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Study of Trapped Magnetic Field Relaxation in a Cylindrical Micron-Sized HTS

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Abstract. Relaxation curves of a cylindrical $Bi_2Sr_2CaCu_2O_{8-\delta}$ superconductor several microns in diameter have been calculated by means of the Monte Carlo method within the model of a layered high-temperature superconductor in magnetic fields parallel to the cylinder axis and at different temperatures. Relaxation rates have been obtained for various sample diameters and temperatures. Power-law nature of the relaxation rate vs temperature dependences has been demonstrated for all the considered sample diameters

Keywords: magnetic relaxation, Abrikosov vortices, HTS, Monte Carlo method.

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Introduction

Nowadays, high-temperature superconductors (HTSs) of micron and submicron dimensions are increasingly becoming the objects of experimental and theoretical research. At the same time, it is often necessary to study the dynamics of trapped magnetic flux in a superconductor on time, in particular, the relaxation processes occurring in the vortex system are of great interest.

Relaxation properties of HTSs are considered in a number of papers dedicated to the construction of magnets based on stacks of HTS tapes: in [1], an electric motor with superconducting winding cooled by liquid helium in an alternating field was studied. The main result was that the exit rate of the trapped magnetic flux decreases with time.

The aim of work [2] was to obtain data on the parameters of a stack of HTS tapes in a tilted field using numerical modeling in Comsol Multiphysics and experimental methods. It was found that, for each combination of strength and direction of the magnetizing field, there is an optimal number of tapes at which the magnetization on the tape is maximum.

The authors of [3] studied the properties of superconducting stacks in a rotating mechanism using physical modeling methods in comparison with experimental data. Such devices can also

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be used in synchrotron studies. Deviations are associated with incomplete knowledge about the properties of the studied materials.

When considering relaxation in HTSs, it is impossible to ignore the work [4]. The experimental data for $YBa_2Cu_3O_{7+x}$ presented there allows us to compare the general nature of the behavior of the main HTS characteristics, including the relaxation rate.

In not too high fields (of the order of hundreds or thousands of Oe. Such fields are generally used in experiments with micron-sized superconductors), the sample is penetrated by about several thousand vortices. The vortex lattice in a superconductor with pinning centers and Meissner and transport currents is a complex system that generally does not allow an analytical description. Therefore, numerical modeling is of particular value. Samples smaller than the London penetration depth of the magnetic field can be effectively modeled by solving the Ginzburg-Landau equations. For macroscopic samples, modeling at the level of individual vortices is not possible. In the intermediate case of samples tens or hundreds of λ in size, Monte Carlo and molecular dynamics modeling is possible. In our work, we have investigated the processes of magnetic flux exit from a superconductor in the shape of a cylinder from 2 to 5 microns in diameter and in parallel field. The choice of this geometry is due to the fact that superconducting nanowires [5] are often used for the construction of micro-bridges. Also, superconductors of similar dimensions can be used in the manufacture of superconducting sensors.

1. Description of the model

The calculations were performed using the Monte Carlo method for a vortex system within the framework of the two-dimensional model of a layered HTS [6]. In this model, a system of flat layered pancake vortices can be represented as an ensemble of classical particles with a long-range potential. The energy of the vortex system can be represented in the following form:

$$G = \sum_{ij} \frac{\Phi_0^2 h}{8\pi^2\lambda^2}\,\mathrm{K}_0(r_{ij}/\lambda) + \sum_{i} \frac{\Phi_0^2 h}{16\pi^2\lambda^2}(0.52 + \log(\lambda/\xi)) + E_{\text{boundary}} + \\ + \sum_{id} \alpha \frac{\exp(-r_{id}/2\xi)}{1 + r_{id}/\xi} + \sum_{ib} H \frac{\Phi_0 h}{4\pi} \frac{\mathrm{I}_0(r_{ib}/\lambda)}{\mathrm{I}_0(d/2\lambda)} + E_{\text{current}}. \tag{1}$$

The Gibbs energy of the system takes into account the following terms: the repulsion and attraction of vortex pairs (respectively, of identical or opposite signs), the intrinsic energy of vortices, their interaction with sample boundaries, the pinning energy on defects or impurities; the influence of external magnetic field, and the contribution of electric current. Here, d and h are the diameter and thickness of the HTS sample respectively, r_{ij} is the distance between interacting vortices, r_{id} is the distance between a vortex and a defect, r_{ib} is the distance from a vortex to the boundary, K_0 is the modified Bessel function of the second kind, I_0 is the modified Bessel function of the first kind, $\Phi_0 = hc/2e$ is the magnetic flux quantum.

The superconductor parameters are calculated as follows:

$$\lambda = \lambda_0 \left(1 - \left(\frac{T}{T_c}\right)^{3.3}\right)^{-1/2}, \tag{2a}$$

$$\xi = \xi_0 \left(1 - \left(\frac{T}{T_c}\right)^{3.3}\right)^{-1/2}. \tag{2b}$$

The magnetization was calculated from the number of trapped vortices:

$$|4\pi M| = \left| H - \frac{\Phi_0 N}{S} \right|,$$

where N is the number of vortices, S — the cylinder circular face area, H — the external magnetic field strength.

Schematically, the mutual arrangement of the vortex filaments and the external magnetic field is shown in Fig. 1 (here, the magnetic field is directed perpendicular to the HTS layers). In this geometry, the Meissner currents flow in circles inside the cylinder, so the Lorentz force acting on the vortices is directed towards the center of the cylinder.

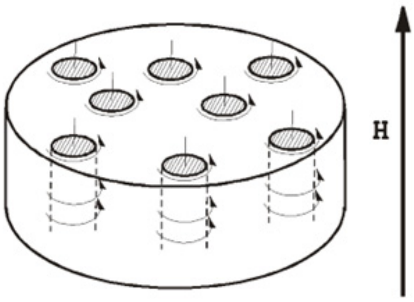

Fig. 1. Cylindrical geometry for a two-dimensional model of layered HTS. The magnetic field is oriented perpendicular to the superconducting CuO_2 layer. Vortices line up along the field

The defects are randomly scattered over the sample with a constant density of $10^1 1$ cm^{-2}. The effective depth α of potential wells of defects took values from 0.01 to 0.06 eV (their distribution obeyed the Gauss law). The selected potential well depth limits the pinning to one vortex per defect.

In our calculations, we neglect any changes in the temperature of superconductor arising from the motion of vortices.

An analogue of the relaxation time is the Monte Carlo (MC) step. Based on the results of earlier works [7] and [8], one may argue that one MC step corresponds to 10^{-8}–10^{-9} s. In this paper, we have considered relaxation processes lasting up to several milliseconds.

Calculations were performed for the typical parameters of bismuthic HTSs (for certainty, we took the parameters of $Bi_2Sr_2CaCu_2O_{8-\delta}$).

2. Relaxation of the trapped magnetic flux

In our work, we have calculated the dependence of the magnitude of the trapped magnetic flux on the effective time (given in Monte Carlo (MC) steps). The relaxation calculations were performed for different diameters of the cylindrical sample and for temperatures from 10 to 40 K. The sample diameter varied from 2 to 5 microns. At the same time, the minimum diameter of 2 μm is about 10λ for the considered material, which suggests a noticeable influence of the boundaries on the exit of vortices from the sample.

The modeling of magnetic relaxation was carried out in two stages:

1. At a field of 3000 Gauss, vortices entered the sample forming the initial state (the first curve in the left-hand image of Fig. 2). This process lasted for 12500 MC steps.

2. The magnetic field was instantly switched off, after which the residual magnetization was collected every 50 physical MC steps.

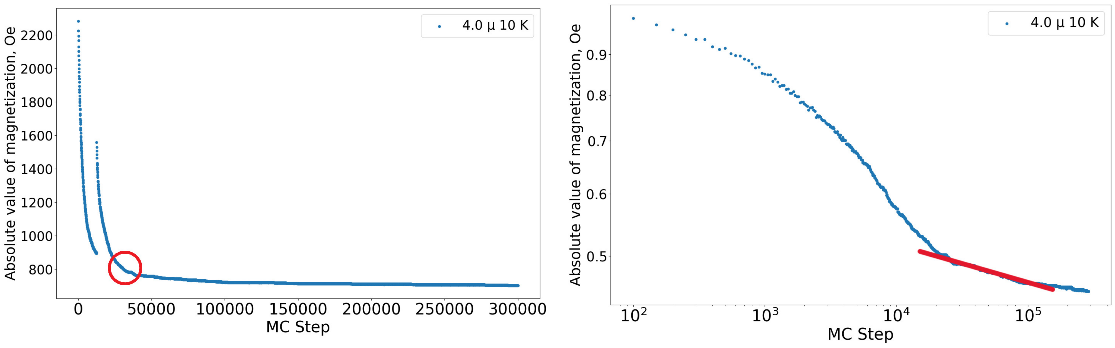

Fig. 2. Time-dependences of the absolute value of magnetization. The image on the left is presented on a linear scale, the one on the right — on a double logarithmic scale

The non-physical discontinuity is caused by the instant turning off of the field. The width of this discontinuity is less than 3000 Gs due to a delay of 50 MC steps between adjacent points of the graph, during which part of the trapped magnetic flux manages to exit the sample.

Relaxation curves were obtained for samples from 2.0 to 5.0 microns in diameter with an increment of 0.5 microns and at temperatures from 10 to 40 K with an increment of 5 K.

In Fig. 3, an example of parts of calculations is given for the entire temperature range at fixed sample diameters. The curves are normalized to the initial value and given on a double logarithmic scale. As expected, after the temperature increases, the decay rate of the residual magnetization rises — the curves are arranged sequentially in accordance with the temperature during the simulations.

Observing similar dependences at constant temperatures Fig. 4, one can also see sequentially arranged curves, albeit in accordance with the sample dimensions. The presence of steps at small diameters and high temperatures indicates the exit of single vortices from the sample. At this stage, there is a deviation from the collective flux creep theory — there is no decrease in the relaxation rate over time, which is especially noticeable on large time spans Fig. 5.

Based on the previously obtained results, we can confirm the absence of deviations from the described patterns in the behavior of residual magnetization for all temperatures and sample diameters 2 microns and greater in cylindrical geometry.

Now let us draw our attention to the relaxation rate (3) which is defined as the slope of the tangent to residual magnetization on the double logarithmic scale (Fig. 2 on the right).

$$v = \frac{\partial log(M)}{\partial log(t)}. \tag{3}$$

For certainty, the relaxation rate was calculated for the part of the curve marked in Fig. 2 with a circle (on the left) and a straight line (on the right). On a double logarithmic scale, the plot can be approximated by a straight line. It should be noted that the relaxation rate for a micron-diameter superconductor does not remain constant with time. This is especially noticeable for the curve corresponding to the sample diameter of 2.5 microns: its slope is maximal at times of the order of $5 \cdot 10^3$ MC steps, then the slope gradually decreases (Fig. 6).

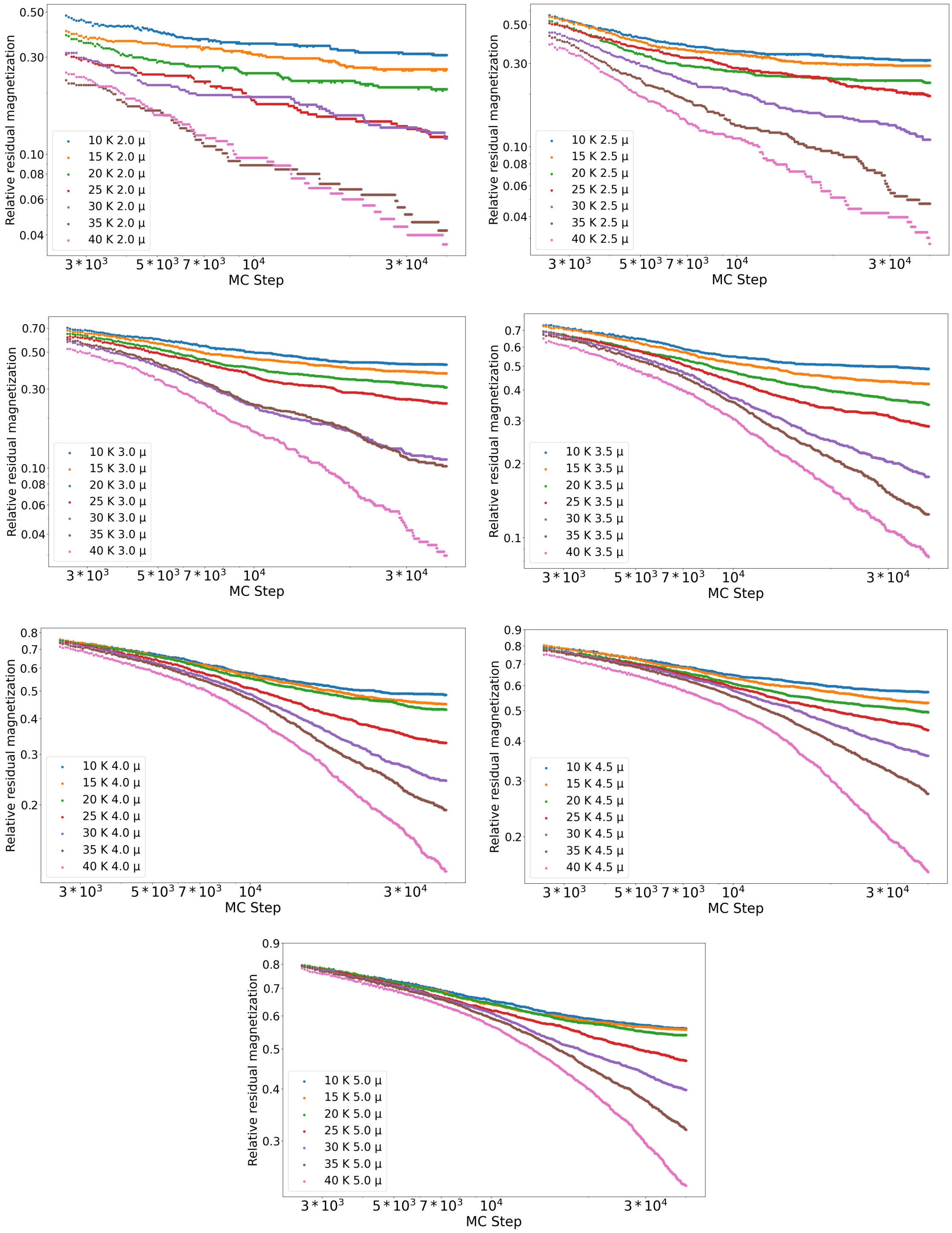

Fig. 3. Dependence of relative residual magnetization on time (in MC steps) at fixed sample sizes

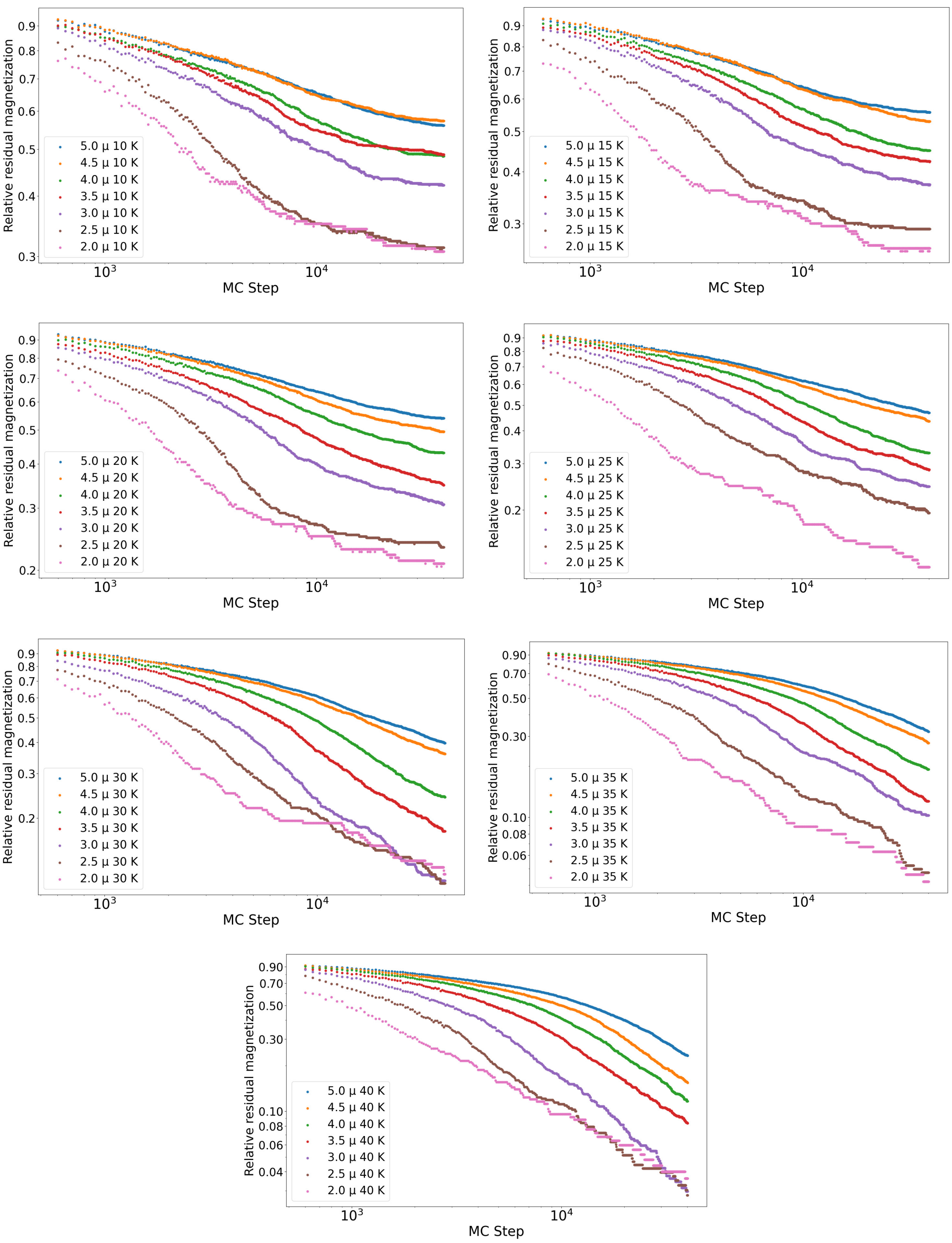

Fig. 4. Dependence of relative residual magnetization on time (in MC steps) at constant temperatures

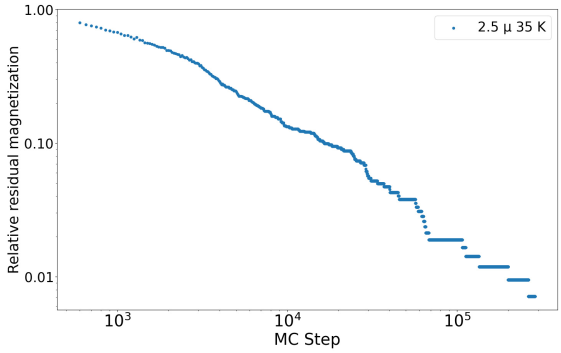

Fig. 5. A full relaxation loop after switching off of the field for a sample diameter of 2.5 microns and at a temperature of 35 K

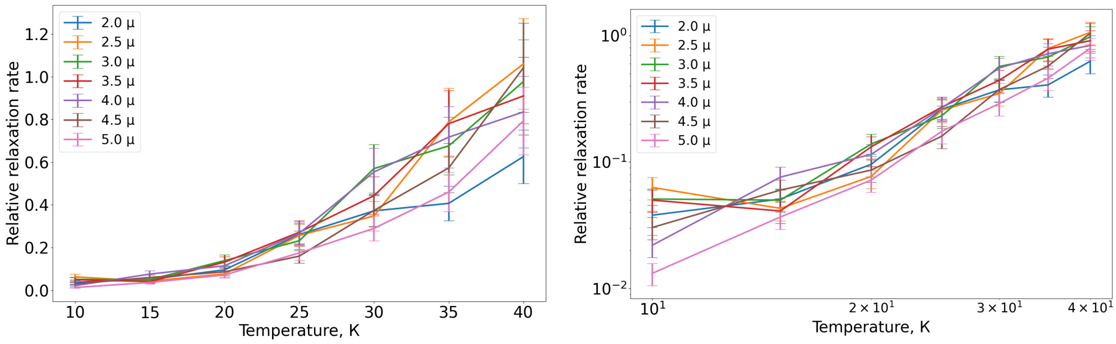

Fig. 6. Temperature dependence of the relaxation rate for samples of different diameters. The image on the left is given on a linear scale, the one on the right — on a double logarithmic scale

Fig. 6 shows the relaxation rate values for all temperatures and sample diameters studied – each line corresponds to a separate sample diameter.

When constructed on a double logarithmic scale, the relaxation rate shows a straight line, which means that there is an apparent power-law dependence of relaxation rate on temperature. It will be interesting to continue the calculations on a larger parameter span, e. g. consider different defect concentrations and pinning types.

Conclusion

The process of relaxation of the trapped magnetic flux in a cylindrical micron-diameter superconductor (analogous to a superconducting nanowire) has been studied by the Monte Carlo method within the framework of the two-dimensional model of a layered HTS. Temperature dependences of the relaxation rate have been obtained from the calculated time-dependences of the trapped magnetic field on a double logarithmic scale. It has been shown that, as the temperature increases, the decay rate of the residual magnetization also increases. The influence of the diameter of the superconducting cylinder on the relaxation processes has also been studied.

The obtained results suggest that the stability of trapped magnetic flux increases with the increase in the sample geometric dimensions. It has been shown that the temperature dependence of the relaxation rate has a power-law shape regardless of the sample size within the considered parameter ranges. The presence of steps on the dependence of trapped magnetic field on time has also been shown, which was caused by the exit of single vortices from the sample (at the same time, no more than several dozen vortices are trapped in the sample). This behavior of magnetization at the considered times was observed at temperatures higher than 30 K and in samples with a diameter less than 3 microns. The results obtained in the present paper can be used in planning experiments related to micro-bridges and magnetic field sensors based on superconducting nanowires.

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Исследование релаксации захваченного магнитного поля в цилиндрических микронных ВТСП

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Аннотация. Методом Монте-Карло в рамках модели слоистого высокотемпературного сверхпроводника были рассчитаны релаксационные кривые цилиндрического сверхпроводника $Bi_2Sr_2CaCu_2O_{8-\delta}$ диаметром в несколько микрометров в магнитных полях, параллельных оси цилиндра, и при разных температурах. Получены величины скорости релаксации для разных диаметров образца и температур. Продемонстрирован степенной характер зависимости скорости релаксации от температуры для всех рассмотренных диаметров образца.

Ключевые слова: магнитная релаксация, вихри Абрикосова, ВТСП, метод Монте-Карло.