EDN: WYJIOS
УДК 510.665; 510.643

Interval Multi-agent Logic with Reliability Operator

Vladimir R. Kiyatkin*
Vladimir V. Rybakov†
Siberian Federal University
Krasnoyarsk, Russian Federation

Received 10.04.2024, received in revised form 19.05.2024, accepted 24.07.2024

Abstract. We study intransitive temporal multi-agent logic with agents' multi-valuations for formulas letters and relational models representing reliable states. This logic is defined in a semantic as a set of formulas which are true at linear models with multi-valued variables. We propose a background for such approach and a technique for computation truth values of formulas. Main results concerns solvability problem, we prove that the resulting logic is decidable.

Keywords: modal logic, temporal logic, common knowledge, deciding algorithms, multi-agent logic.

Citation: V.R. Kiyatkin, V.V. Rybakov, Interval Multi-agent Logic with Reliability Operator, J. Sib. Fed. Univ. Math. Phys., 2024, 17(5), 679–683. EDN: WYJIOS.

Introduction

Mathematical logics widely applied in research concerning computer science and information sciences overall. We can observe the both side interaction. Tasks and problems in computer science generate new areas in mathematical logic and induce creation new technique and tools in mathematical logic itself. Conception of knowledge, which arose in the analysis of distributed systems, leaded to development multi-agent and multi-valued logical systems. More details about this can be found in the works of Halpern, Vardy (Reasoning About Knowledge [1]), Rybakov (Refined common knowledge logics or logics of common information, [2]).

It concern also from the certain point of view approaches to omniscience, monotonicity, justified knowledge, etc (cf. for example Artemov (Evidence-Based Common Knowledge [3]), S. Artemov (Evidence-Based Common Knowledge, (Technical Report TR-2004018) [4]), S. Artemov (Explicit Generic Common Knowledge, [5]), S. Artemov (Justification awareness, [5]). And it also was implemented in research concerning uncertainty and plausibility (cf. V. Rybakov Temporal Multi-Agent's Logic, Knowledge, Uncertainty, and Plausibility [6] Agents and Multi-Agent Systems: Technologies and Applications, LNCS, 2021, 2005–2014. Later some works were done towards consolidation such technique and to improve hybrid cooperation of the agents [7–9]. Also technique for formalization of knowledge was enriched by research in description logics (cf. Balder and Staler, [10]), first-order logic was also implemented (cf. F. Selaginella, A. Lombroso [11]). Various semantic technique was used (cf. Horrors, Settler, — A Description Logic with Transitive and Inverse Roles and Role Hierarchies [12]; Horrors, Geese, Karamu, Waller, — Using Semantic Technology to Tame the Data Variety Challenge, [13]).

Nowadays research concerning knowledge was combined with implementation of temporal logic (cf. Rybakov [14–17]). An automata-theoretic approach to multi-agent planning was evolved at Footbridge, [18].

*kiyatkinvr@mail.ru
†Vladimir_Rybakov@mail.ru https://orcid.org/0000-0002-6654-9712

In this our short paper we study intransitive temporal multi-agent logic with agents' multi-valuations for formulas letters. Common knowledge in [1] was modelled at Triple models. This brought interesting strong results, correlating well with observed examples and intuition. Here we wish to develop this approach towards modelling knowledge with Triple frames which are linear time models and treating reliable states of models. Here time is intransitive and it acts to only finite intervals. Main results concerns solvability problem, we prove that the resulting logic is decidable, prove existence of sone deciding algorithm.

1. Notation, Preliminary facts

Formulas of our logic $\mathcal{L}(\mathcal{M}_{\mathbb{N}})$ will be introduced as the set of special formulas, which are true at states of certain special relational Kripke-like models.

Alphabet for the language of our logic $\mathcal{L}(M_{\mathbb{N}})$ is defined in a standard way and consists of denumerable set of propositional letters (variables), parenthesises, logical Boolean operators, modal operators $\Box$, $\Diamond$, logical reliability operator $\mathcal{S}$ and also special time operator $\mathcal{N}$.

We remind, that every modal operation $\Box$ can be defined by means of modal operation $\Diamond$ as follows $\Box = \neg\Diamond\neg$. Now we give inductive definition of the formulas in the language of our logic $\mathcal{L}(\mathcal{M}_{\mathbb{N}})$.

1. Any propositional variable $p \in Prop$ is formula.
2. If α is formula, then $\neg\alpha$ is formula also.
3. If α and β are formulas, then $(\alpha \wedge \beta)$, $(\alpha \vee \beta)$ and $(\alpha \to \beta\beta)$ are formulas as well.
4. If α is formula, then $\Box\,\alpha$ is a formula also.
5. If α is formula, then $\Diamond\,\alpha$ is a formula also.
6. If α is formula, then $\mathcal{S}\alpha$ is formula as well.
7. If α is formula, then $\mathcal{N}\alpha$ is formula also.

There is no other formulas in the language of logic $\mathcal{L}(\mathcal{M}_{\mathbb{N}})$.

No we turn to describe relational models for our logic. We take as the basic set of the model $\mathcal{M}_{\mathbb{N}}$ the set $\mathbb{N}$ of all natural numbers. Here we suppose $\mathbb{N} = \bigcup\limits_{j=1}^{\infty} Int_j$, where Int_j are not intersecting intervals on $\mathbb{N}$ possibly of different length. Each interval Int_j can have inside some intervals $Int_{j1}, Int_{j2}, \ldots, Int_{js}$ of "reliable states" inside. Denote $C(Int_j) = \bigcup\limits_{t=1}^{s} Int_{jt}$. Binary relation $\preccurlyeq$ coincides with the standard linear order $\leqslant$ only inside but not outside every interval Int_j.

$Next$ is the binary relation inside every interval Int_j such that if $a \in Int_j$ and $aNextb$, then b is the first number of the interval Int_{j+1} (first following after Int_j, that is $a + 1 = b$ holds). We keep it to connect subsequently following intervals. We can write $Next(a) = b$. That makes connection between neighboring intervals. Linear multi-agent model is the model of the form:

$$\mathcal{M}_{\mathbb{N}} = \langle \mathbb{N}, \preccurlyeq, Next, V_1, \ldots, V_k \rangle\,,$$

where valuations $V_i,\ i \in [1,\ k]$ of every propositional variable p are some subsets $V_i(p)$ from $\mathbb{N}$.

Now we precisely define the truth values of formulas in our model.

For any a, b, $c \in \mathcal{M}$ the truth relations are as follows:

$$\forall p \in Prop: \ a \Vdash_{V_i} p \iff a \in V_i(p),$$
$$a \Vdash_{V_i} \neg\alpha \iff a \nVdash_{V_i} \alpha,$$
$$a \Vdash_{V_i} (\alpha \wedge \beta) \iff a \Vdash_{V_i} \alpha \quad \text{and} \quad a \Vdash_{V_i} \beta,$$
$$a \Vdash_{V_i} \Box\,\alpha \iff \forall\, b\,[(a \preccurlyeq b) \Rightarrow (b \Vdash_{V_i} \alpha)]\,,$$
$$a \Vdash_{V_i} \Diamond\,\alpha \iff \exists\, b\,[(a \preccurlyeq b) \wedge (b \Vdash_{V_i} \alpha)]\,.$$
$$a \Vdash_{V_i} \mathcal{S}\,\alpha \iff (\,a \in Int_j \Rightarrow (\exists\, b \in C(Int_j)\,[(a \ \preccurlyeq \ b) \Rightarrow b \Vdash_{V_i} \alpha])),$$
$$a \Vdash_{V_i} \mathcal{N}\,\alpha \iff \forall\, b\,[(a\,Next\,b) \Rightarrow b \Vdash_{V_i} \alpha]\,.$$

Formula α is said to be refutable in the logic , if there exist a state $a \in \mathcal{M}_{\mathbb{N}}$ such as $a \nVdash_{V_i} \alpha$. Formula α is said to be true in model $\mathcal{M}_{\mathbb{N}}$ if it is true at any state a from $\mathbb{N}$.

The set of all formulas, which are true in all our models is said to be the logic $\mathcal{L}(\mathcal{M}_{\mathbb{N}})$ generated by model $\mathcal{M}_{\mathbb{N}}$.

2. Decidability of logic $\mathcal{L}(\mathcal{M}_{\mathbb{N}})$

To solve the problem of decidability of logic $\mathcal{L}(\mathcal{M}_{\mathbb{N}})$ we shell transform models $\mathcal{M}_{\mathbb{N}}$ to get special finite like models, named $\mathcal{M}_C$, which, in a sense, are equivalent to $\mathcal{M}_{\mathbb{N}}$. That means that formula α belongs to the logic $\mathcal{L}(\mathcal{M}_{\mathbb{N}})$ if and if only it is true at any state from any model $\mathcal{M}_C$. The details will be given later.

Now we begin to subsequently describe undertaken transformation. First step.

1. For any state $a \in \mathcal{M}_{\mathbb{N}}$ and for valuation $V_i \quad i \in [1,\ k]$ we define the following theory:

$$Sub_i(a) = \{\beta \in Sub(\alpha) \mid b \Vdash_{V_i} \beta\}.$$

Evidently, there exist at most $2^{\|Sub(\alpha)\|}$ such different theories.

2. The set of theories:

$$T(a) = \{Sub_1(a), Sub_2(a), \dots, Sub_k(a)\}$$

corresponds to every state $a \in \mathcal{M}_{\mathbb{N}}$.

There exists only

$$d = 2^{\|Sub(\alpha)\|} \times \dots \times 2^{\|Sub(\alpha)\|} = 2^{k\cdot\|Sub(\alpha)\|}$$

such different sets of theories.

3. We shell obtain model $\mathcal{M}_C$ from $\mathcal{M}_{\mathbb{N}}$ with the help of the procedure of *rarefaction*.

Consider one arbitrary interval Int_j.

The set of all states in interval Int_j we denote $\mathcal{A}(Int_j)$, the set of all of reliable states in Int_j— $\mathcal{C}(Int_j)$ and the set of all not reliable states — $\mathcal{B}(Int_j)$. The character of the reliable states differs from the character of the other states, that is why we apply such rarefaction procedure for $\mathcal{B}(Int_j)$ and $\mathcal{C}(Int_j)$ separately.

Let us represent $\mathcal{B}(Int_j) = B_1 \cup B_2 \cup, \dots, \cup B_s$, where the any set $B_i,\ i \in [1,\ s]$ consists of the states b only, which have the same set $T(b)$ of theories.

First of all, we remove from Int_j all the states from B_1, except one the largest state b. We name that state *representative* of B_1, and denote $\overline{b}$. That is procedure of *rarefaction of states.*

Then we rarify in such manner all $B_2,\ B_3, \dots, B_s$ from $\mathcal{B}(Int_j)$.

After such transformations of the interval Int_j there leaved fixed only (some) s non-reliable states with pairwise different set of theories.

Further, we represent reliable states as follows – $\mathcal{C}(Int_j) = C_1 \cup C_2 \cup, \ldots, \cup C_r$, where the set $C_j,\ j \in [1,\ r]$ of states c, which have the same set $T(c)$ of theories. Then we apply procedure of rarefaction to every set $C_1,\ C_2, \ldots, C_r$ of reliable states as before we did for non-reliable states.

After such transformation of the interval Int_j inside it there were be leaved fixed only a finite (computable bounded size) reliable states with pairwise different sets of theories. So we obtain totally rarified interval with reliable and non-reliable states.

We denote this interval $\overline{Int_j}$.

If in the all our model we will replace the intervals Int_j by intervals $\overline{Int_j}$, and else will leave in any intervals the smallest and biggest states (re-deifying Next relation appropriately, to keep connection), then the states of intervals $\overline{Int_j}$ will have the same truth values of formulas as in the initial models (may be shown by usual induction by temporal and modal length o formulas).

To complete our result, we only need to clarify now many intervals $\overline{Int_j}$ subsequently maybe be chosen and inserted to support truth values of the formulas.

Theorem 1. *For any formula α with temporal degree t and any given modal degree this formula maybe be disproved at a model $\mathcal{M}_C = \langle \overline{N}, \preccurlyeq, Next, V_1, \ldots, V_k \rangle$, iff α may be disproved in the model obtained from intervals $\overline{Int_j}$ (described earlier above) by subsequent concatenation of at most $t+1$ finite intervals So we get the logic in decidable.*

Proof. Straightforward through induction by t using the described above construction. □

Conclusion

In this paper we considered problem of decidability of a logic with models including reliable states. We investigated temporal modal logic $\mathcal{L}(\mathcal{M}_{\mathbb{N}})$ for description of reliability information. We considered intervals of stable truth values of formulas and their interaction. The techniques is constructed and by it we wind an algorithm which may recognize decidability that logic.

This work is supported by the Krasnoyarsk Mathematical Center and financed by the Ministry of Science and Higher Education of the Russian Federation (Agreement No. 075-02-2024-1429).

References

[1] R.Fagin, J.Y.Halpern, Yoram Moses and Moshe Vardi), MIT, 1995.

[2] V.V.Rybakov, Refined common knowledge logics or logics of common information, *Archive for Mathematical Logic*, **42**(2003), 179–200. DOI:1 0.1007/s001530100134

[3] S.Artemov, Evidence-Based Common Knowledge, Technical Report TR-2004018 CUNY Ph.D. Program in Computer Science (revised version), 2006.

[4] S.Artemov, Explicit Generic Common Knowledge, Lect/ Notes in CS, LFCS 2013: Logical Foundations of Computer Science, 2013, 16–28.

[5] S.Artemov, Justification awareness, *Journal of Logic and Computation*, **30**(2020), no. 8, 1431–1446.

[6] V.Rybakov, Temporal Multi-Agent's Logic, Knowledge, Uncertainty, and Plausibility. – Agents and Multi-Agent Systems: Technologies and Applications, LNCS, 2021, 2005 - 2014.

[7] S.Babenyshev, V.Rybakov, Decidability of Hybrid Logic with Local Common Knowledge Based on Linear Temporal Logic LTL, CiE 2008: Lecture Notes in Computer Science, vol. 5028, 2008, 32-41. DOI: 10.1007/978-3-540-69407-6_4

[8] S.Babenyshev, V.Rybakov, Logic of Discovery and Knowledge: Decision Algorithm, KES (2), Lecture Notes in Computer Science, vol. 5178, 2008, 711–718.
DOI: 10.1007/978-3-540-85565-1_88

[9] S.Babenyshev, V.Rybakov, Describing Evolutions of Multi-Agent Systems, KES (1), Lecture Notes in Computer Science, vol. 5711, 2009, 38–45.
DOI: 10.1007/978-3-642-04595-0_5

[10] F.Baader, ,Sattler Expressive Number Restrictions in Description Logics, *J. Log. Comput.*, **9**(1999), no.3, 319–350.

[11] F.Belardinelli, A.Lomuscio, Interactions between Knowledge and Time in a First-Order Logic for Multi-Agent Systems: Completeness Results, *Journal of Artificial Intelligence Research*, **45**(2012), 1–45.

[12] I.Horrocks, U.Sattler, A Description Logic with Transitive and Inverse Roles and Role Hierarchies, Description Logics, 1998.

[13] I.Horrocks, M.Giese, E.Kharlamov, A.Waaler, Using Semantic Technology to Tame the Data Variety Challenge, *IEEE Internet Computing*, **20**(2016), no. 6, 62–66.
DOI: 10.1109/MIC.2016.121

[14] V.V.Rybakov, Non-transitive linear temporal logic and logical knowledge operations, *J. Logic and Computation*, Oxford University Press, **26**(2016), no. 3, 945–958.
DOI: 10.1093/logcom/exv016

[15] V.V.Rybakov, Temporal logic with overlap temporal relations generated by time states themselves, *Siberian Mathematical Reports*, **17**(2020), 923–932.

[16] V.V.Rybakov, Multi-agent temporal nontransitive linear logics and the admissibility problem, *Algebra and Logic*, **59**(2020), 87–100. DOI: 10.33048/alglog.2020.59.108

[17] V.V.Rybakov, Branching Time Logics with Multiagent Temporal Accessibility Relations, *Siberian Mathematical Journal*, **62**(2021), no. 3, 503–510. DOI: 10.33048/smzh.2021.62.313

[18] M.Wooldridge, An Automata-theoretic approach to multi-agent planning, Proceedings of the First European Workshop on Multi-agent Systems (EUMAS), December 2003, Oxford University.

Интервальная многоагентная логика с оператором надёжности

Владимир Р. Кияткин
Владимир В. Рыбаков
Сибирский федеральный университет
Красноярск, Российская Федерация

Аннотация. В предлагаемой статье мы изучаем нетранзитивную временную многоагентную логику с мультиозначиванием агентов и реляционные модели, представляющие надёжные состояния. Эти логики определяются семантически, как множества формул, истинных на линейных моделях с мультиозначиванием. В работе мы предложили основу для такого подхода и разработали технику для вычисления истинностных значений формул. Основной результат касается проблемы разрешимости. Доказано, что рассматриваемая логика разрешима.

Ключевые слова: модальные логики, модели Крипке, многоагентные логики, проблема разрешимости.