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Hysteresis Effects in the Critical Behavior of Heisenberg Thin Films in an External Oscillating Field

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Abstract. In this work, we simulated hysteresis effects in thin Heisenberg films subjected to an external oscillating field by Monte Carlo methods. It was observed that the system exhibits different types of phase transitions below the Curie temperature, depending on the rate of field influence. Relaxation features of the system have been identified, which may also impact the nature of the dynamic phase transition.

Keywords: Heisenberg model, dynamic phase transition, Monte Carlo methods, hysteresis effects.

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The phenomenon of dynamic phase transition is widespread across all fields of human activity. It has been observed that dynamic phase transitions can be used to describe biological [1, 2] and chemical systems [3], as well as processes associated with human social behavior [4]. The exploration of dynamic phase transitions in promising materials such as nanographene [5] or $\mathrm{LiMn_{0.5}Fe_{0.5}PO_4}$ [6] opens new opportunities in energy engineering and design.

A dynamic phase transition in magnetic systems occurs when the speed of influence of an external oscillating field changes. At a high half-period of the external field, magnetization follows the cyclic changes in the field and remains in a dynamically disordered state. However, at a low half-period value, magnetization cannot qualitatively follow the oscillations and transitions into a dynamically ordered state. The transition between a dynamically ordered state and a dynamically disordered state is referred to as a dynamic phase transition.

The classic model for studying dynamic phase transitions is the Ising kinetic model. Work [7] was the first to prove the existence of a dynamic phase transition in magnetic structures.

In early experimental works [8, 9], a dynamic phase transition was observed in the hysteresis response with a change in the amplitude of the external field H_0. Recent experimental studies on the magnetization reversal of thin films [10–14] have indicated that the system can undergo a qualitative transition from one ordered state to another by introducing an additional field in conjunction with an external oscillating field. As demonstrated in previous works [13, 14], this additional field in dynamic phase transitions is comparable to the influence of the field $H(t)$ in thermodynamic phase transitions.

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Hysteresis effects in dynamic phase transitions are being actively studied numerically modeling and experimentally. However, many questions remain open and require additional study. The studies were carried out using the Heisenberg model, which is a more complex system compared to the Ising model [15], including the influence of anisotropy and three-dimensional spin. Considering the dynamic phase transition within a more complex model can provide additional insight into this phenomenon.

1. Model and methods

In this work, a thin magnetic film in an external oscillating field $H(t)$ with an amplitude H_0 below the Curie temperature was studied by Monte Carlo methods, in particular by the Metropolis algorithm using the anisotropic Heisenberg model.

The Hameltonian of the anisotropic Heisenberg model was chosen as:

$$H = -J \sum_{i,j} [(1 - \Delta(N))(S_i^{\mathrm{x}} S_j^{\mathrm{x}} + S_i^{\mathrm{y}} S_j^{\mathrm{y}}) + S_i^{\mathrm{z}} S_j^{\mathrm{z}}] - H(t) \sum_i S_i^{\mathrm{z}}, \tag{1}$$

where $S_i = (S_i^{\mathrm{x}}, S_i^{\mathrm{y}}, S_i^{\mathrm{z}})$ is the three-dimensional spin at the i-th node of all the systems; N is number of monolayers; $L \times L \times N$— total number of spins of the system; J is the exchange integral of the interaction between nearest spins S_i.

$\Delta(N)$ is an anisotropy parameter depending on the number of monolayers, the value of which was chosen based on the article [16], the anisotropy value $\Delta(N = 5) = 0.75$. In this work, "easy axis" anisotropy was studied. The external magnetic field was directed perpendicular to the plane of the ferromagnetic film.

The dynamic order parameter Q is defined as:

$$Q = \frac{1}{2t_{1/2}} \int_0^{2t_{1/2}} m_z(t) dt. \tag{2}$$

In the dynamically disordered phase, the order parameter Q is close to zero, and in the ordered phase it is nonzero. The parameter that acts as an analogue of temperature in the transition is $\Theta = t_{1/2}/\langle\tau\rangle$, where $t_{1/2}$ is the half-period of the external field, $\langle\tau\rangle$ is the time of the metastable state, defined as the time at which the magnetization first crosses zero during the relaxation process. The magnetization of the z component was calculated using the formula:

$$m_z(t) = \frac{1}{L^2} \sum_{i=1}^{L^2} s_i^z. \tag{3}$$

The field bias value was introduced as a low additional field to the oscillating external field. As a result, uncompensation leads to asymmetrical oscillation of the field relative to zero:

$$H_b = \langle H(t) \rangle = \frac{1}{2t_{1/2}} \int_0^{2t_{1/2}} H(t) dt. \tag{4}$$

Simulation of the magnetic film was carried out for linear size $L = 128$ with external field amplitude $H_0 = 0.2$ and temperature $T = 0.6T_c(N)$, where $T_c(N = 5) = 1.31J$ [17]. The field bias H_b changed in steps of 0.001 and at time relaxation (the number of cycles) $P = 1000$. During the simulation, the number of monolayers $N = 5$ was considered. The spin system represents a cubic structure, based on the type of substrate anisotropy in experiments [12, 13].

2. Results and discussion

In this work, a thorough investigation of the various consequences of hysteresis in the critical region was carried out. The behavior of the order parameter $Q(H_b)$ is considered as the half-period $t_{1/2}$ increases to detect a dynamic phase transition. Figures 1 shows the change in the order parameter Q as a function of the field bias H_b at low field frequencies $t_{1/2} = 20$ MCS/s (Fig. 1(a)) with a clear existence of the first-order phase transition. With the appearance of the half-period $t_{1/2}$ (Fig. 1(b,c)), a sharp jump in the parameter was observed for a long time. A continuous phase transition occurs only at a critical half-period definition (Fig. 1(d)), when magnetization can follow a change in the oscillating field. In this case, collapse of the hysteresis loops is observed.

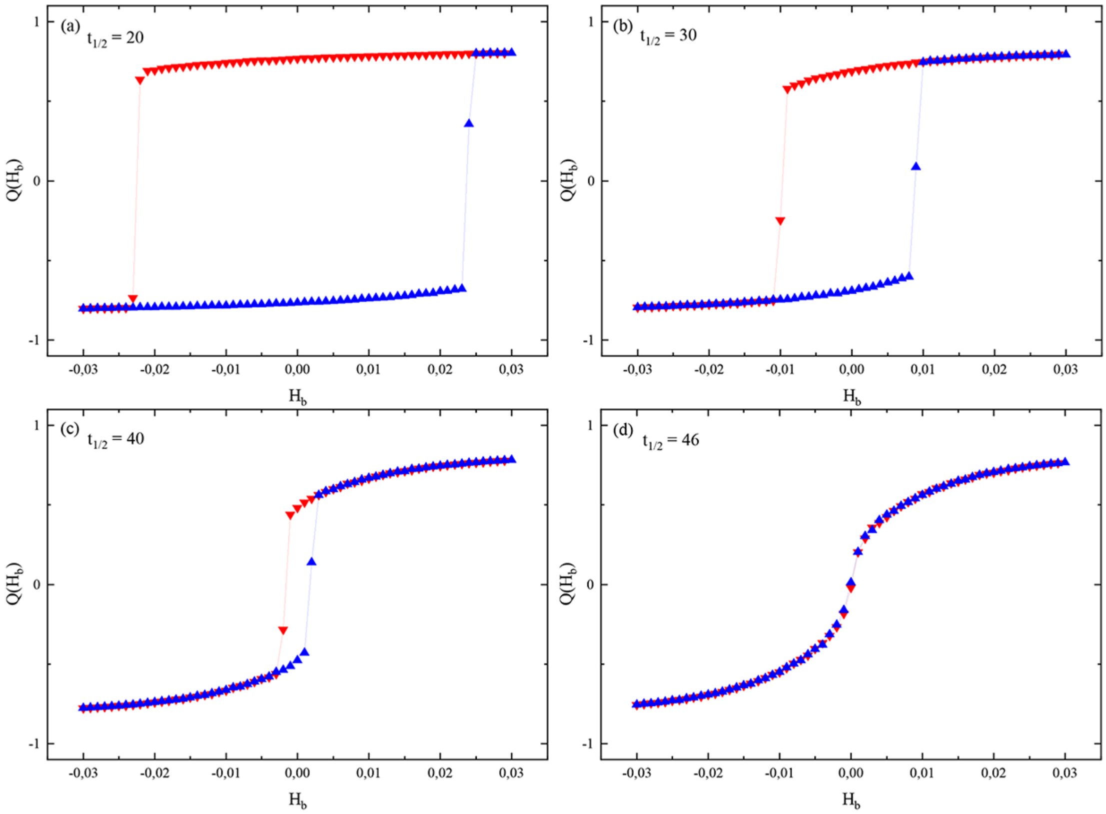

Fig. 1. Hysteresis loops for different values of half-period $t_{1/2}$. For case (a), a wide loop is formed, where the half-period is equal to $t_{1/2} = 20$ MCS/s, (b) an increase in the half-period $t_{1/2} = 30$ MCS/s contributes to the narrowing of the loop, as well as for case (c) $t_{1/2} = 40$ MCS/s, (d) $t_{1/2} = 46$ MCS/s the hysteresis loop collapses

The type of phase transition when considering hysteresis effects can also depend on the relaxation features of the model. Fig. 2 shows the relaxation dependence of the parameter Q on H_b on the oscillation cycles P of the system. A gradual increase in observation time leads to the fact that the hysteresis loops begin to collapse. In the region of dynamic phase transition, taking into account relaxation effects can play an important role. The simulation data qualitatively correlate with the experimental results of [11].

To consider the peculiarities of the behavior of the system after a dynamic phase transition (Fig. 3), a sufficiently large half-period $t_{1/2} = 100$ MCS/s for $H_0 = 0.2$ was chosen. We changed

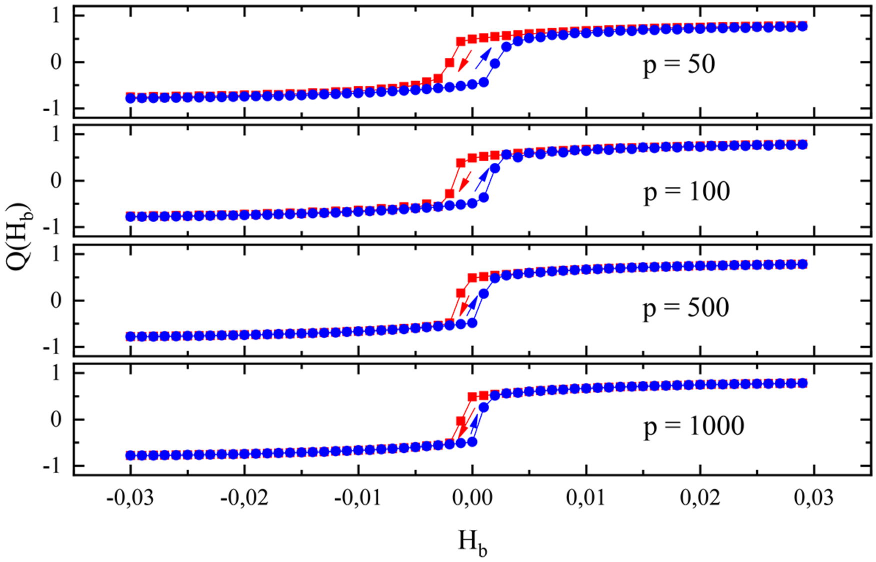

Fig. 2. Hysteresis effect depending on bias field H_b with increasing oscillation cycles P of the influence of the external field $H(t)$ at half-period $t_{1/2} = 40$ MCS/s

the value of the field amplitude from $H_0 = 0.16 \div 0.22$ in steps of 0.02 to more clearly show changes in the behavior of magnetization with distance from the multicritical point. Thus, at $H_0 = 0.16$ a collapsed hysteresis loop is represented, but fluctuations appear as the field increases. The magnitude of the field amplitude significantly affects the area of fluctuations and their magnitude.

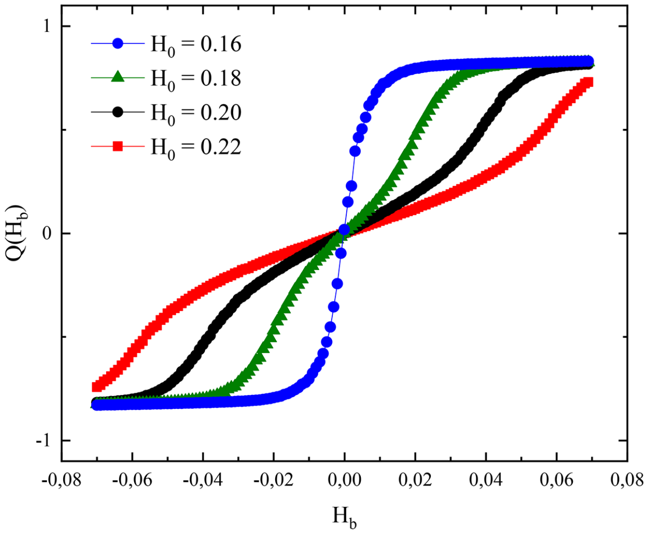

Fig. 3. Dependence of the hysteresis effect on changes in the field bias H_b for different field amplitudes $H_0 = 0.16 - 0.22$. Linear size of the system $L = 128$, half-period $t_{1/2} = 100$ MCS/s. At $H_0 = 0.16$ the hysteresis loop collapses (blue dots); at $H_0 = 0.20$ the loop bends (black dots)

Conclusion

We have carried out numerical modeling of hysteresis effects in a dynamic phase transition using the Heisenberg model in an external oscillating field by Monte Carlo methods. The dependence of the hysteresis loops on the $t_{1/2}$ was studied. As a result of which the existence of a first-order phase transition was revealed at less than values of the half-period $t_{1/2} = 46$ MCS/s, when the magnetization changes its state in an abrupt manner. A second-order phase transition occurs at higher values of $t_{1/2} = 46$ MCS/s. An increase in the half-period of the field leads to the fact that the magnetization of the system can follow the oscillations of the external field and consistently changes its values.

The collapse of the hysteresis loop occurs at a field amplitude $H_0 \geqslant H_0^c(t_{1/2}) = 0.16$ and $t_{1/2} = 100$ MCS/s. Curvature of the loops is observed with increasing H_0 [18]. This type indicates fluctuations that arise due to the stronger influence of $H(t)$ far from the critical region. The destruction of the phase transition with the formation of a metastable phase with a change in $H(t)$ is also observed, which also leads to a shift in the critical point.

Relaxation processes for hysteresis effects have been identified. The type of phase transition can be strongly influenced by the time of observation of the system. The results show a narrowing of the hysteresis loops with increasing oscillation cycles.

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Эффекты гистерезиса в критическом поведении тонких гейзенберговских пленок во внешнем осциллирующем поле

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Аннотация. В данной работе методами Монте-Карло моделировались эффекты гистерезиса в тонких пленках Гейзенберга во внешнем осциллирующем поле. Было обнаружено, что в системе ниже температуры Кюри наблюдаются различные типы фазовых переходов в зависимости от скорости воздействия осциллирующего поля. Выявлены релаксационные особенности системы, которые влияют на характер динамического фазового перехода.

Ключевые слова: Модель Гейзенберга, динамический фазовый переход, методы Монте-Карло, эффекты гистерезиса.