

EDN: RPCLDC
УДК 536.25

Modeling of Stationary Flows of a Liquid-Gas System in an Inclined Channel Subject to Evaporation

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Received 28.06.2022, received in revised form 06.08.2022, accepted 20.10.2022

Abstract. Two-layer flows of liquid and gas-vapor mixture in an inclined channel are modeled based on the system of the Oberbeck–Boussinesq convection equations taking into account the evaporation and thermodiffusion effects. A new exact solution of the problem of evaporative convection is constructed in the statement with non-deformable interface and zero vapor flux on the upper channel wall. The analytical form of the required functions is presented in the case of heating of the channel boundaries linearly with respect to the longitudinal coordinate. Algorithms of calculation of the integration constants are described in detail. Examples of flow types are provided for the ethanol-nitrogen fluid system.

Keywords: exact solution, two-layer flow, convection, evaporation, interface, inclined channel.

Citation: E.E. Makarov, Modeling of Stationary Flows of a Liquid-Gas System in an Inclined Channel Subject to Evaporation, J. Sib. Fed. Univ. Math. Phys., 2023, 16(1), 110–120. EDN: RPCLDC



Introduction

Mathematical modeling of two-layer systems with interfaces is motivated by intensive development of knowledge-intensive technologies and experimental approach to research of features of joint convective flows of liquids and gases [1]. Most such convective processes are quite difficult to study due to the existence of a large number of factors affecting the flow nature.

The problems with evaporation or condensation are of particular interest for investigation. The Ostroumov–Birikh type solutions (see [1–3]) are prominent ones among exact solutions of evaporative convection problems, since they are realized in reality. One of the feature of these solutions is that they allow us to test different types of boundary conditions for vapor concentration and temperature functions.

Historically, the problem of unidirectional two-layer flows induced by gravitational and Marangoni forces was first considered in [4]. The first results of the study the flows with evaporation in a two-layer system based on an analogue of the Ostroumov–Birikh solution were presented in [5].

The impact of the reciprocal thermodiffusion effects on the parameters of convective regimes in the two-layer system was investigated in [6–8] based on the Ostroumov–Birikh type solution. The Soret effect (or thermodiffusion effect) is related to the molecular transport of matter in the presence of a temperature gradient. The Dufour effect (or diffusive thermal conductivity effect)

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determines the occurrence of temperature differences due to differences in the concentrations of impurity components (see [9, 10]).

In the present work a two-layer flow with evaporation in an inclined channel is modeled, taking into account various factors affecting the flow structure, temperature and vapor concentration distributions.

1. Governing equations and form of exact solution

The joint flow of viscous incompressible liquid and gas-vapor mixture in an infinite channel is considered (see Fig. 1). The liquid and gas-vapor layers have constant thicknesses l and h . The upper and lower walls of the channel are solid impenetrable boundaries. Vapor is a passive admixture in the upper layer containing gas. The Cartesian coordinate system is oriented so that the non-deformable interface is given by the equation $y = 0$ and the gravity force vector \mathbf{g} is directed at an angle φ to the substrate ($\mathbf{g} = (g \cos \varphi, -g \sin \varphi)$). The system of the Navier–

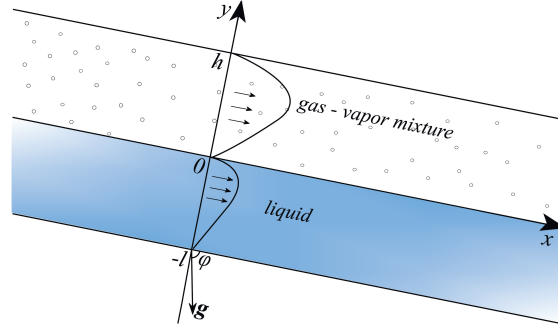


Fig. 1. Flow scheme

Stokes equations in the Oberbeck–Boussinesq approximation is utilized as a mathematical model to describe flows in the bilayer system. The Soret and the Dufour effects are taken into account in the gas. The system of equations for finding the functions that determine velocity, temperature, pressure and vapor concentration is written in the following form:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p'}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - g \cos \varphi (\beta T + \underline{\gamma C}), \quad (1)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p'}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g \sin \varphi (\beta T + \underline{\gamma C}), \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \chi \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \delta \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) \right). \quad (4)$$

The vapor transfer in the gas phase is governed by the convective diffusion equation, which is the result of the Fick's law [11]:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \right). \quad (5)$$

In equations (1)–(5) the following notations are used: u , v are the velocity vector projections on the axes of the Cartesian coordinate system Ox and Oy , respectively; p' is the function defined

deviation from the hydrostatic pressure ($p' = p - \rho \mathbf{g} \cdot \mathbf{x}$, $\mathbf{x} = (x, y)$, p is the pressure), T is the temperature, C is the vapor concentration, ρ is the density (relative density value), ν is the coefficient of kinematic viscosity, β is the coefficient of thermal expansion, γ is the concentration coefficient of density, χ is the coefficient of heat diffusivity, D is the coefficient of vapor diffusion in the gas, parameters α and δ are the Soret and Dufour coefficients respectively. The underlined terms in equations (1), (2) and (4) as well as equation (5) are used only in the modeling of heat and mass transfer.

System (1)–(5) admits an exact solution of the special form [2,3]:

$$u_i = u_i(y), \quad v_i = 0, \quad T_i = Ax + \vartheta_i(y), \quad C = -Bx + \psi(y), \quad p'_i = p'_i(x, y). \quad (6)$$

Here, A and B are constant longitudinal temperature and vapor concentration gradients; ϑ_i and ψ are functions depending on the variable y included in the expressions setting the temperature and vapor concentration functions, respectively. Index i denotes characteristics of working fluid: $i = 1$ and $i = 2$ regard to the liquid in the lower layer and the gas-vapor mixture in the upper one, respectively.

2. Conditions for the desired functions on the solid walls and the interface

The no-slip conditions on the rigid walls are valid for the velocity functions:

$$u_1|_{y=-l} = 0, \quad u_2|_{y=h} = 0, \quad (7)$$

and the temperature distribution is linear with respect to the longitudinal coordinate:

$$T_1|_{y=-l} = Ax + \vartheta^-, \quad T_2|_{y=h} = Ax + \vartheta^+. \quad (8)$$

Here, ϑ^- and ϑ^+ are considered to be of given constant values.

The vapor concentration satisfies the condition of zero vapor flux at the upper boundary $y = h$:

$$\left(\frac{\partial C}{\partial y} + \alpha \frac{\partial T_2}{\partial y} \right) \Big|_{y=h} = 0. \quad (9)$$

The conditions of continuity of longitudinal velocities and temperature should be fulfilled at the thermocapillary interface $y = 0$:

$$u_1|_{y=0} = u_2|_{y=0}, \quad T_1|_{y=0} = T_2|_{y=0}. \quad (10)$$

Note, the equality of the values determining the longitudinal temperature gradients and the equation $\vartheta_1(0) = \vartheta_2(0)$ will provide the temperature continuity condition.

Kinematic and dynamic conditions are also required to be set at the interface. Kinematic condition ($v_1 = 0$ and $v_2 = 0$) is fulfilled automatically due to the type of the exact solution form (6). Dynamic condition is written as follows:

$$\rho_1 \nu_1 u_{1y} = \rho_2 \nu_2 u_{2y} + \sigma_T \frac{\partial T_1}{\partial x} \Big|_{y=0}. \quad (11)$$

Dynamic condition expresses the tangential stress balance at the interface. The constant σ_T is the temperature coefficient of the surface tension σ , $\sigma_T < 0$. The linear dependence of surface tension on temperature is assumed: $\sigma = \sigma_0 + \sigma_T(T - T_0)$, σ_0 is the surface tension at some initial temperature T_0 .

The heat transfer condition at the interface, including the terms corresponding to the diffusive mass flux M and the Dufour effect, is set in the form:

$$\kappa_1 \frac{\partial T_1}{\partial y} - \kappa_2 \frac{\partial T_2}{\partial y} - \delta \kappa_2 \frac{\partial C}{\partial y} \Big|_{y=0} = -LM, \quad M = -D\rho_2 \left(\frac{\partial C}{\partial y} \Big|_{y=0} + \alpha \frac{\partial T_2}{\partial y} \Big|_{y=0} \right). \quad (12)$$

Here, L is the latent heat of evaporation, M is the mass velocity of liquid evaporating from a unit surface area per unit time ($M = const$), κ_1 and κ_2 are thermal conductivity coefficients.

The condition for C function on the phase boundary sets the saturated vapor concentration and presents the linearized form of the equation which is the consequence of the Clapeyron–Clausius and Mendeleev–Clapeyron equations [6]:

$$C \Big|_{y=0} = C_* [1 + \varepsilon (T_2 \Big|_{y=0} - T_0)], \quad (13)$$

where $\varepsilon = L\mu_0/(RT_0^2)$, μ_0 is the molar mass of the evaporating liquid, R is the universal gas constant, C_* is the saturated vapor concentration at $T_2 = T_0$.

Note, that the equations describing motion (1), (2), heat (4) and vapor transfer (5), as well as the temperature continuity condition (see the second relation in (10)), dynamic condition (11), and heat transfer condition (12) admit substitution of the temperature function in the form $\tilde{T}_i = T_i - T_0$ with respond to introduction of modified pressure \tilde{p} ($\tilde{p} = p' - \rho\beta T_0 \mathbf{g} \cdot \mathbf{x}$). Condition (13) can be written in the form:

$$C \Big|_{y=0} = C_* (1 + \varepsilon T_2 \Big|_{y=0}). \quad (14)$$

The corresponding substitution in the conditions setting the thermal boundary regime (8) leads to relations $\tilde{T}_1 = Ax + \tilde{\vartheta}^-$, $\tilde{T}_2 = Ax + \tilde{\vartheta}^+$, where $\tilde{\vartheta}^- = \vartheta^- - T_0$, $\tilde{\vartheta}^+ = \vartheta^+ - T_0$. For convenience, we will omit the symbol “tilde” over T_i , p_i , ϑ^- , ϑ^+ in the next sections.

The problem is solved at given gas flow rate Q , defined by relation:

$$Q = \int_0^h \rho_2 u_2(y) dy, \quad (15)$$

and under assumption of the closed flow condition in the liquid layer:

$$\int_{-l}^0 u_1(y) dy = 0. \quad (16)$$

3. Exact solutions under condition of zero vapor flux on the upper solid wall of the channel

The derivation of solution for equations (1)–(5) in form (6) results in the explicit expressions for the required functions which define longitudinal velocity $u_i(y)$, temperature $T_i(x, y)$ and vapor concentration in gas phase $C(x, y)$.

In the case of $A > 0$ we obtain the following relations:

$$\begin{aligned}
u_1(y) &= C_1 \sin(k_1 y) + C_2 \cos(k_1 y) + C_3 \operatorname{sh}(k_1 y) + C_4 \operatorname{ch}(k_1 y), \\
u_2(y) &= \bar{C}_1 \sin(m_1 y) \operatorname{sh}(m_1 y) + \bar{C}_2 \cos(m_1 y) \operatorname{sh}(m_1 y) + \bar{C}_3 \sin(m_1 y) \operatorname{ch}(m_1 y) + \\
&\quad + \bar{C}_4 \cos(m_1 y) \operatorname{ch}(m_1 y), \\
T_1(x, y) &= Ax + \frac{F_1}{k_1^2} (-C_1 \sin(k_1 y) - C_2 \cos(k_1 y) + C_3 \operatorname{sh}(k_1 y) + C_4 \operatorname{ch}(k_1 y)) + C_5 y + C_6, \\
T_2(x, y) &= Ax + \frac{F_2}{2m_1^2} \left(-\bar{C}_1 \cos(m_1 y) \operatorname{ch}(m_1 y) + \bar{C}_2 \sin(m_1 y) \operatorname{ch}(m_1 y) - \right. \\
&\quad \left. - \bar{C}_3 \cos(m_1 y) \operatorname{sh}(m_1 y) + \bar{C}_4 \sin(m_1 y) \operatorname{sh}(m_1 y) \right) + \bar{C}_5 y + \bar{C}_6, \\
C(x, y) &= -Bx + \frac{G}{2m_1^2} \left(-\bar{C}_1 \cos(m_1 y) \operatorname{ch}(m_1 y) + \bar{C}_2 \sin(m_1 y) \operatorname{ch}(m_1 y) - \right. \\
&\quad \left. - \bar{C}_3 \cos(m_1 y) \operatorname{sh}(m_1 y) + \bar{C}_4 \sin(m_1 y) \operatorname{sh}(m_1 y) \right) + \bar{C}_7 y + \bar{C}_8.
\end{aligned} \tag{17}$$

The functions u_1 and u_2 are solutions of the corresponding equations $u_{1y}^{(4)} + \lambda_1 u_1 = 0$ and $u_{2y}^{(4)} + \lambda_2 u_2 = 0$ [12], which follow from equations (1), (2) as a result of series of consecutive actions: substitution of solutions (6), cross differentiation (in order to exclude pressure function), differentiation on y . If $A > 0$ then the inequality $\lambda_1 < 0$ is valid; inequality $\lambda_2 > 0$ will be fulfilled for liquid – gas system like "ethanol – nitrogen" because of $E < 0$.

When condition $A < 0$ is satisfied then the required functions (6) take the form:

$$\begin{aligned}
u_1(y) &= C_1 \sin(k_2 y) \operatorname{sh}(k_2 y) + C_2 \cos(k_2 y) \operatorname{sh}(k_2 y) + C_3 \sin(k_2 y) \operatorname{ch}(k_2 y) + C_4 \cos(k_2 y) \operatorname{ch}(k_2 y), \\
u_2(y) &= \bar{C}_1 \sin(m_2 y) + \bar{C}_2 \cos(m_2 y) + \bar{C}_3 \operatorname{sh}(m_2 y) + \bar{C}_4 \operatorname{ch}(m_2 y), \\
T_1(x, y) &= Ax + \frac{F_1}{2k_2^2} \left(-C_1 \cos(k_2 y) \operatorname{ch}(k_2 y) + C_2 \sin(k_2 y) \operatorname{ch}(k_2 y) - \right. \\
&\quad \left. - C_3 \cos(k_2 y) \operatorname{sh}(k_2 y) + C_4 \sin(k_2 y) \operatorname{sh}(k_2 y) \right) + C_5 y + C_6, \\
T_2(x, y) &= Ax + \frac{F_2}{m_2^2} \left(-\bar{C}_1 \sin(m_2 y) - \bar{C}_2 \cos(m_2 y) + \bar{C}_3 \operatorname{sh}(m_2 y) + \bar{C}_4 \operatorname{ch}(m_2 y) \right) + \bar{C}_5 y + \bar{C}_6, \\
C(x, y) &= -Bx + \frac{G}{m_2^2} \left(-\bar{C}_1 \sin(m_2 y) - \bar{C}_2 \cos(m_2 y) + \bar{C}_3 \operatorname{sh}(m_2 y) + \bar{C}_4 \operatorname{ch}(m_2 y) \right) + \bar{C}_7 y + \bar{C}_8.
\end{aligned} \tag{18}$$

Here, u_1 and u_2 also satisfy the equations $u_{1y}^{(4)} + \lambda_1 u_1 = 0$ and $u_{2y}^{(4)} + \lambda_2 u_2 = 0$. The inequalities $\lambda_1 > 0$ and $\lambda_2 < 0$ hold if $A < 0$.

Coefficients $\lambda_1, \lambda_2, k_s, m_s, F_1, F_2, G, E$ are calculated via geometric, physical and chemical parameters of the problem. Here, index s denotes the solution for positive ($s = 1$) or negative ($s = 2$) values of the longitudinal temperature gradient A . Coefficients C_i and \bar{C}_i ($i = 1, \dots, 8$) are the integration constants. They are different for each of the solutions. Exact representations of the listed parameters $\{\lambda_i, k_s, m_s, F_i, G, E\}$ ($i = 1, 2; s = 1, 2$) are given in Appendix 1.

We have to note the following: the equation determining the saturated vapor concentration at the interface (14) dictates the compatibility condition connecting the values A and B : $B = -C_* \varepsilon A$. The pressure functions p_i are found on the basis of their partial derivatives from (1), (2).

4. The general scheme for finding the integration constants

Let the gas flow rate Q (see (15)), the parameters determining the external thermal impact on the channel walls A and ϑ^- , ϑ^+ be preset. The fulfilment of boundary conditions (7)–(12) and relationships (15), (16) results in a systems of linear algebraic equations for calculation the integration constants. Determining the constants, it is possible to plot the velocity profiles, temperature and vapor concentration distribution according to formulas (17) or (18), depending on the sign of parameter A .

The algorithm for finding all the unknown integration constants is outlined below.

- (i) Conditions on the solid walls and the interface (7)–(12), expression defining the closed flow in the liquid layer (16), and equality (15) giving the gas flow rate lead to systems of linear algebraic equations to find the constants $C_1, C_2, C_3, C_4, C_5, C_6$ and $\bar{C}_1, \bar{C}_2, \bar{C}_3, \bar{C}_4, \bar{C}_5, \bar{C}_6, \bar{C}_7$. The resulting systems contain 13 unknowns with 11 equations.
- (ii) The integration constants C_6, \bar{C}_6 are set equal to zero in order to close the systems of linear algebraic equations.
- (iii) The systems of equations are solved by the Gaussian method using program code, which also provides a data set for plotting velocity profiles, temperature and vapor concentration distributions.
- (iv) The coefficient \bar{C}_8 is expressed through the coefficient \bar{C}_1 (when $A > 0$) or through the coefficients \bar{C}_2 and \bar{C}_4 (when $A < 0$) by virtue of condition (14) for defining the vapor concentration at the interface.

The systems of equations themselves, which determine the unknown parameters of integration, are presented in Appendix 2 (see (19) and (21)).

5. Examples of two-layer flows

The effect of changing of the channel inclination angle, the interface temperature gradient and transversal temperature drop on the structure of the flow, temperature and vapor concentration distribution has been studied. The ethanol and nitrogen are chosen as working liquid and gas respectively. The ethanol liquid volatilizes from the lower layer, so that a mixture of nitrogen and ethanol vapor is in the upper layer. The physical parameters of the working media are given below according to [13] in order {ethanol, nitrogen} (or ethanol only): $\rho = \{7.89 \cdot 10^2, 1.2\}$ kg/m³, $\nu = \{0.15 \cdot 10^{-5}, 0.15 \cdot 10^{-4}\}$ m²/s, $\beta = \{1.079 \cdot 10^{-3}, 3.67 \cdot 10^{-3}\}$ K⁻¹, $\chi = \{8.9 \cdot 10^{-8}, 0.3 \cdot 10^{-4}\}$ m²/s, $\kappa = \{0.1672, 0.02717\}$ W/(m·K), $\sigma_T = -0.8 \cdot 10^{-4}$ N/(m·K), $D = 0.135 \cdot 10^{-4}$ m²/s, $L = 8.55 \cdot 10^5$ W·s/kg, $C_* = 0.1$ (corresponds to equilibrium temperature $T_0 = 20^\circ\text{C}$), $\gamma = -0.62$, $\varepsilon = 0.059$ K⁻¹.

The value of the gas flow rate Q is assumed to be equal to $3.6 \cdot 10^{-5}$ kg/(m·s). The following parameters are fixed for all the cases under consideration: thicknesses of the liquid and gas layers are $l = 5$ mm and $h = 5$ mm, respectively, and gravity acceleration $g = 9.81$ m/s². The values of the Soret and Dufour coefficients are assumed to be equal to $\alpha = 10^{-4}$ K⁻¹, $\delta = 10^{-4}$ K, respectively. The parameters defining the external thermal loads applied on the channel walls are the longitudinal temperature gradient A and values ϑ^- , ϑ^+ (see condition (8)). Let us note that the values of functions T_i and terms ϑ^- , ϑ^+ in Figs. 2–4 refer to the deviation from the reference

temperature T_0 due to substitution made in Section 2 (here, $T_0 = 20^\circ\text{C}$). Figure captions contain values of these deviations.

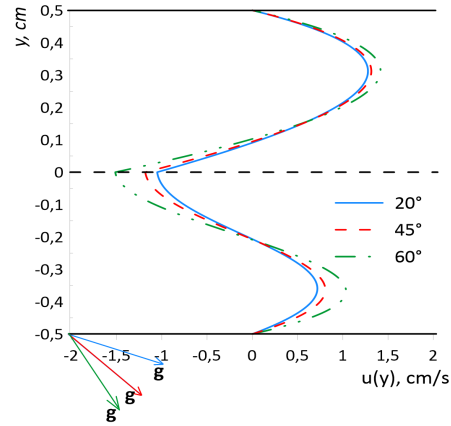


Fig. 2. Velocity profiles in the ethanol–nitrogen system: $Q = 3.6 \cdot 10^{-5} \text{ kg}/(\text{m}\cdot\text{s})$, $A = 10 \text{ K}/\text{m}$, $\vartheta^- = -4^\circ\text{C}$, $\vartheta^+ = 3^\circ\text{C}$; $\varphi = 20^\circ$ (blue line), 45° (red line), 60° (green line)

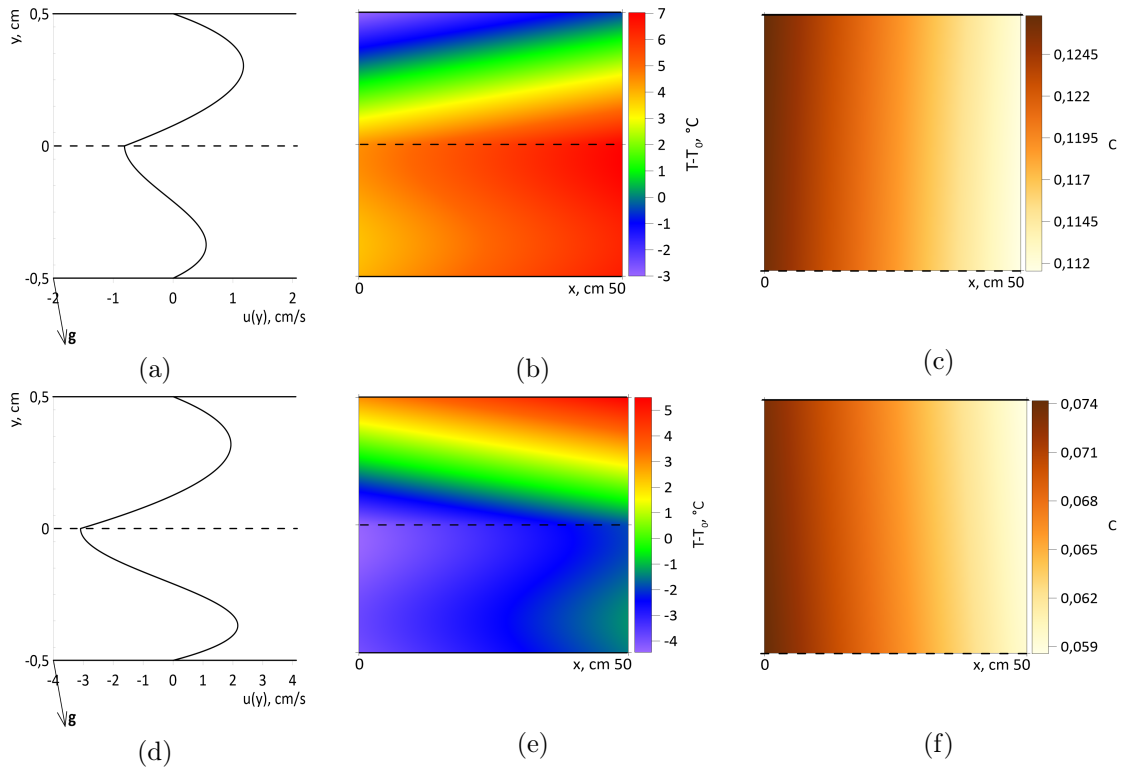


Fig. 3. Velocity profiles (a, d), thermal field (b, e), vapor concentration distribution (c, f) in the ethanol–nitrogen system: $Q = 3.6 \cdot 10^{-5} \text{ kg}/(\text{m}\cdot\text{s})$, $A = 5 \text{ K}/\text{m}$, $\varphi = 80^\circ$; (a, b, c) – $\vartheta^- = 4^\circ\text{C}$, $\vartheta^+ = -3^\circ\text{C}$, $M = 1.026 \cdot 10^{-6} \text{ kg}/(\text{m}^2\cdot\text{s})$, (d, e, f) – $\vartheta^- = -4^\circ\text{C}$, $\vartheta^+ = 3^\circ\text{C}$, $M = 1.062 \cdot 10^{-6} \text{ kg}/(\text{m}^2\cdot\text{s})$

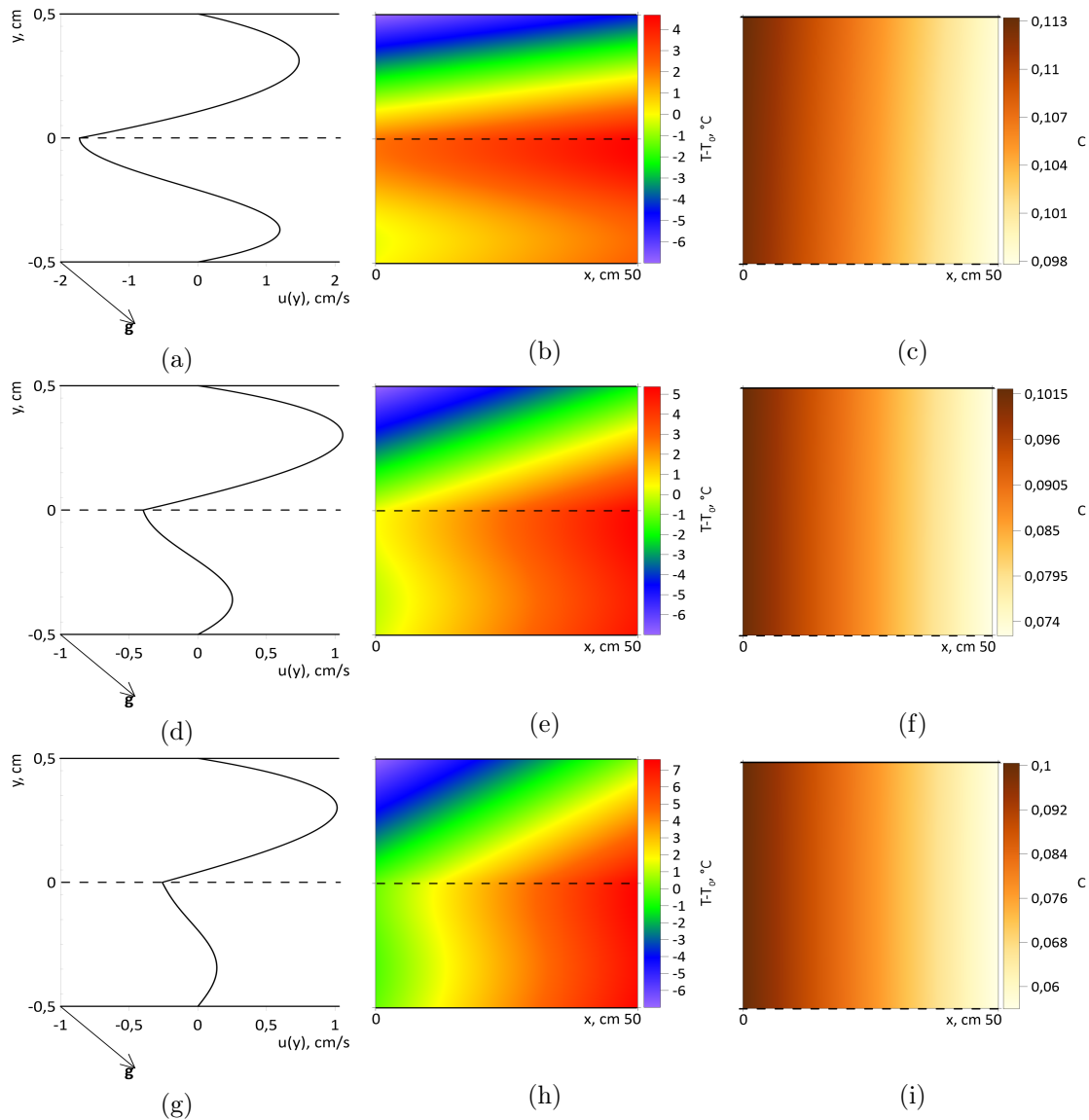


Fig. 4. Velocity profiles (a, d, g), thermal field (b, e, h), vapor concentration distribution (c, f, i) in the ethanol–nitrogen system: $Q = 3.6 \cdot 10^{-5}$, kg/(m·s), $\varphi = 45^\circ$, $\vartheta^- = 0^\circ\text{C}$, $\vartheta^+ = -7^\circ\text{C}$; (a,b,c) – $A = 5$ K/m, $M = 1.062 \cdot 10^{-6}$ kg/(m²·s), (d,e,f) – $A = 10$ K/m, $M = 2.124 \cdot 10^{-6}$ kg/(m²·s), (g,h,i) – $A = 15$ K/m, $M = 3.186 \cdot 10^{-6}$ kg/(m²·s)

Velocity profiles at the inclination angle φ , having values 20° , 45° and 60° are presented in Fig. 2. The reverse flow near the interface intensifies at larger values of the angle φ .

The effect of intensification of the thermal regime created on the channel walls on the character of the flow has been studied. The change of coefficients ϑ^- , ϑ^+ (see Fig. 3) led to the formation of transverse temperature drop and, therefore, to the change of flow characteristics. In the first case, their values are assumed to be $\vartheta^- = 4^\circ\text{C}$, $\vartheta^+ = -3^\circ\text{C}$ (Fig. 3 (a, b, c)), in another case: $\vartheta^- = -4^\circ\text{C}$, $\vartheta^+ = 3^\circ\text{C}$ (Fig. 3 (d, e, f)). The maximum velocity values are observed in the liquid layer in the second case (Fig. 3 (d)). The vapor distributions are preserved in Fig. 3 (c, f).

Values of the evaporation mass flow rate M are equal for both cases.

The dependence of the velocity field, temperature and vapor concentration distributions on changes of the longitudinal temperature gradient A is demonstrated in Fig. 4. The parameter A takes the values $\{5, 10, 15\}$ K/m. The velocity field is restructured due to changing of the thermocapillary effect intensity. At the same time more complicated temperature pictures and the growth of the evaporation mass flow rate are observed with increase of A (see Fig. 4 (b, e, h)).

Conclusions

New exact solution of a special type of convection equations has been constructed. It allows one to model a two-layer flow in an inclined channel. Evaporation at a non-deformable thermocapillary interface, the Soret and Dufour effects in the gas-vapor layer are taken into account. The condition of zero vapor flux on the upper solid wall of the channel is set. Exact solution has been obtained for the cases of positive and negative longitudinal temperature gradients at the channel boundaries. Scheme for determining the parameters of the problem and unknown constants is outlined. Examples of characteristics for bilayer flows in the ethanol-nitrogen system are presented. The influence of the channel inclination angle and the thermal load at the system boundaries on the flow pattern has been studied.

The work was carried out in accordance with the State Assignment of the Russian Ministry of Science and Higher Education entitled ‘Modern methods of hydrodynamics for environmental management, industrial systems and polar mechanics’ (Govt. contract code: FZMW-2020-0008).

Appendix 1. Formulas for calculating the parameters in expressions (17) and (18)

Coefficients k_s, m_s, F_i, G :

$$k_1 = \sqrt[4]{\frac{Ag \cos \varphi \beta_1}{\chi_1 \nu_1}}, \quad m_1 = \sqrt[4]{-Ag \cos \varphi \frac{E}{4}}, \quad \lambda_1 = -\frac{Ag \cos \varphi \beta_1}{\chi_1 \nu_1},$$

$$k_2 = \sqrt[4]{-\frac{Ag \cos \varphi \beta_1}{4\chi_1 \nu_1}}, \quad m_2 = \sqrt[4]{Ag \cos \varphi E}, \quad \lambda_2 = -Ag \cos \varphi E,$$

$$F_1 = \frac{A}{\chi_1}, \quad F_2 = \frac{A(D - \delta \chi_2 C_* \varepsilon)}{\chi_2 D(1 - \alpha \delta)}, \quad G = \frac{A(\alpha D - \chi_2 C_* \varepsilon)}{\chi_2 D(\alpha \delta - 1)}, \quad E = \frac{D(\beta_2 - \alpha \gamma) - \chi_2 C_* \varepsilon(\delta \beta_2 - \gamma)}{\chi_2 \nu_2 D(1 - \alpha \delta)}.$$

Appendix 2. Systems of linear algebraic equations to find integration coefficients in (17) and (18)

System in the case of negative value of the parameter A :

$$\begin{aligned}
& \sin(k_2 l) \operatorname{sh}(k_2 l) C_1 - \cos(k_2 l) \operatorname{sh}(k_2 l) C_2 - \sin(k_2 l) \operatorname{ch}(k_2 l) C_3 + \cos(k_2 l) \operatorname{ch}(k_2 l) C_4 = 0, \\
& \sin(m_2 h) \bar{C}_1 + \cos(m_2 h) \bar{C}_2 + \operatorname{sh}(m_2 h) \bar{C}_3 + \operatorname{ch}(m_2 h) \bar{C}_4 = 0, \\
& \frac{F_1}{2k_2^2} \left(-\cos(k_2 l) \operatorname{ch}(k_2 l) C_1 - \sin(k_2 l) \operatorname{ch}(k_2 l) C_2 + \cos(k_2 l) \operatorname{sh}(k_2 l) C_3 + \right. \\
& \left. + \sin(k_2 l) \operatorname{sh}(k_2 l) C_4 \right) - l C_5 = \vartheta^-, \\
& \frac{F_2}{m_2^2} \left(-\sin(m_2 h) \bar{C}_1 - \cos(m_2 h) \bar{C}_2 + \operatorname{sh}(m_2 h) \bar{C}_3 + \operatorname{ch}(m_2 h) \bar{C}_4 \right) + h \bar{C}_5 = \vartheta^+, \\
& \frac{G + \alpha F_2}{m_2} \left(-\cos(m_2 h) \bar{C}_1 + \sin(m_2 h) \bar{C}_2 + \operatorname{ch}(m_2 h) \bar{C}_3 + \operatorname{sh}(m_2 h) \bar{C}_4 \right) + \alpha \bar{C}_5 + \bar{C}_7 = 0, \\
& C_4 - \bar{C}_2 - \bar{C}_4 = 0, \\
& -\frac{F_1}{2k_2^2} C_1 + \frac{F_2}{m_2^2} (\bar{C}_2 - \bar{C}_4) = 0, \\
& \rho_1 \nu_1 k_2 C_2 + \rho_1 \nu_1 k_2 C_3 - \rho_2 \nu_2 m_2 \bar{C}_1 - \rho_2 \nu_2 m_2 \bar{C}_3 = \sigma_T A, \\
& \kappa_1 \frac{F_1}{2k_2} (C_2 - C_3) + \kappa_1 C_5 + \frac{\kappa_2 (F_2 + \delta G) + LD \rho_2 (G + \alpha F_2)}{m_2} (\bar{C}_1 - \bar{C}_3) - \\
& - (\kappa_2 + LD \rho_2 \alpha) \bar{C}_5 - (\delta \kappa_2 + LD \rho_2) \bar{C}_7 = 0, \\
& (1 - \cos(m_2 h)) \bar{C}_1 + \sin(m_2 h) \bar{C}_2 + (\operatorname{ch}(m_2 h) - 1) \bar{C}_3 + \operatorname{ch}(m_2 h) \bar{C}_4 = \frac{m_2 Q}{\rho_2}, \\
& (\sin(k_2 l) \operatorname{ch}(k_2 l) - \cos(k_2 l) \operatorname{sh}(k_2 l)) C_1 + (1 - \sin(k_2 l) \operatorname{sh}(k_2 l) - \\
& - \cos(k_2 l) \operatorname{ch}(k_2 l)) C_2 + (-1 - \sin(k_2 l) \operatorname{sh}(k_2 l) + \cos(k_2 l) \operatorname{ch}(k_2 l)) C_3 + \\
& + (\cos(k_2 l) \operatorname{sh}(k_2 l) + \sin(k_2 l) \operatorname{ch}(k_2 l)) C_4 = 0.
\end{aligned} \tag{19}$$

The coefficient \bar{C}_8 is expressed through the coefficients \bar{C}_2 and \bar{C}_4 by virtue of the condition (14):

$$\bar{C}_8 = C_* + \frac{G + C_* \varepsilon F_2}{m_2^2} \bar{C}_2 - \frac{G - C_* \varepsilon F_2}{m_2^2} \bar{C}_4. \tag{20}$$

System in the case of positive value of the parameter A :

$$\begin{aligned}
& -\sin(k_1 l)C_1 + \cos(k_1 l)C_2 - \operatorname{sh}(k_1 l)C_3 + \operatorname{ch}(k_1 l)C_4 = 0, \\
& \sin(m_1 h) \operatorname{sh}(m_1 h)\bar{C}_1 + \cos(m_1 h) \operatorname{sh}(m_1 h)\bar{C}_2 + \sin(m_1 h) \operatorname{ch}(m_1 h)\bar{C}_3 + \\
& + \cos(m_1 h) \operatorname{ch}(m_1 h)\bar{C}_4 = 0, \\
& \frac{F_1}{k_1^2} \left(\sin(k_1 l)C_1 - \cos(k_1 l)C_2 - \operatorname{sh}(k_1 l)C_3 + \operatorname{ch}(k_1 l)C_4 \right) - lC_5 = \vartheta^-, \\
& \frac{F_2}{2m_1^2} \left(-\cos(m_1 h) \operatorname{ch}(m_1 h)\bar{C}_1 + \sin(m_1 h) \operatorname{ch}(m_1 h)\bar{C}_2 - \cos(m_1 h) \operatorname{sh}(m_1 h)\bar{C}_3 + \right. \\
& \left. + \sin(m_1 h) \operatorname{sh}(m_1 h)\bar{C}_4 \right) + h\bar{C}_5 = \vartheta^+, \\
& \frac{G + \alpha F_2}{2m_1} \left((-\cos(m_1 h) \operatorname{sh}(m_1 h) + \sin(m_1 h) \operatorname{ch}(m_1 h))\bar{C}_1 + (\sin(m_1 h) \operatorname{sh}(m_1 h) + \right. \\
& \left. + \cos(m_1 h) \operatorname{ch}(m_1 h))\bar{C}_2 + (-\cos(m_1 h) \operatorname{ch}(m_1 h) + \sin(m_1 h) \operatorname{sh}(m_1 h))\bar{C}_3 + \right. \\
& \left. + (\sin(m_1 h) \operatorname{ch}(m_1 h) + \cos(m_1 h) \operatorname{sh}(m_1 h))\bar{C}_4 \right) + \alpha\bar{C}_5 + \bar{C}_7 = 0, \\
& C_2 + C_4 - \bar{C}_4 = 0, \\
& \frac{F_1}{k_1^2} \left(-C_2 + C_4 \right) + \frac{F_2}{2m_1^2} \bar{C}_1 = 0, \\
& \rho_1 \nu_1 k_1 C_1 + \rho_1 \nu_1 k_1 C_3 - \rho_2 \nu_2 m_1 \bar{C}_2 - \rho_2 \nu_2 m_1 \bar{C}_3 = \sigma_T A, \\
& \kappa_1 \frac{F_1}{k_1} \left(-C_1 + C_3 \right) + \kappa_1 C_5 - \frac{\kappa_2 (F_2 + \delta G) LD \rho_2 (G + \alpha F_2)}{2m_1} (\bar{C}_2 - \bar{C}_3) - \\
& - (\kappa_2 + \alpha LD \rho_2) \bar{C}_5 - (\delta \kappa_2 + LD \rho_2) \bar{C}_7 = 0, \\
& (\sin(m_1 h) \operatorname{ch}(m_1 h) - \cos(m_1 h) \operatorname{sh}(m_1 h))\bar{C}_1 + (-1 + \sin(m_1 h) \operatorname{sh}(m_1 h) + \\
& + \cos(m_1 h) \operatorname{ch}(m_1 h))\bar{C}_2 + (1 + \sin(m_1 h) \operatorname{sh}(m_1 h) - \cos(m_2 h) \operatorname{ch}(m_1 h))\bar{C}_3 + \\
& + (\cos(m_1 h) \operatorname{sh}(m_1 h) + \sin(m_1 h) \operatorname{ch}(m_1 h))\bar{C}_4 = \frac{2m_1 Q}{\rho_2}, \\
& (-1 + \cos(k_1 l))C_1 + \sin(k_1 l)C_2 + (1 - \operatorname{ch}(k_1 l))C_3 + \operatorname{sh}(k_1 l)C_4 = 0.
\end{aligned} \tag{21}$$

The coefficient \bar{C}_8 in this case is expressed through the coefficient \bar{C}_1 by virtue of the condition (14) and has a form:

$$\bar{C}_8 = C_* + \frac{G - C_* \varepsilon F_2}{2m_1^2} \bar{C}_1. \tag{22}$$

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Моделирование стационарных течений системы жидкость-газ в наклонном канале с учетом испарения

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Аннотация. Двухслойные течения жидкости и газопаровой смеси в наклонном канале моделируются на основе системы уравнений конвекции Обербека–Буссинеска с учетом эффектов испарения и термодиффузии. Построено новое точное решение задачи испарительной конвекции в постановке с недеформируемой границей раздела и при условии отсутствия потока пара на стенке канала. Представлен аналитический вид искомых функций в случае линейного по продольной координате нагрева границ канала. Подробно описаны алгоритмы расчета констант интегрирования. Для системы жидкостей этанол-азот приведены примеры типов течений.

Ключевые слова: точное решение, двухслойное течение, конвекция, испарение, граница раздела, наклонный канал.