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Approximate Solution to a Model of the far Momentumless Axisymmetric Turbulent Wake

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Abstract. The flow in the far axisymmetric momentumless turbulent wake is described with the use of a mathematical model based on $k-\varepsilon$ semi-empirical model of turbulence. A group-theoretical analysis of the mathematical model of the wake is performed. The similarity reduction of the model to a system of ordinary differential equations is obtained. Asymptotic expansion of the solution in the vicinity of a singular point is used to construct approximate solution of corresponding boundary value problem.

Keywords: far momentumless axisymmetric turbulent wake, approximate solution, asymptotic expansion.

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Introduction

Turbulent momentumless wake behind body of revolution was considered in many publications (see, e.g., [1–17] and references therein). The turbulent axisymmetric wake has been studied experimentally [1–6]. These experiments showed that wake asymptotically tends to self-similarity at a relatively small distance from the body.

Theoretical analysis of the self-similarity of the wake was performed in [7–12]. In these works asymptotic behaviour of the far wake was investigated. The non-linear eigenvalue problem for turbulent energy, its dissipation rate and velocity deficit was solved numerically Hassid [10]. Exponents in the power law were also obtained. The asymptotic behaviour of the wake was analysed [12] using the theory of self-similar solutions of the second kind [18]. The similarity solution of the second-order turbulence model was obtained analytically and the process of transition to self-similarity was studied numerically. It was found that a single-point spectrum of solutions of corresponding eigenvalue problem for turbulent energy and dissipation rate exists. Moreover, it was shown that wake parameters is weakly dependent on empirical constant $C_{\varepsilon 2}$.

Numerical modelling of the axisymmetric momentumless turbulent wake was carried out using different semi-empirical turbulence models [13–17].

Mathematical model based on $k-\varepsilon$ semi-empirical model of axisymmetric momentumless wake was used to tackle the problem of degeneration of the far turbulent wake behind a self-propelled body in a passively stratified medium [19–22]. The model was reduced [20–22] to a system of ordinary differential equations using group-theoretical analysis [23] and the B–determining equations method [24]. The boundary-value problem for the reduced system was solved numerically

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using shooting method. Self-similarity index was determined during calculation process. An approach to determine self-similarity index has been suggested [25] where approximate solution to a model of the far plane momentumless turbulent wake was constructed using asymptotic expansion of the solution in a vicinity of the singular point.

This work is a continuation of studies presented in [20–22, 25]. In this paper an approximate solution was constructed to describe flow in the far axisymmetric momentumless turbulent wake.

1. Similarity reduction

The following semi–empirical model of turbulence is used to describe flow in the far axisymmetric momentumless turbulent wake

$$U_0\frac{\partial U_1}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(C_\mu r\frac{e^2}{\varepsilon}\frac{\partial U_1}{\partial r}\right), \tag{1}$$

$$U_0\frac{\partial e}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(C_\mu r\frac{e^2}{\varepsilon}\frac{\partial e}{\partial r}\right)-\varepsilon, \tag{2}$$

$$U_0\frac{\partial \varepsilon}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{C_\mu}{\sigma}r\frac{e^2}{\varepsilon}\frac{\partial \varepsilon}{\partial r}\right)-C_{\varepsilon 2}\frac{\varepsilon^2}{e}. \tag{3}$$

Here $U_1 = U - U_0$ is the deficit of the mean longitudinal velocity component, k is the kinetic energy of turbulence, and ε is the kinetic energy dissipation rate. It is assumed that fluid is incompressible and the flow is steady. Moreover, in what follows the undisturbed flow velocity U_0 is taken to be unity.

The empirical constants are as follows

$$C_\mu = 0.136,\ \sigma = 1.3,\ C_{\varepsilon 2} = 1.92.$$

The empirical constant C_μ has a modified value of 0.136 because model (1)–(3) was constructed as a simplification of more complicated algebraic model of Reynolds stresses [26–29].

The consequences of equation (1) is the following law of conservation of total excess momentum

$$J=\int_0^\infty rU_1dr=0. \tag{4}$$

A theoretical-group analysis [23] is used to construct self-similar solution. The Lie algebra basis of equations (1)–(3) consists of the following infinitesimal generators

$$X_1=\frac{\partial}{\partial x},\ X_2=\frac{\partial}{\partial U_1},\ X_3=U_1\frac{\partial}{\partial U_1},\ X_4=x\frac{\partial}{\partial x}-2e\frac{\partial}{\partial e}-3\varepsilon\frac{\partial}{\partial \varepsilon},\ X_5=r\frac{\partial}{\partial r}+2e\frac{\partial}{\partial e}+2\varepsilon\frac{\partial}{\partial \varepsilon}.$$

Using linear combination of operators X_3, X_4 and X_5 it is not difficult to obtain the following representation for solution of (1)–(3)

$$U_1 = x^\beta U_2(t),\ e = x^{2\alpha-2}K(t),\ \varepsilon = x^{2\alpha-3}E(t),\ t = r/x^\alpha, \tag{5}$$

here t is the self-similar variable, α and β are arbitrary constants appearing in the linear combination of operators X_3, X_4 and X_5.

Using representation (5) the initial mathematical model (1)–(3) can be reduced to the following system of ordinary differential equations

$$C_\mu\frac{K^2U_2''}{E}+\left(C_\mu\frac{K}{E}\left(2K'-\frac{KE'}{E}+\frac{K}{t}\right)+\alpha t\right)U_2'-\beta U_2=0, \tag{6}$$

$$C_\mu \frac{K^2 K''}{E} + 2C_\mu \frac{KK'^2}{E} - \left(C_\mu \frac{K^2}{E} \left(\frac{E'}{E} - \frac{1}{t} \right) + \alpha t \right) K' - 2(\alpha - 1)K - E = 0, \tag{7}$$

$$\frac{C_\mu}{\sigma} \frac{K^2 E''}{E} - \frac{C_\mu}{\sigma} \frac{K^2 E'^2}{E^2} + \left(\frac{C_\mu}{\sigma} \frac{K}{E} \left(2K' + \frac{K}{t} \right) + \alpha t \right) E' - C_{\varepsilon 2} \frac{E^2}{K} - (2\alpha - 3)E = 0. \tag{8}$$

Solution of reduced system (6)–(8) has to satisfy the following conditions

$$U_2'(0) = K'(0) = E'(0) = 0, \tag{9}$$

$$U_2(a) = K(a) = E(a) = 0. \tag{10}$$

The first group of conditions take into account that flow is symmetric with respect to the Ox axis. The second group of conditions follow from the requirement that flow is undisturbed outside the turbulent wake domain. The value of a is related to the turbulent wake semi-width and it can be set equal to unity in the following calculations by virtue of the invariance of equations of reduced system (6)–(8) with respect to the scaling transformation. It should also be noted that coefficients of system (6)–(8) have singularities in the boundary conditions.

2. Approximate solution

According to the results presented in [25] to construct approximate solution of boundary-value problem (6)–(10) asymptotic expansion of a solution of equations (6)–(8) near the singular point $t = 1$

$$\begin{gathered} U_2 = u_1(1-t)^{10/7} + u_2(1-t)^{17/7} + u_3(1-t)^{20/7} + u_4(1-t)^{24/7} + u_5(1-t)^{27/7} + \\ + u_6(1-t)^{30/7} + u_7(1-t)^{31/7} + o(|1-t|^{31/7}), \end{gathered} \tag{11}$$

$$\begin{gathered} K = k_1(1-t)^{10/7} + k_2(1-t)^{17/7} + k_3(1-t)^{20/7} + k_4(1-t)^{24/7} + k_5(1-t)^{27/7} + \\ + k_6(1-t)^{30/7} + k_7(1-t)^{31/7} + o(|1-t|^{31/7}), \end{gathered} \tag{12}$$

$$\begin{gathered} E = e_1(1-t)^{13/7} + e_2(1-t)^{20/7} + e_3(1-t)^{23/7} + e_4(1-t)^{27/7} + e_5(1-t)^{30/7} + \\ + e_6(1-t)^{33/7} + e_7(1-t)^{34/7} + o(|1-t|^{34/7}) \end{gathered} \tag{13}$$

is patched at the point $t = 0$ with an expansion of the solution near $t = 0$

$$U_2 = U_0 + \alpha_2 t^2 + \alpha_4 t^4 + \alpha_6 t^6 + \alpha_8 t^8 + o(t^8), \tag{14}$$

$$K = K_0 + \beta_2 t^2 + \beta_4 t^4 + \beta_6 t^6 + \beta_8 t^8 + o(t^8), \tag{15}$$

$$E = E_0 + \gamma_2 t^2 + \gamma_4 t^4 + \gamma_6 t^6 + \gamma_8 t^8 + o(t^8), \tag{16}$$

where

$$\begin{aligned} \alpha_2 = & \frac{125\beta U_0 E_0}{68K_0^2}, \quad \alpha_4 = \frac{125\beta U_0 E_0^2}{18496K_0^4} \left(\frac{124E_0}{K_0} - 600\alpha + 125\beta + 25 \right), \\ \alpha_6 = & -\frac{125\beta U_0 E_0^3}{45278208K_0^6} \left(469488\frac{E_0^2}{K_0^2} + \frac{100E0(30062\alpha - 2480\beta - 20481)}{K_0} - \right. \\ & \left. - 4185000\alpha^2 + 1075000\alpha\beta - 62500\beta^2 - 868750\alpha - 50000\beta + 1344375 \right), \end{aligned}$$

$$\alpha_8 = -\frac{125\beta U_0 E_0^4}{49262690304 K_0^8}\Bigg(149379072\frac{E_0^3}{K_0^3} - \frac{400E0^2(15958860\alpha - 911685\beta - 5144117)}{K_0^2} -$$
$$-\frac{2500E0(10062956\alpha^2 - 1156330\alpha\beta + 31000\beta^2 - 12644966\alpha + 724275\beta + 3240202)}{K_0} +$$
$$+ 13444250000\alpha^3 - 3935625000\alpha^2\beta + 312500000\alpha\beta^2 - 7812500\beta^3 + 15625187500\alpha^2 -$$
$$- 707656250\alpha\beta - 15625000\beta^2 - 23595718750\alpha + 1174453125\beta + 5909437500\Bigg),$$

$$\beta_2 = \frac{125E_0}{68K_0^2}(2K_0(\alpha - 1) + E_0),$$
$$\beta_4 = \frac{125E_0^2}{36992K_0^3}\left(\frac{872E_0^2}{K_0^2} + \frac{E_0}{K_0}(446\alpha - 1921) - 100(14\alpha + 9)(\alpha - 1)\right),$$
$$\beta_6 = \frac{125E_0^3}{22639104K_0^5}\Bigg(\frac{386136E_0^3}{K_0^3} - 2\frac{E_0^2}{K_0^2}(1091094\alpha + 44131) - 25\frac{E_0}{K_0}(97608\alpha^2 - 262642\alpha +$$
$$+ 61109) + 625(\alpha - 1)(3656\alpha^2 + 4190\alpha - 1911)\Bigg),$$
$$\beta_8 = \frac{125E_0^4}{24631345152K_0^7}\Bigg(\frac{113548800E_0^4}{K_0^4} - 4\frac{E_0^3}{K_0^3}(464953518\alpha - 134090393) +$$
$$+ 25\frac{E_0^2}{K_0^2}(582220340\alpha^2 - 282803792\alpha + 15406827) + 625\frac{E_0}{K_0}(31897184\alpha^3 - 87603958\alpha^2$$
$$+ 49562507\alpha - 7217483) - 31250(\alpha - 1)(216336\alpha^3 + 630596\alpha^2 - 596608\alpha + 113937)\Bigg),$$

$$\gamma_2 = \frac{13E_0^2}{136K_0^2}\left(\frac{48E_0}{K_0} + 50\alpha - 75\right),$$
$$\gamma_4 = \frac{13E_0^3}{73984K_0^4}\left(\frac{59808E_0^2}{K_0^2} + \frac{200E_0}{K_0}(86\alpha - 663) - 625(28\alpha + 37)(2\alpha - 3)\right),$$
$$\gamma_6 = \frac{13E_0^4}{45278208K_0^6}\Bigg(\frac{52068864E_0^3}{K_0^3} - \frac{600E_0^2}{K_0^2}(176050\alpha + 179049) - \frac{1250E_0}{K_0}(56324\alpha^2 -$$
$$- 234744\alpha - 26877) + 15625(2\alpha - 3)(1912\alpha^2 + 3460\alpha - 471)\Bigg),$$
$$\gamma_8 = \frac{13E_0^4}{45278208K_0^6}\Bigg(\frac{52068864E_0^4}{K_0^4} - \frac{19200E_0^3}{K_0^3}(13948348\alpha + 4637301) + \frac{2500E_0^2}{K_0^2}(226322592\alpha^2 +$$
$$+ 144240370\alpha + 705225) + \frac{15625E_0}{K_0}(35006416\alpha^3 - 122923916\alpha^2 + 29552956\alpha - 1486725) -$$
$$- (390625(2\alpha - 3))(236544\alpha^3 + 833020\alpha^2 - 455825\alpha + 49755)\Bigg).$$

Representing (11)–(13) as a power series at $t = 0$

$$U_2 = \bar\alpha_0 + \bar\alpha_1 t + \bar\alpha_2 t^2 + \bar\alpha_3 t^3 + \bar\alpha_4 t^4 + \bar\alpha_5 t^5 + \bar\alpha_6 t^6 + \bar\alpha_7 t^7 + \bar\alpha_8 t^8 + o(t^8), \tag{17}$$
$$K = \bar\beta_0 + \bar\beta_1 t + \bar\beta_2 t^2 + \bar\beta_3 t^3 + \bar\beta_4 t^4 + \bar\beta_5 t^5 + \bar\beta_6 t^6 + \bar\beta_7 t^7 + \bar\beta_8 t^8 + o(t^8), \tag{18}$$
$$E = \bar\gamma_0 + \bar\gamma_1 t + \bar\gamma_2 t^2 + \bar\gamma_3 t^3 + \bar\gamma_4 t^4 + \bar\gamma_5 t^5 + \bar\gamma_6 t^6 + \bar\gamma_7 t^7 + \bar\gamma_8 t^8 + o(t^8), \tag{19}$$

where

$$\bar\alpha_0 = u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7,\ \bar\alpha_1 = -\frac{1}{7}(10u_1 + 17u_2 + 20u_3 + 24u_4 + 27u_5 + 30u_6 + 31u_7),$$
$$\bar\alpha_2 = \frac{1}{49}(15u_+ 85u_2 + 130u_3 + 204u_4 + 270u_5 + 345u_6 + 372u_7),$$

$$\bar{\alpha}_3 = \frac{1}{343}(20u_1 - 85u_2 - 260u_3 - 680u_4 - 1170u_5 - 1840u_6 - 2108u_7),$$
$$\bar{\alpha}_4 = \frac{1}{2401}(55u_1 - 85u_2 - 65u_3 + 510u_4 + 1755u_5 + 4140u_6 + 5270u_7),$$
$$\bar{\alpha}_5 = \frac{1}{16807}(198u_1 - 187u_2 - 104u_3 + 408u_4 + 351u_5 - 1656u_6 - 3162u_7),$$
$$\bar{\alpha}_6 = \frac{1}{117649}(825u_1 - 561u_2 - 260u_3 + 748u_4 + 468u_5 - 1380u_6 - 2108u_7),$$
$$\bar{\alpha}_7 = \frac{1}{5764801}(26400u_1 - 14025u_2 - 5720u_3 + 13464u_4 + 7020u_5 - 16560u_6 - 23188u_7),$$
$$\bar{\alpha}_8 = \frac{1}{40353607}(128700u_1 - 6100u_2 - 20735u_3 + 42075u_4 + 19305u_5 - 39330u_6 - 52173u_7),$$
$$\bar{\beta}_0 = k_1 + k_2 + k_3 + k_4 + k_5 + k_6 + _7, \quad \bar{\beta}_1 = -\frac{1}{7}(10k_1 + 17k_2 + 20k_3 + 24k_4 + 27k_5 + 30k_6 + 31k_7),$$
$$\bar{\beta}_2 = \frac{1}{49}(15k_1 + 85k_2 + 130k_3 + 204k_4 + 270k_5 + 345k_6 + 372k_7),$$
$$\bar{\beta}_3 = \frac{1}{343}(20k_1 - 85k_2 - 260k_3 - 680k_4 - 1170k_5 - 1840k_6 - 2108k_7),$$
$$\bar{\beta}_4 = \frac{1}{2401}(55k_1 - 85k_2 - 65k_3 + 510k_4 + 1755k_5 + 1840k_6 - 2108k_7),$$
$$\bar{\beta}_5 = \frac{1}{16807}(198k_1 - 187k_2 - 104k_3 + 408k_4 + 351k_5 - 1656k_6 - 3162k_7),$$
$$\bar{\beta}_6 = \frac{1}{117649}(825k_1 - 561k_2 - 260k_3 + 748k_4 + 468k_5 - 1380k_6 - 2108k_7),$$
$$\bar{\beta}_7 = \frac{1}{5764801}(26400k_1 - 14025k_2 - 5720k_3 + 13464k_4 + 7020k_5 - 16560k_6 - 23188k_7),$$
$$\bar{\beta}_8 = \frac{1}{40353607}(128700k_1 - 56100k_2 - 20735k_3 + 42075k_4 + 19305k_5 - 39330k_6 - 52173k_7),$$
$$\bar{\gamma}_0 = e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7, \quad \bar{\gamma}_1 = -\frac{1}{7}(13e_1 + 20e_2 + 23e_3 + 27e_4 + 30e_5 + 33e_6 + 34e_7),$$
$$\bar{\gamma}_2 = \frac{1}{49}(39e_1 + 130e_2 + 184e_3 + 270e_4 + 345e_5 + 429e_6 + 459e_7),$$
$$\bar{\gamma}_3 = \frac{1}{343}(13e_1 - 260e_2 - 552e_3 - 1170e_4 - 1840e_5 - 2717e_6 + 3060e_7),$$
$$\bar{\gamma}_4 = \frac{1}{2401}(26e_1 - 65e_2 + 276e_3 + 1755e_4 + 4140e_5 + 8151e_6 + 9945e_7),$$
$$\bar{\gamma}_5 = \frac{1}{16807}(78e_1 - 104e_2 + 276e_3 + 351e_4 - 1656e_5 - 8151e_6 - 11934e_7),$$
$$\bar{\gamma}_6 = \frac{1}{117649}(286e_1 - 260e_2 + 552e_3 + 468e_4 - 1380e_5 - 2717e_6 - 1989e_7),$$
$$\bar{\gamma}_7 = \frac{1}{5764801}(8294e_1 - 5720e_2 + 10488e_3 + 7020e_4 - 16560e_5 - 24453e_6 - 15912e_7),$$
$$\bar{\gamma}_8 = \frac{1}{40353607}(37323e_1 - 20735e_2 + 34086e_3 + 19305e_4 - 39330e_5 - 48906e_6 - 29835e_7),$$

and equating like powers of t in (14)–(16) and (17)–(19), the system of 27 algebraic equations with 20 unknowns α, β, U_0, K_0, E_0, u_i, k_i, e_i, $i = 1, \ldots, 7$ is obtained. The equation for E at t^8 is omitted. This system of algebraic equations is solved numerically. The solution of this system is facilitated because (6) is split off from (7) and (8). The described procedure is initially applied to equations (7) and (8) to find

$$\alpha = 0.2208287460, \quad K_0 = 0.7998977201, \quad E_0 = 0.9205281496, \quad k_1 = 4.111142059,$$

$$k_2 = -22.05686118, \quad k_3 = 40.76497218, \quad k_4 = -49.00284702, \quad k_5 = 43.42154950,$$
$$k_6 = -31.82206053, \quad k_7 = 15.38400271, \quad e_1 = 10.09175704, \quad e_2 = -85.15426605,$$
$$e_3 = 173.9330323, \quad e_4 = -224.1225187, \quad e_5 = 205.3920322, \quad e_6 = -154.4430672,$$
$$e_7 = 75.22355867.$$

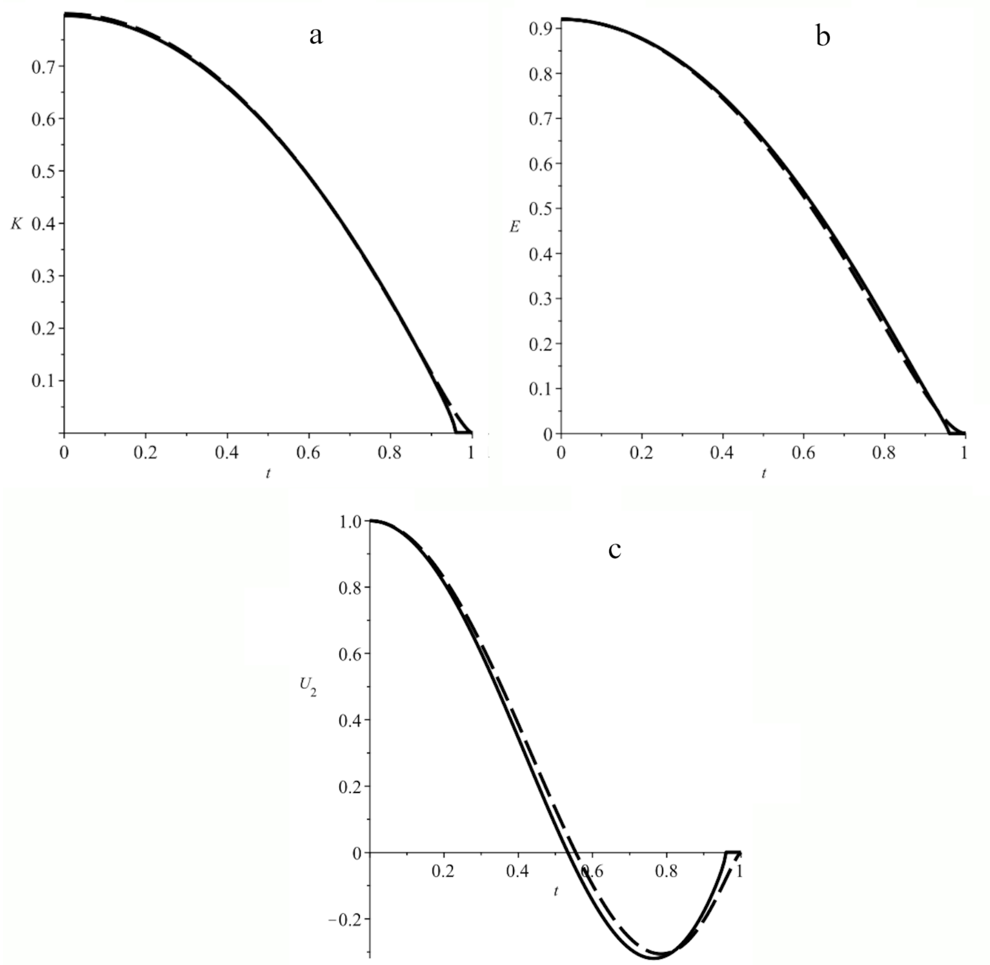

Fig. 1. Profiles of approximate and numerical solutions: a — the kinetic energy of turbulence; b — the kinetic energy dissipation rate; c — the deficit of the longitudinal averaged velocity component; solid lines — numerical solution, dotted lines — approximate solution

Obtained values are unique, taking into account (9), (10) and conditions

$$\alpha, K_0, E_0 > 0; \quad K'(t), E'(t) < 0, \quad t \in (0, 1).$$

Further, equation (6) is considered in a similar way and the following values are determined:

$$U_0 = 1, \quad \beta = -1.698508059, \quad u_1 = -10.17461628, \quad u_2 = 101.0215753, \quad u_3 = -191.1549873,$$
$$u_4 = 238.1557643, \quad u_5 = -197.6563167, \quad u_6 = 79.67680195, \quad u_7 = -18.86822130.$$

In order to increase accuracy one of the algebraic equations for determining coefficients of asymptotic expansion (11) is replaced by integral relation (4).

The obtained values α, K_0, and E_0 are used to solve boundary value problem (6)–(10) by the shooting method. As a result of numerical calculations the following values were found: $K_0 = 0.79617$, $E_0 = 0.92053$, and $\beta = -1.822$. The difference between approximate and numerical solutions does not exceed 5% (see Fig. 1).

Thus, at large distance behind the body the flow in an axisymmetric momentumless turbulent wake is characterized by the following laws of similarity degeneration: $U_1(x,0) \sim x^{-1.822}$, $e(x,0) \sim x^{-1.558}$, $\varepsilon(x,0) \sim x^{-2.558}$, $l \sim x^{0.221}$ (l is the width of the wake). The established laws are consistent with those presented in [12,17,21,22,].

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Приближенное решение модели дальнего безымпульсного осесимметричного турбулентного следа

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Аннотация. Для описания течения в дальнем осесимметричном безымпульсном турбулентном следе привлекается модель, основанная на $k - \varepsilon$ модели турбулентности. Выполнен теоретико-групповой анализ модели. Получена автомодельная редукция уравнений модели к системе обыкновенных дифференциальных уравнений. Для построения приближенного решения соответствующей краевой задачи используется асимптотическое разложение решения в окрестности особой точки.

Ключевые слова: дальний безымпульсный осесимметричный турбулентный след, приближенное решение, асимптотическое разложение.