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A Multifaceted Approach to Forming Mathematical Digital **Competency of Future Engineers in Teaching Applied Mathematics**

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Abstract. The paper deals with the problem of forming mathematical digital competency of engineering students. Authors suggest a comprehensive approach to solve the problem which is implemented in a course of applied mathematics for future engineers. The concept of mathematical digital competency of an engineering specialist, the formation of which is the aim of students' training, is articulated. The paper provides rationale for the use of professionally oriented task system for mastering methods of mathematical modelling, computer simulators as well as game simulation models for teaching applied mathematics with the use of +ACI-Teacher-Student+ACI- automated working space. It facilitates quick mastering of basic methods of applied mathematics, computer science, some elements of algorithmization and programming by engineering students. The contents of the virtual laboratory complex for the course of applied mathematics powered by AnyLogic platform for simulation model implementation is described. The description of the developed system for managing student individual work is given. Tools for diagnosing the formation of mathematical digital competency are also presented.

Keywords: applied mathematics, mathematical digital competency, future engineers, mathematical modeling, digital education tools.

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Introduction

The model-based process of developing science and technology requires strong mathematical foundation. It allows one to build models, elaborating algorithms using the apparatus of computational mathematics for calculations and estimating model accuracy in analysis and optimization problems. It means that teaching mathematical modelling based on integration of mathematics and applied science combined with digital technologies is a current trend of development in modern engineering education [1].

One of the important subjects combining mathematical and computational modelling is applied mathematics. Its main objective is to develop competency to use mathematical apparatus to solve problems in professional activity of future engineers, including computer modelling. It is applied mathematics that is considered to be one of the main subjects forming mathematical competency of future engineers.

We regard mathematical digital competency of an engineering specialist as a competency constituting in the knowledge and understanding of mathematical language and digital tools to be used in engineering. Both mathematical and digital competencies determine readiness and ability to solve engineering problems by the means of mathematical and computer modelling [2].

Having created a special system of teaching applied mathematics that combines mathematical and applied knowledge based on information and communication technologies and digital tools, we aim to develop mathematical digital competency in future engineers.

The objective of the article is to describe an approach to the process of teaching future engineers a course of applied mathematics that combines mathematical and computer modelling aimed to develop mathematical digital competency.

1. Materials and methods

We chose the research methods based on the approaches to teach applied mathematics combining algorithmic and heuristic approaches to solve problems. The algorithmic approach suggests searching solution of a problem in accordance with the predetermined order of actions. In the heuristic approach the problem solving is based on the adopted strategy of finding solutions [3]). This approach agrees with developing mathematical competencies for future engineers.

A major problem that emerges in the course of teaching applied mathematics to engineering students is forming the mathematical mindset, the ability to discover and experience new patterns, and the interest in studying mathematical models. The listed features are mainly developed in the course of solving professionally oriented tasks (POT).

We regard the professionally oriented task system for mastering methods of mathematical modelling by engineering students as a combination and sequence of professional content tasks in the courses of further and applied mathematics which facilitate the development of mathematical digital competency of future engineers.

The tool that helps students to solve professionally oriented tasks which constitute the basis of mathematical and computer modelling of actual technical processes is chosen to be an organizational and technical system containing:

• computer simulators allowing for interaction with students via built-in control elements (button, check box, combo box, link label, radio button, text box, numeric up-down, and more);

• edutainment models for teaching applied mathematics embedded in "" Teacher–Student" automated working space" software [4], mathematical suites simplifying routine calculations.

We regard *computer simulators* as imitation computer models which represent actual or hypothetical technological processes. They can be used to study dynamic changes in parameters of initial technical processes or to build up a hypothesis to explore them.

Such models for teaching applied mathematics are presented as:

- a simulator of interactive curve plotting set in explicit, parametric form and polar coordinates;
- a simulator of the interactive analysis of a queueing system;
- a simulator of the graphic method of solving linear programming problems (including Gomory's cutting plane method for integer programming problems);
- a simulator of the graphic method of solving game models with the dimension of $2 \times n$ etc.

The simulators allow one to simulate various operations from construction of lines to modelling complex production processes. Students use them while searching for a mathematical model, analyse them in the course of applied mathematics, and apply them in studying technical processes that allows one to form digital competencies of students.

The key component of teaching applied mathematics is "" Teacher–Student" automated working space" software (AWS). The use of this software was particularly described in articles [4–6] etc.

Students are offered to apply the author-developed software for individual work. It can be used both during a lab session under the supervision of the instructor and in home conditions following this procedure: Stage 1 — model testing; Stage 2 — working with demonstration program; Stage 3 — individual model building and implementation check; Stage 4 — controlling student learning achievements.

The developed AWS can be considered as a tool to quickly master basic methods of applied mathematics, computer science, elements of algorithmization and programming. Further, the skills developed can widely be used to state and solve complex problems with the help of highend mathematical suites. These skills also help students create game models while solving applied mathematics problems.

Imitation models can be ergonomically developed on AnyLogic platform as a virtual laboratory complex for the course of applied mathematics. The complex is focused on training road transport engineer and contains laboratory works.

The main objective of laboratory works is to meet the demands of actual technological processes, and to form students+AJI- ability to carry out scientific activity.

For instance, in the laboratory work «Queueing systems» it is suggested to combine the elements of classical form of subject presentation with developing the imitation model based on AnyLogic, namely, to get the visualization of the model with the opportunity to present actual (simulated) processes of queueing systems after providing the mathematical rationale. The main thing that students are supposed to do in the course of performing laboratory works is organizing their own project research activity, for example, to design gas filling stations capable of bringing service to a big number of automobiles.

Scientific research activities of future engineers have prime importance as they develop engineering mindset, and form professional competency [7]. This is the reason why students present

the laboratory work completion report to a teacher as a design of a gas filling station for a specific residential area. Students present the best research projects at scientific conferences.

Using the above mentioned materials and methods not only allows establishing hierarchy of studying applied mathematics but also paves the way to the use of integrative, research, and practice oriented approaches to teach students, and to form their mathematical and digital competencies.

2. Research results

A pedagogical experiment in developing mathematical digital competency in future engineers was carried out at Gorlovka Automobile and Highway Institute of Donetsk National Technical University over the period of 2017–2021. The experiment was focused on testing, adjustment, and implementation of the mathematical modeling teaching system in the course of applied mathematics in the scope of education digitalization.

In particular, in the course of organizing applied mathematics learning activity in the experimental group interactive methods of digital didactics (heuristic methods, gaming methods, problem based learning methods) were used; conventional organizational forms of teaching applied mathematics were combined with hybrid and mixed forms of training.

For example, in the course of learning applied mathematics, a classical model called «travelling salesman» with the purpose to search for the most advantageous closed walk going through a net of given points (locations) [8, p. 438].

Students are offered to study the lecture on the topic "Branch and Bound Method" on their own, the lecture is presented in a Moodle-based e-learning course. Students must go over the lecture material and review the solution to the problem implementing the travelling salesman model. Let us provide an example of methodology for solving applied problems.

Problem 1. A bus tour route over Donetsk is organized. The tour must start at the "Donbass Palace" hotel. The main points are: "Nemo" dolphinarium, Donetsk Forged Figures Park, planetarium, botanical garden, "Donbass Arena" stadium. Plan the tour route with the shortest bus path length [9].

In order to implement this model we use two algorithms of branch and bound method:

a) Algorithm 1 - binary partition;

b) Algorithm 2 - random partition.

After the problem analysis it is suggested that students answer the following comprehension questions:

1) define "combinatorial optimization";

2) define "the upper bound of the problem";

3) define "the lower bound of the subset";

4) continue the following statement: "if the lower bound for a subset is greater than the upper bound, then ...";

5) explain how a smaller distance matrix is built;

6) describe the stages of lower bound building;

7) characterize "branch and bound" mathematical model;

8) explain the algorithm of choosing the starting point;

9) explain the difference in using binary partition and random partition in branch and bound method;

10) describe the stages of building a route tree in branch and bound method;

11) provide the rationale for the applicability of the travelling salesman model in your specialist field.

12) analyze the stages of implementing the breach and bound method in logistics and highway transport management.

Further, students are suggested to perform the following activities:

- study the lecture material on this topic;
- answer the questions;
- make the list of problematic questions which need discussing during a practical session in class.

The class session starts with the problem statement: "How do we choose the most advantageous closed walk going through a net of given points"? Many students immediately suggest using Google Maps (Yandex Karty) with intuitive interface to specify the starting, the ending, and the intermediate points and also providing automated search of the optimal route. Some students suggest identifying the distance matrix as an array of data with a subsequent processing of the matrix as an array of data. Speculations and discussion that follow go to show that the future engineers are making active use of the theoretical material to create model situations.

Continuing the heuristic dialogue with the students we pose another problem: "Can you obtain a large distance matrix (big data)?" Many students reply that it will increase labour intensity even from the standpoint of performance, brute forcing all possible closed walks with further specifying the optimal (shortest) route. Some students who put forward this technology agree with the difficulty of big data processing.

The teacher gives various examples (for instance, with the use of the dichotomy method and the chord method) where the number of iterations can be drastically reduced (in iterative methods). Based on the discussion and the examples presented the teacher leads the students to the idea of using the branch and bound method, the essence of which is determining the upper bound, the lower bounds for subsets with the follow-up application of the statement: *if the lower bound on the subset of the initial set is greater than the upper bound, the solution lies in the opposite subset.*

Thus, a precise algorithm of operations (calculations) is formed in the student's mind. We present the implementation of the algorithm in "Travelling Salesman Model" laboratory work.

Any valid route can be taken as the upper bound. To calculate the lower bounds we suggest using Lower Bound Calculator (Fig. 1).

Students start performing the laboratory work applying the breach and bound method to combinatorial models.

As a result of the laboratory work students obtain the result as a solution tree (Fig. 2).

There are no more interrupted branches left (all of them have been crossed out), so the route obtained is the shortest.

The main objective of the "Travelling Salesman Model" laboratory work is not obtaining a final solution in the first place, but consistent work on applying the branch and bound method to combinatorial models.

To self-check the performed task, the student is given the macro of the automatic search of the optimal route.



Fig. 1. Lower Bound Calculator



Fig. 2. Search Tree for the Most Optimal and Shortest Route

Besides, in experimental groups virtual laboratory works were constantly carried out, some of these sessions were described in the articles [6, 10, 11]. The tool to perform these works was the virtual laboratory complex.

An important factor of assuring the quality of studying the course of applied mathematics is advisably arranged student independent work (SIW). According to O. L. Prokhorova, SIW provides consistent gain of knowledge and its qualitative complication, mastering rational methods and techniques of absorbing new ways of educational activity [12].

In the course of independent work with the use of digital technologies student activity becomes more deliberate being characterized by concentration, velocity and accuracy, completeness and meaningfulness of mathematical model building process.

Each student fills Textbox in Form of the dialogue mode of "Teacher–Student" automated working space in order to receive an individual task for independent work (Fig. 3).

SIW is managed with the help of "Teacher-Student" AWS, which allows students to go



Fig. 3. Receiving SIW Form and Step-by-step Implementation of Algorithm Scheme

through each step of the model algorithm interactively.

In the end, the program reports the correctness of task performance or current mistakes, filling the Textbox background green and red respectively (Fig. 4).



Fig. 4. Colour Prompts of SIW Management System

We have introduced learning material acquisition tests into the management system of the student independent work. They are distributed as follows:

- basic test taken in class (occasional selective);
- self-check (self-control), performed by the student individually (with the use of AWS);
- written reproduction of the material;
- training exercises and check points.

Due to the self-check with the use of AWS, the management of the student independent work within the course of applied mathematics takes on the form of cognitive, task-oriented, and creative activity.

At the end of the course of applied mathematics we organized a test work to determine the *mathematical and digital competency* proficiency level in the control group (CG) and the experimental group (EG) with the total number of 110 students (CG – 56 ppl., EG – 54 ppl.).

The tasks for the test work were picked from the study guide for higher vocational education institution students titled "Applied Aspects of Mathematics" [9].

At this stage of the experiment, with the maximum score of 20, transitioning from the ratio scale measurement to the ordinal scale measurement, the following scale of levels of the mathematical and digital competency (MDC) was used: low (with *the score* \leq 5); middle (5 <*the score* \leq 14); high (14 < *the score* \leq 20). The results of the test work from the ratio scale to the ordinal scale are shown in Tab. 1.

Table 1. Student Distribution by Academic Achievement Levels in EG and CG after Completing Test Work.

MDC levels	Number of students CG, ppl.	Number of students EG, ppl.
Low	8	5
Middle	31	22
High	15	29
Total:	54	56

The visualization of the test work results as a percentage of the total number of students of each group is given in Fig. 5.



Fig. 5. Results of Applied Mathematics Final Test Work in EG and CG (level of mathematical and digital competency, %)

At the end of the experiment, the control and experimental groups were offered a questionnaire that characterizes the attitude of engineering students to the need to study mathematical modeling for future professional activities.

The experimental group students' replies were more conscious.

For instance, to the question "Express your attitude towards the necessity to study applied mathematics for a future engineer" 56% of the students of the experimental group replied: "I am

happy to study applied mathematics and mathematical modeling, because it is necessary for my future engineering activities".

To the question: "In your opinion, does a future engineer need to master the methods of mathematical modeling?" 72% of the students of the experimental group gave a positive answer, the students of the control group mostly (65%) gave the following answer: "No, because there are modern computer programs that allow modeling complex systems".

To the question "Is the level of your knowledge and skills in mathematical modeling sufficient to create professional models?" all respondents studying in both control and experimental groups gave similar answers: "Rather no than yes" — answered the students of the control group; "Rather yes than no" — replied the students of the experimental group. These answers make it possible to conclude that the approach to teaching mathematical modeling should be multifaceted; using the example of just one course, it is impossible to form the concepts of mathematical models effectively and learn how to deal with them.

The question "Do you know how mathematical models of technical objects, phenomena, and processes are created?" was answered negatively by the majority of students in both groups. This answer goes to show that realizing the importance of studying the methods of mathematical modeling it is necessary to utilize not only applied mathematics, but also various professional subjects to expand the essence of modeling, to familiarize with virtual models using the examples of engineering processes, to use mathematical and computer modeling in researching while working upon a graduation thesis.

"Can the computer be applied to solve problems with the use of mathematical modeling?" — a unanimous positive answer was received from the experimental group students, as they all built models based on computer experiments when studying applied mathematics in integrated and virtual laboratory works.

Thus, the student questionnaire to identify their attitude to the need to study mathematical modeling for its use in future professional activities showed that the effect of changes is caused precisely by the use of experimental teaching methods.

Based on the results of the experiment, a conclusion was made about the effectiveness of the author-developed methodology for teaching mathematical modeling to engineering students in the course of applied mathematics aimed at developing students' mathematical digital competency.

3. Discussion

As engineering education researchers point out, teaching future engineers must be aimed at their mastering both mathematical and digital competencies necessary for designing and solving technical systems modeling problems [13–16].

Applied mathematics is one of the subjects in which is it possible to combine the processes of studying mathematical and computer modeling. We agree with the authors of applied mathematics educational manuals who state the necessity of studying the methods of mathematical and computer modeling to develop professional competency of students [17–20]. At the same time, there have been many articles that characterize the importance of implementing high-end training aids [21], virtual laboratory works [22], computer simulators [23,24], designed with the help of digital tools. However, the multifaceted approach to teaching applied mathematics designed for maintaining the process of teaching mathematical and computer modeling in various forms of classes and student independent work is poorly presented in research and methodological literature. The experiment carried out at Gorlovka Automobile and Highway Institute of Donetsk National Technical University has shown the effectiveness of the represented teaching system. As a result, students' mathematical and digital competency is developed, which corresponds to the idea of the engineer of the new technological paradigm.

Conclusion

In our opinion, the course of applied mathematics must be organized as a system of developing of mathematical digital competency of future engineers which provides for the integration of mathematical and computer modeling.

The main components of the multifaceted approach to developing mathematical digital competency of future engineers in the course of applied mathematics are:

- a system of professionally oriented tasks aimed at mastering the techniques of mathematical modeling;
- laboratory works based on students' mastering mathematical modeling operations with the use of ICT;
- virtual laboratory works that facilitate mastering imitation modeling and digital tools;
- a management system of the student independent work based on "Teacher Student" automated working space.

The implementation of such a system into the process of teaching applied mathematics in engineering educational institutions will enable researching complex processes and phenomena in real time, using application program suites to perform engineering calculations when solving problems, as well as facilitating the development of student mathematical and digital competency.

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Комплексный подход к формированию математической цифровой компетентности будущих инженеров в обучении прикладной математике

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Аннотация. В статье рассматривается проблема формирования математической цифровой компетентности студентов технических направлений подготовки. Авторами предложен комплексный подход решения проблемы, реализуемый в обучении будущих инженеров дисциплине «Прикладная математика». Раскрыто понятие математической цифровой компетентности специалиста в области инженерии, формирование которой у студентов является целью обучения. Обосновано применение системы профессионально ориентированных задач по овладению приемами математического моделирования, компьютерных симуляторов, а также игровых имитационных моделей обучения прикладной математике на основе автоматизированного рабочего места «Преподаватель – студент», являющегося средством для достаточно быстрого овладения студентами технических направлений подготовки базовыми методами прикладной математики, информатики, элементами алгоритмизации и программирования. Описано содержание авторского виртуального лабораторного комплекса по дисциплине «Прикладная математика», разработанного на платформе AnyLogic, предназначенного для реализации имитационных моделей.

Ключевые слова: прикладная математика, математическая цифровая компетентность, будущие инженеры, математическое моделирование, цифровые средства обучения.