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Mathematical Modeling of the Thin Liquid Layer Runoff Process Based on Generalized Conditions at the Interface: Parametric Analysis and Numerical Solution

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Abstract. The problem of a thin layer of liquid flowing down an inclined substrate under conditions of a co-current gas flow is considered. Mathematical modeling is carried out on the basis of the Navier–Stokes and heat transfer equations, as well as generalized conditions at the thermocapillary boundary. Parametric analysis of the problem is made. An algorithm of numerical solution is constructed for the evolution equation determining the thickness of the liquid layer. A comparison of numerical calculations for ethanol and HFE-7100 liquids is presented. The influence of an additional term in the interface energy equation on the dynamics of the liquid layer is shown.

Keywords: Navier–Stokes equations, interface, thin layer approximation, evaporation, parametric analysis, numerical solution.

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Introduction

Currently, film flows are used in many fields of activity, such as the space industry, chemical and pharmaceutical industries, etc. This is largely due to the trend of device miniaturization. Thin layers of liquid are used as heat carriers and heat stabilizers, solvents and much more. In this regard, interest in the theoretical study of such flows has increased. The main difficulties in their analytical study are associated with a large number of factors affecting thin liquid layers and the nonlinearity of the processes under study. In some cases, when the processes described by the mathematical model have strong nonlinearity, it is permissible to use some simplifications of the model or its solutions that do not entail significant distortions of the results. One of the methods can be a parametric analysis of the problem, which makes it possible to detect elements that do not make a significant contribution to the processes under study.

Quite a large number of works are devoted to mathematical modeling of flows of thin liquid layers, taking into account additional factors that govern the nature of processes [1–6]. One of the important effects in the study of such flows is evaporation [7–11]. As a rule, mathematical models

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of problems in the thin layer approximation are based on the Navier–Stokes equations [4,5] or Oberbeck–Bussinesq equations [1,3,12]. When modeling flows with interface particular attention is paid to the formulation of boundary conditions [8,9,13]. The numerical simulation of the flows of thin liquid layers are preformed in [12,14,15].

This paper presents a mathematical model of the motion for a thin film of a viscous incompressible liquid driven by a gas flux along an inclined unevenly heated solid substrate. Gravitational, thermocapillary effects, evaporation, as well as the impact of additional shear stresses from the external environment are taken into account in the model. The fulfillment of the laws of conservation of mass, impulse and energy is ensured by the kinematic, dynamic and energy conditions set at the interface. Modeling of liquid motion is based on the Navier–Stokes and heat transfer equations. The Hertz–Knudsen kinetic equation is used to determine the dependence of the local vapor mass flux on the temperature at the interface. A parametric analysis of the problem is performed based on the use of two types of liquids: ethanol and HFE-7100. Analytical solutions for the main terms of the expansion in powers of a small parameter and an evolution equation that determines the position of the interface are obtained. An algorithm for the numerical solution of the evolution equation is constructed. Numerical results on the study of the influence of the liquid nature on change in the liquid layer thickness over time are shown. Numerical results are obtained in the case when the energy condition is written taking into account an additional term.

1. Problem statement

Let us consider the flow of a thin layer of a viscous incompressible liquid over an inclined, unevenly heated substrate. A gas moves over the layer. The problem is considered in one-sided formulation when dynamic processes in the gas are not considered. However, the shear stresses created by the gas can be taken into account when modeling the flow at the interface. It is assumed that evaporation occurs at the thermocapillary interface. A solid impenetrable substrate is inclined at an angle α to the horizon, coincides with the coordinate axis Ox and is defined by the expression $z = 0$ (see Fig. 1). The position of the interface is given by the equation $z = h(x,t)$. The gravity vector has the form $\mathbf{g} = (g_1, g_2) = (g\sin\alpha, -g\cos\alpha)$, $g = |\mathbf{g}|$.

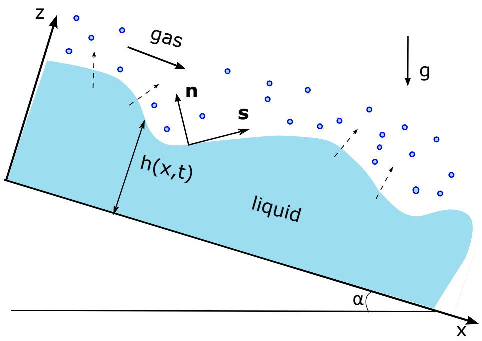

Fig. 1. Geometry of the flow area.

As a rule, the characteristic deformation length of the free surface exceeds the deformation amplitude. Therefore, two different length scales are often distinguished when considering problems about the flow of a thin layer. Let l be the longitudinal characteristic length and d is the transverse characteristic scale such that $l \gg d$. Then $\varepsilon = \dfrac{d}{l}$ is a small dimensionless parameter of the problem. Characteristic longitudinal and transverse velocities u_* and w_* must also be related: $w_* = \varepsilon u_*$. The characteristic time of the process t_* is related to other parameters of the problem as follows: $l = u_* t_*$. The characteristic pressure is given by the formula $p_* = \dfrac{\rho u_* \nu l}{d^2}$.

The Navier-Stokes and heat transfer equations are used as a mathematical model. The system of equations in dimensionless form is written as follows:

$$Re\varepsilon^2(u_t + uu_x + wu_z) - \varepsilon^2 u_{xx} = u_{zz} - p_x + \gamma_1 \sin\alpha, \tag{1}$$

$$Re\varepsilon^4(w_t + uw_x + ww_z) - \varepsilon^4 w_{xx} - \varepsilon^2 w_{zz} = -p_z - \gamma_2 \cos\alpha, \tag{2}$$

$$u_x + w_z = 0, \tag{3}$$

$$RePr\varepsilon^2(T_t + uT_x + wT_z) - \varepsilon^2 T_{xx} = T_{zz}. \tag{4}$$

Here, $\mathbf{v} = (u, w)$ is the liquid velocity vector, p is the pressure, T is the temperature, $Re = \dfrac{u_* l}{\nu}$ is the Reynolds number, $Pr = \dfrac{\nu}{\chi}$ is the Prandtl number, $\gamma_1 = \dfrac{Gr}{BuRe\varepsilon}$, $\gamma_2 = \dfrac{Gr}{BuRe}$, $Gr = \dfrac{Bugd^3}{\nu^2}$ is the Grashof number, $Bu = \beta T_*$ is the Boussinesq number, ν and χ are the kinematic viscosity and thermal diffusivity coefficients, ρ is the liquid density, T_* is the characteristic temperature difference.

On a solid impermeable substrate, the no-slip conditions are satisfied:

$$u|_{z=0} = 0, \quad w|_{z=0} = 0. \tag{5}$$

The temperature at the boundary $z = 0$ is distributed according to some given law:

$$T|_{z=0} = \Theta_0(x, t). \tag{6}$$

The kinematic, dynamic and energetic conditions fulfilled at the interface are consequences of the laws of conservation of mass, impulse and energy [8,10,11]. Coordinates of the normal vector to the boundary (n_1, n_2), the curvature of the free boundary H and the speed of its movement in the direction of the external normal D_n are given by the relations:

$$n_1 = -\frac{\varepsilon h_x}{\sqrt{1+\varepsilon^2 h_x^2}}, \quad n_2 = \frac{1}{\sqrt{1+\varepsilon^2 h_x^2}}, \quad 2H = \frac{\varepsilon h_{xx}}{\sqrt{(1+\varepsilon^2 h_x^2)^3}}, \quad D_n = -\frac{\varepsilon h_t}{\sqrt{1+\varepsilon^2 h_x^2}}.$$

Then, the kinematic condition in dimensionless form will be written as follows:

$$-\varepsilon(h_t + h_x u - w)\frac{1}{\sqrt{1+\varepsilon^2 h_x^2}} = J_{ev}\bar{J}. \tag{7}$$

The projections of the dynamic condition on the normal and tangent vector have the following form:

$$\begin{gathered} -p + \frac{2\varepsilon^2}{1+\varepsilon^2 h_x^2}[\varepsilon^2 h_x^2 u_x + w_z - h_x(u_z + \varepsilon^2 w_x)] = \\ = -p^g + \frac{\bar{\rho}\bar{\nu}\bar{\upsilon}}{\bar{h}}\frac{2\varepsilon^2}{1+\varepsilon^2 h_x^2}\left[\varepsilon^2 h_x^2 u_x^g + w_z^g - \varepsilon h_x(u_z^g + w_x^g)\right] + Re\varepsilon^2(1-\frac{1}{\rho})J_{ev}^2\bar{J}^2 + 2\sigma H\frac{\varepsilon^2}{Ca}, \end{gathered} \tag{8}$$

$$\frac{2}{1+\varepsilon^2 h_x^2}\Big[-\varepsilon h_x u_x+\varepsilon h_x w_z-\frac{1}{2\varepsilon}(1-\varepsilon^2 h_x)(u_z+\varepsilon^2 w_x)\Big]-$$
$$-\frac{\bar\rho\bar\nu\bar\upsilon}{\bar h}\frac{2}{1+\varepsilon^2 h_x^2}\Big[-\varepsilon h_x u_x^g+\varepsilon h_x w_z^g+\frac{1}{2}(1-\varepsilon^2 h_x^2)(u_z^g+w_x^g)\Big]= \tag{9}$$
$$=-\frac{Ma}{RePr}\Big[\frac{1}{\sqrt{1+\varepsilon^2 h_x^2}}(T_x+h_xT_z)\Big].$$

Let us represent the energy condition in the following dimensionless form:

$$\frac{\partial T}{\partial n}+\beta_2\{Tdiv_\Gamma v\}=\beta_3\bar J J_{ev}+\beta_4\bar J J_{ev}\Big\{-p+\frac{2\varepsilon^2}{1+\varepsilon^2h_x^2}[\varepsilon^2h_x^2u_x+w_z-h_x(u_z+\varepsilon^2w_x)]\Big\}+$$
$$+\frac{1}{2}\beta_5\bar J^3J_{ev}^3+\beta_6\sigma\Bigg\{\frac{\varepsilon h_{xx}}{\sqrt{(1+\varepsilon^2h_x^2)^3}}\Bigg\}\bar J J_{ev}, \tag{10}$$

where $\dfrac{\partial T}{\partial n}$ and $div_\Gamma\mathbf{v}$ are calculated as follows:

$$\frac{\partial T}{\partial n}=\frac{1}{\varepsilon}\frac{1}{\sqrt{1+\varepsilon^2h_x^2}}(-\varepsilon^2H_xT_x+T_z),$$

$$div_\Gamma\mathbf{v}=\sum_{i=1}^{2}\frac{\partial v_i}{\partial x_i}-\sum_{i=1}^{2}n_i(n\cdot\nabla v_i)=$$
$$=(u_x+w_z)|_\Gamma-\Big\{\frac{\varepsilon^2h_x^2}{1+\varepsilon^2h_x^2}u_x-\frac{\varepsilon h_x}{1+\varepsilon^2h_x^2}u_z--\frac{\varepsilon h_x}{1+\varepsilon^2h_x^2}w_x+\frac{1}{1+\varepsilon^2h_x^2}w_z\Big\}.$$

Here $\bar\nu$, $\bar\rho$ are the ratios of kinematic viscosity coefficients and densities of gas and liquid, respectively $\Big(\bar\nu=\dfrac{\nu^g}{\nu};\bar\rho=\dfrac{\rho^g}{\rho}\Big)$, $\bar\upsilon=\dfrac{u_*^g}{u_*}$ is the the ratio of the characteristic longitudinal velocity of the gas to the characteristic velocity of the liquid u_*, p^g is the gas pressure. $Ma=\dfrac{\sigma_TT_*l}{\rho\nu\chi}$ is the Marangoni number, $Ca=\dfrac{u_*\rho\nu}{\sigma_0}$ is the capillary number, $\beta_2=\dfrac{Ma}{Re^2PrE\overline{U}}$, $\beta_3=\dfrac{1}{E}$, $\beta_4=(\dfrac{1}{\bar\rho}-1)\dfrac{1}{E\overline{U}}$, $\beta_5=(1-\dfrac{1}{\bar\rho})^2\dfrac{1}{E\overline{U}}$, $\beta_6=(1-\dfrac{1}{\bar\rho})\dfrac{1}{ReCaE\overline{U}}$, $\overline{U}=\dfrac{\lambda_U}{u_*^2}$, $E=\dfrac{\kappa T_*}{\lambda_U\rho\nu}$ is the evaporation parameter [6], κ is the coefficient of thermal conductivity, λ_U is the latent heat of vaporization, $\bar J=\dfrac{J_*^{ev}}{\rho u_*}$ or $\bar J=\dfrac{E}{Re}$, where J_*^{ev} is the characteristic value of vapor mass flux $\Big(J_*^{ev}=\dfrac{\kappa T_*}{\lambda_U\rho\nu}\Big)$. The first term on the left side of the condition (10) is responsible for the heat defect during its transfer through the interface. The remaining terms define the contribution of individual physical phenomena that create this defect. The second term on the left side is for the energy spent to overcome surface deformation by thermocapillary forces along the surface. The first term on the right side specifies the heat consumption for vaporization, the second — for boundary deformation, the third — for the change in the kinetic energy of the substance during the phase transition, the fourth — for the work performed by the liquid substance during evaporation (condensation) due to a change in specific volume [7, 9].

A linear dependence of the surface tension coefficient on temperature is assumed in this paper. In the dimensionless form, this dependence is written as follows: $\sigma=1-\alpha_\sigma T$, $\alpha_\sigma=\dfrac{MaCa}{RePr}$.

The value of the local vapor mass flux at the interface J_{ev} determined by the ratio (see [5,8]):

$$J_{ev} = \alpha_J T|_{z=h(x,t)}, \qquad \alpha_J = \alpha \rho_s \lambda_U \frac{T_*}{J_*} \Big(\frac{M}{2\pi R_g T_s^3}\Big)^{1/2}. \tag{11}$$

Here, α is the accommodation coefficient, ρ_s is the vapor density, M is the molecular weight, R_g is the universal gas constant, T_s is the saturated vapor temperature.

Let the characteristic velocity u_* be equal to the characteristic relaxation velocity of viscous stresses $u_\nu = \dfrac{\nu}{l}$. Then, the Reynolds number is $Re = 1$. In the present paper modeling is carried out for the case of moderate Reynolds numbers ($Re = O(1)$).

2. Obtaining an equation that determines the position of the interface

To determine the desired functions u, w, T, p, as well as the thickness of the liquid layer h, system of equations (1)–(4) in the long-wave approximation is considered. The solution of the problem is sought in the form of expansions in powers of a small parameter ε.

Equations (1)–(4) written for the principal terms of the expansion take the form

$$p_x^0 = u_{zz}^0 + \gamma_1 \sin\alpha, \qquad p_z^0 = -\gamma_2 \cos\alpha,$$

$$w_z^0 = -u_x^0, \qquad T_{zz}^0 = 0.$$

Consequences of the no-slip conditions (5) on the boundary $z = 0$ are the relations

$$u^0|_{z=0} = 0, \quad w^0|_{z=0} = 0, \tag{12}$$

temperature condition (6) results in following requirement:

$$T^0|_{z=0} = \Theta_0. \tag{13}$$

Consequences of the conditions at the interface (7)–(11) are the relations:

$$p^0 = p^g - \alpha_{Ca} h_{xx}(1 - \alpha_\sigma \Theta^0) + \alpha_D \alpha_J (\Theta^0)^2, \tag{14}$$

$$u_z^0 = -\alpha_{Ma} \tilde{\Theta}, \tag{15}$$

$$T_z^0 + \bar{\beta}_2 \Theta^0 (u_x^0) = \bar{\beta}_3 J_0 + \bar{\beta}_6 J_0 h_{xx}. \tag{16}$$

Here, $\Theta^0 = T^0|_{z=h(x,t)}$, $\tilde{\Theta} = (T_x^0 + h_x T_z^0)|_{z=h(x,t)}$.

A large number of effects are taken into account when the flow of a thin liquid layer is modelled at a thermocapillary boundary. Therefore, to simplify the problem, it makes sense to evaluate the contribution of each of them. During the parametric analysis of the problem, estimates of the values of dimensionless parameters α_f and β_i were obtained in cases when ethanol and HFE-7100 were selected as working liquids for the characteristic temperature values T_* equal to 1 and 10 K (see Tabs. 1–3).

Taking into account the parametric analysis of the problem, the solutions for the main terms of the decomposition are the functions u^0, w^0, p^0, T^0 of the form:

$$u^0 = (C_0)_x \frac{z^2}{2} - \gamma_1 \sin\alpha \frac{z^2}{2} + C_1 z, \tag{17}$$

Table 1. Physico-chemical parameters of the problem

Parameter	Ethanol	HFE-7100
$\rho \cdot 10^{-3}$, kg/m^3	0.79	1.5
$\nu \cdot 10^6$, m^2/sec	1.5	0.38
$\sigma_0 \cdot 10^{-2}$, N/m	2.2	1.24
$\sigma_T \cdot 10^{-4}$, N/(m K)	0.8	1.14
$\lambda_U \cdot 10^{-6}$, W · sec/kg	0.9085	0.111
$\kappa, W/(m \cdot K)$	0.1675	0.07
$\chi \cdot 10^7$, m^2/sec	0.89	0.4
$c_p \cdot 10^{-3}$, W· sec/(kg· K)	2.97	1.3

Table 2. The values of the parameters α_f in the systems "ethanol – nitrogen" and "HFE 7100 – nitrogen"

α_f parameter	values ($T_* = 1\ K$) ethanol	values ($T_* = 10\ K$) ethanol	values ($T_* = 1\ K$) HFE-7100	values ($T_* = 10\ K$) HFE-7100
$\alpha_\sigma = \dfrac{MaCa}{RePr}$	10^{-2}	10^{-1}	10^{-3}	10^{-2}
$\alpha_{Ca} = \dfrac{\varepsilon^3}{Ca}$	$10^5\varepsilon^3$	$10^5\varepsilon^3$	$10^6\varepsilon^3$	$10^6\varepsilon^3$
$\alpha_D = \varepsilon^2(\dfrac{1}{\bar{\rho}} - 1)\bar{J}^2$	$10^{-5}\varepsilon^2$	$10^{-3}\varepsilon^2$	$10^{-3}\varepsilon^2$	$10^{-1}\varepsilon^2$
$\alpha_\tau = \dfrac{\bar{\rho}\,\bar{\nu}\,\bar{v}\,\varepsilon}{\bar{h}}$	$\varepsilon; 10\varepsilon$	$\varepsilon; 10\varepsilon$	$\varepsilon; 10\varepsilon$	$\varepsilon; 10\varepsilon$
$\alpha_{Ma} = \dfrac{\varepsilon Ma}{RePr}$	$10^3\varepsilon$	$10^4\varepsilon$	$10^4\varepsilon$	$10^5\varepsilon$

$$w^0 = -(C_0)_{xx}\frac{z^3}{6} - (C_1)_x\frac{z^2}{2}, \tag{18}$$

$$p^0 = -\gamma_2 \cos\alpha z + C_0, \tag{19}$$

$$T^0 = A(x,t)z + \Theta_0(x,t). \tag{20}$$

Here, the coefficients $C_0(x,t)$, $C_1(x,t)$, $A(x,t)$ satisfy the following relations:

$$C_0(x,t) = p^g - \alpha_{Ca}h_{xx}(1 - \alpha_\sigma\Theta^0) + \gamma_2\cos\alpha h,$$

$$C_1(x,t) = -\alpha_{Ma}\tilde{\Theta} - (C_0)_x h + \gamma_1 \sin\alpha h,$$

$$A = \frac{(-\bar{\beta}_2(C_1)_x h + \bar{\beta}_3\alpha_J + \bar{\beta}_6 h_{xx}\alpha_J)\Theta_0}{1 + \bar{\beta}_2(C_1)_x h^2 - \bar{\beta}_3\alpha_J h - \bar{\beta}_6\alpha_J h_{xx} h},$$

where $\Theta^0 = Ah + \Theta_0$, $\tilde{\Theta} = A_x h + (\Theta_0)_x + h_x A$.

Note that the dynamic condition and the energy balance condition at the interface do not consider additional tangential stresses and the divergent term.

Using the formula (7), we obtain the following equation for determining the thickness of the liquid layer:

$$h_t + uh_x - w + \frac{E}{\varepsilon}J_{ev} = 0. \tag{21}$$

Table 3. The values of the parameters β_i in the systems "ethanol – nitrogen" and "HFE 7100 – nitrogen"

β_i parameter	values ($T_* = 1\ K$) ethanol	values ($T_* = 10\ K$) ethanol	values ($T_* = 1\ K$) HFE-7100	values ($T_* = 10\ K$) HFE-7100
$\beta_2 = \dfrac{Ma}{Re^2 Pr E\overline{U}}$	$10 \cdot \varepsilon^{-2}$	$10 \cdot \varepsilon^{-2}$	ε^{-2}	ε^{-2}
$\beta_3 = \dfrac{1}{E}$	10^4	10^3	10^3	10^2
$\beta_4 = (\dfrac{1}{\overline{\rho}} - 1)\dfrac{1}{E\overline{U}}$	10	1	1	0.1
$\beta_5 = (1 - \dfrac{1}{\overline{\rho}})^2 \dfrac{1}{E\overline{U}}$	10^3	10^2	10^3	10^2
$\beta_6 = (1 - \dfrac{1}{\overline{\rho}})\dfrac{1}{ReCaE\overline{U}}$	$-10^6\varepsilon^{-1}$	$-10^5\varepsilon^{-1}$	$-10^6\varepsilon^{-1}$	$-10^5\varepsilon^{-1}$
$\overline{\beta}_2 = \varepsilon\beta_2$	$10\varepsilon^{-1}$	$10\varepsilon^{-1}$	ε^{-1}	ε^{-1}
$\overline{\beta}_3 = \varepsilon\beta_3\overline{J}$	ε	ε	ε	ε
$\overline{\beta}_6 = \varepsilon^2\beta_6\overline{J}$	$-10^2\varepsilon$	$-10^2\varepsilon$	$-10^2\varepsilon$	$-10^2\varepsilon$

3. Results of numerical calculations

Taking into account the form of the solution for the principal terms (17)–(20), equation (21) takes the form

$$h_t + h_x\Big[(C_0)_x\frac{h^2}{2} - \gamma_1 \sin\alpha\frac{h^2}{2} + C_1 h\Big] - \Big[-(C_0)_{xx}\frac{h^3}{6} - (C_1)_x\frac{h^2}{2}\Big] + \frac{E}{\varepsilon}J_{ev} = 0. \qquad (22)$$

Here, $J_{ev} = \alpha_J[A(x,t)h + \Theta_0(x,t)]$. The problem must be supplemented with initial conditions $h(x,0) = h_0(x) = 1 - 0.1\cos(kx)$ (see [5]) and conditions at infinity. The temperature distribution on a solid substrate is determined as follows:

$$\Theta_0 = 1 + \delta_0\cos(k_1 x)\cos(k_2 t). \qquad (23)$$

Equation (23) allows us to model a periodic heating.

For the numerical solution of the problem of periodic flowing of a thin liquid layer, one considers the segment $x \in [-L; L]$. The following periodic conditions are assumed to be fulfilled at the ends of the test cell:

$$h|_{x=-L} = h|_{x=L}, \qquad h_x|_{x=-L} = h_x|_{x=L}, \qquad h_{xx}|_{x=-L} = h_{xx}|_{x=L}. \qquad (24)$$

An implicit finite-difference scheme to determine the liquid layer thickness is constructed for the equation (22) of the form

$$\frac{h^{k+1} - h^k}{\tau} + A_4^k h_{xxxx}^{k+1} + A_3^k h_{xxx}^{k+1} + A_2^k h_{xx}^{k+1} + A_1^k h^{k+1} + D^k = 0. \qquad (25)$$

Finite-difference analogues of the second order of approximation are used for all derivatives with respect to x included in (25). The problem is reduced to solving a system of linear algebraic equations by the method of five-point sweep and sweep with the parameter [12, 14].

Periodic runoff of two different liquids, ethanol and HFE-7100, has been numerically investigated. Physico-chemical parameters of liquids are given in Tab. 1. The following values of characteristic quantities were used for calculations: $l = 0.1$ m, $d = 0.01$ m, $T_* = 10$ K, $u_* = 0.15 \cdot 10^{-4}$ m/sec for ethanol, $u_* = 0.38 \cdot 10^{-5}$ m/sec for HFE-7100.

Let the energy condition at the interface (10) be used in the classical formulation, i.e. $\bar{\beta}_2 = \bar{\beta}_6 = 0$. The temperature on an inclined substrate is distributed inhomogeneously and the heating changes over time according to formula (23). Ethanol and HFE-7100 were used as working media, nitrogen was used as the gas. Fig. 2 shows the dependence of the change in the liquid layer thickness over time on the type of liquid. For both media, the alignment of the interface with time is observed (see lines 1, 2, 3 for ethanol and lines 1, 4, 5 for HFE-7100). Note that with a similar qualitative picture, the thickness of the HFE-7100 layer decreases more inetsively than the ethanol layer.

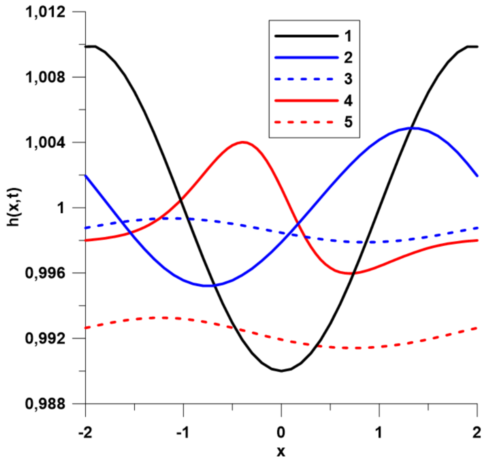

Fig. 2. Changing the position of the interface with time, non-stationary heating of the substrate, $\bar{\beta}_2 = 0$, $\bar{\beta}_6 = 0$. 1: initial position of the interface; 2: $t = 10^{-3}$, ethanol; 3: $t = 10^{-2}$, ethanol; 4: $t = 10^{-3}$, HFE-7100; 5: $t = 10^{-2}$, HFE-7100

Let us consider the case when the energy condition (10) at the thermocapillary boundary is written taking into account the term responsible for the energy consumption to overcome the surface deformation by thermocapillary forces along the surface ($\bar{\beta}_2 \neq 0$). Numerical calculations were carried out for the case of uniform heating of the substrate. Fig. 3 shows the change in the position of the interface over time in the case of using different types of liquids. Accounting for the additional term in the energy condition significantly affects the flow nature qualitatively and quantitatively. The previously shown effect of the influence of the liquid type on intensity of decrease in the liquid layer thickness is preserved: in the case HFE-7100 fluid, values of function $h(x, t)$ are smaller than those for the ethanol liquid at the same time moment (see lines 4 and 5 of Fig. 3).

Conclusion

The presented mathematical model describes the dynamics of a thin liquid layer moving along an inclined substrate. The conditions at the thermocapillary interface provide the fulfillment of the laws of conservation of mass, impulse and energy. The influence of various effects on the flow pattern is estimated using parametric analysis. Analytical solutions are obtained for the

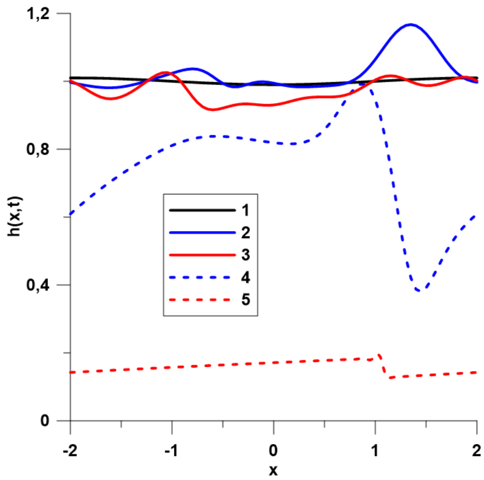

Fig. 3. Changing the position of the interface with time, homogeneous heating of the substrate, $\bar{\beta}_6 = 0$. 1: nitial position of the interface; 2: $t = 10^{-4}$, ethanol; 3: $t = 10^{-4}$, HFE-7100; 4: $t = 10^{-3}$, ethanol; 5: $t = 10^{-3}$, HFE-7100

principal terms of the decomposition. The evolution equation of the thickness of the liquid layer allows to take into account the influence of evaporation, capillary and thermocapillary forces, gravity on the process of liquid flowing. The influence of the type of liquid on the rate of decrease in the liquid layer thickness as well as the impact of an additional term in the energy condition on the nature of the flow are shown using the numerical solution of the evolutionary equation.

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Математическое моделирование процесса стекания тонкого слоя жидкости на основе обобщенных условий на границе раздела: параметрический анализ и численное решение

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Аннотация. Рассматривается задача о стекании тонкого слоя жидкости по наклонной подложке в условиях спутного потока газа. Математическое моделирование проводится на основе уравнений Навье–Стокса и переноса тепла, а также обобщенных для случая ненулевого потока пара условий на термокапиллярной границе. Проведен параметрический анализ задачи. Для эволюционного уравнения, определяющего толщину жидкого слоя, построен алгоритм численного решения. Представлено сравнение численных расчетов для жидкостей типа этанол и HFE-7100. Показано влияние дополнительного слагаемого в энергетическом условии на динамику жидкого слоя.

Ключевые слова: уравнения Навье–Стокса, граница раздела, приближение тонкого слоя, испарение, параметрический анализ, численное решение.