начальных напряжений и метода конечных элементов. Представлены результаты серии вычислительных экспериментов по расчету напряженно-деформированного состояния вертикального сечения пластины под действием цилиндрической нагрузки.

**Ключевые слова:** композитный материал, разномодульная теория упругости, обобщенный реологический метод, сэндвич-пластина, метод конечных элементов.