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Initial Boundary Value Problem on the Motion of a Viscous Heat-Conducting Liquid in a Vertical Pipe

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Abstract. The initial-boundary value problem arising in a modeling an unsteady unidirectional convective flow in vertical heat exchangers with an arbitrary cross section is researched. An a priori estimate in L_2 is obtained and uniqueness of the problem solution is proved. For a rectangular and circular sections solution was found in the form of double Fourier series. Sufficient conditions for stabilization of solution to rest with increasing time are given.

Keywords: initial boundary value problem, a priori estimate, Fourier series, convection.

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Introduction

The equations system for convective motion in the Oberbeck–Boussinesq approximation is given by [1]

$$\mathbf{u_t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \frac{1}{\rho}\nabla p = \nu\Delta\mathbf{u} + g\beta\theta\mathbf{e},$$
$$\operatorname{div}\mathbf{u} = 0, \qquad (1)$$
$$\theta_t + \mathbf{u}\cdot\nabla\theta = \chi\Delta\theta.$$

Here $\mathbf{u}=(u(x,y,z,t), v(x,y,z,t), w(x,y,z,t))$ is velocity vector; $\theta(x,y,z,t)$ is temperature; $p(x,y,z,t)$ is modified pressure; ρ,ν,g,β,χ are positive physical constants of the liquid medium; $\mathbf{e}=(0,0,-1)$. The system (1) admits operator $\partial_z - A(\partial_\theta + \rho g\beta z\partial_p)$ with constant A. Invariant solutions of rank three are sought in the form [1]

$$\mathbf{u} = (u(x,y,t), v(x,y,t), w(x,y,t)),$$
$$p = -\rho g\beta A\frac{z^2}{2} + q(x,y,t), \quad \theta = -Az + T(x,y,t). \qquad (2)$$

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Substitution the form of solution (2) into (1) leads to a system that includes Navier-Stokes equations for the plane motion of a purely viscous fluid for u, v, q and equations

$$\begin{aligned} w_t + uw_x + vw_y &= \nu(w_{xx} + w_{yy}) + \rho g\beta T, \\ T_t + uT_x + vT_y &= Aw + \chi(T_{xx} + T_{yy}). \end{aligned} \tag{3}$$

Suppose that $u = v = 0$, $q = q(t)$, then (3) is converted into linear parabolic equations system

$$\begin{aligned} w_t &= \nu(w_{xx} + w_{yy}) + \rho g\beta T, \\ T_t &= Aw + \chi(T_{xx} + T_{yy}). \end{aligned} \tag{4}$$

The equations (4) are satisfied in a certain region Ω on variables plane x, y with boundary Γ. Here we consider boundary conditions on Γ of the first kind

$$w|_\Gamma = 0, \ T|_\Gamma = 0. \tag{5}$$

The first of them is a no-slip condition, and the second, by virtue of (2), means that a constant temperature gradient is applied along the lateral surface of heat exchanger. For a complete formulation of the problem it is necessary to set initial conditions

$$w|_{t=0} = w_0(x, y), \ T|_{t=0} = T_0(x, y). \tag{6}$$

Of course, for a smooth solution we need to require that the matching conditions $w_0(x, y) = 0$ and $T_0(x, y) = 0$ when $x, y \in \Gamma$.

The problem (4)–(6) is the first initial boundary value problem. We introduce dimensionless variables $\overline{t} = \chi t/d^2$, $\overline{x} = x/d$, $\overline{y} = y/d$, $\overline{w} = dw/\chi$, $\overline{T} = T/Ad$, $\overline{w_0} = dw_0/\chi$, $\overline{T_0} = T_0/Ad$ where $d = diam\ \Omega$. Let be $\overline{\Omega}, \overline{\Gamma}$ is converted Ω and Γ. Then problem (4)–(6) will take the form (bar is omitted)

$$\begin{aligned} w_t &= \frac{1}{P}(w_{xx} + w_{yy}) + GT, \\ T_t &= T_{xx} + T_{yy} + w, \ (x, y) \in \Omega; \\ w|_{t=0} &= w_0(x, y), \ T|_{t=0} = T_0(x, y); \\ w|_\Gamma &= 0, \ \ T|_\Gamma = 0, \end{aligned} \tag{7}$$

where $P = \chi/\nu$ is Prandtl number, $G = \rho g A d^4 \beta/\chi^2$ is Grashof number. So the problem (7) will be subject of our study.

1. A priori estimate

Let us multiply the first equation of system (7) by w, the second by T, integrate over Ω, add results and obtain identity

$$\frac{1}{2}\frac{\partial}{\partial t}\int_\Omega (w^2 + T^2)dxdy = \frac{1}{P}\int_\Omega w\ div\nabla w dxdy + \int_\Omega T\ div\nabla T dxdy + (G+1)\int_\Omega wT dxdy. \tag{8}$$

By virtue of the boundary conditions we have

$$\int_\Omega w\ div\nabla w dxdy = -\int_\Omega |\nabla w|^2 dxdy, \quad \int_\Omega T\ div\nabla T dxdy = -\int_\Omega |\nabla T|^2 dxdy.$$

Hence identity (8) is equivalent to the following

$$\frac{1}{2}\frac{\partial}{\partial t}\int_\Omega (w^2 + T^2)dxdy + \frac{1}{P}\int_\Omega |\nabla w|^2 dxdy + \int_\Omega |\nabla T|^2 dxdy = (G+1)\int_\Omega wT dxdy. \tag{9}$$

Since $w|_\Gamma = 0, T|_\Gamma = 0$, the Friedrichs inequality is satisfied [2]

$$\int_\Omega w^2 dxdy \leqslant C \int_\Omega |\nabla w|^2 dxdy, \quad \int_\Omega T^2 dxdy \leqslant C \int_\Omega |\nabla T|^2 dxdy.$$

In these inequalities constant C depends only on Ω. Moreover, it is known from [3] that $C = 1/\mu$, where μ is smallest eigenvalue of operator $-\Delta$ in region Ω under zero boundary condition.

Now from (9) let us derive the inequality

$$\frac{\partial}{\partial t} \int_\Omega (w^2 + T^2) dxdy \leqslant (|G+1| - 2\alpha) \int_\Omega (w^2 + T^2) dxdy,$$

with constant $\alpha = C^{-1} \min(1, P^{-1}) > 0$. Wherefrom

$$\int_\Omega (w^2 + T^2) dxdy \leqslant e^{\gamma t} \int_\Omega (w_0^2 + T_0^2) dxdy, \tag{10}$$

with $\gamma = |G+1| - 2\alpha$. We get two cases: I) If $G \in (-2\alpha - 1, 2\alpha - 1)$, then $\gamma < 0$; II) If $G \in (-\infty, -2\alpha - 1] \cup [2\alpha - 1, +\infty)$ then $\gamma \geqslant 0$.

It follows from (I), (II) that the solution of initial boundary value problem (7) is unique on a finite time interval $(0, t_0)$ and in case I) additionally exponential damping of the solution in $L_2(\Omega)$ norm with increasing time.

2. Solution of the problem in case of a rectangular section

Consider an arbitrary rectangular area $\{0 < x < l_1, 0 < y < l_2\}$. Without loss of generality we will assume $l_1 < l_2$, so that $d = l_2$. Then Ω and Γ in dimensionless variables are

$$\Omega = \{0 < x < l, 0 < y < 1\}, \quad \Gamma = \{x = 0\} \cup \{x = l\} \cup \{y = 0\} \cup \{y = 1\},$$

where $l = l_1/l_2$. The matching conditions for a smooth solution will have the form

$$\begin{aligned} w_0(0,y) &= 0,\ w_0(l,y) = 0, \quad 0 \leqslant y \leqslant 1, \\ T_0(0,y) &= 0,\ T_0(l,y) = 0, \quad 0 \leqslant y \leqslant 1, \\ w_0(x,0) &= 0,\ w_0(x,1) = 0, \quad 0 \leqslant x \leqslant l, \\ T_0(x,0) &= 0,\ T_0(x,1) = 0, \quad 0 \leqslant x \leqslant l. \end{aligned}$$

Let's estimate solution of the problem by inequality (10); to do this, we find constant C. The smallest of numbers μ for which there exists a solution other than identical zero of the problem

$$-\Delta w = \mu w, \quad w|_\Gamma = 0,$$

in this case is $\mu = \pi^2(1 + l^2)/l^2$ [4]. Thus, solution of the problem in case of a rectangular section is bounded by inequality (10) in the L_2 norm, where $C = l^2/\pi^2(1 + l^2)$. Note that if

$$G \in \left(\frac{-2\pi^2(1 + l^2)\min(1, P^{-1})}{l^2} - 1, \frac{2\pi^2(1 + l^2)\min(1, P^{-1})}{l^2} - 1 \right),$$

then solution tends to zero as $t \to +\infty$ in the $L_2(\Omega)$ norm.

To solve the problem we use the Fourier method. The solution is sought in form

$$w(x,y,t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} S_{nm}(t) \sin \frac{\pi n x}{l} \sin \pi m y, \quad T(x,y,t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} F_{nm}(t) \sin \frac{\pi n x}{l} \sin \pi m y. \tag{11}$$

The boundary conditions are identically satisfied. Substitution (11) into system (7) gives the equalities

$$\begin{gathered}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(S'_{nm}(t)+\frac{\pi^2(n^2+m^2l^2)}{Pl^2}S_{nm}(t)-GF_{nm}(t)\right)\sin\frac{\pi nx}{l}\sin\pi my=0,\\ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(F'_{nm}(t)+\frac{\pi^2(n^2+m^2l^2)}{l^2}F_{nm}(t)-S_{nm}(t)\right)\sin\frac{\pi nx}{l}\sin\pi my=0.\end{gathered}\tag{12}$$

Denote $q_{nm}^2=\pi^2(n^2+m^2l^2)/l^2$. Due to the completeness of basis functions, system (12) reduces to system of ordinary differential equations

$$S'_{nm}+\frac{q_{nm}^2}{P}S_{nm}=GF_{nm},\quad F'_{nm}+q_{nm}^2F_{nm}=S_{nm}.\tag{13}$$

From initial conditions of the problem we determine initial conditions for the system (13) [5]

$$\begin{gathered}S_{nm}(0)=\int_0^l\int_0^1 w_0(x,y)\sin\frac{\pi nx}{l}\sin\pi my\;dxdy\equiv S_{nm}^0,\\ F_{nm}(0)=\int_0^l\int_0^1 T_0(x,y)\sin\frac{\pi nx}{l}\sin\pi my\;dxdy\equiv F_{nm}^0.\end{gathered}\tag{14}$$

The general solution of system (13) has representation

$$S_{nm}(t)=D_{nm}^{(1)}(q_{nm}^2+\lambda_{nm}^{(1)})e^{\lambda_{nm}^{(1)}t}+D_{nm}^{(2)}(q_{nm}^2+\lambda_{nm}^{(2)})e^{\lambda_{nm}^{(2)}t},\quad F_{nm}(t)=D_{nm}^{(1)}e^{\lambda_{nm}^{(1)}t}+D_{nm}^{(2)}e^{\lambda_{nm}^{(2)}t},$$

where

$$\lambda_{nm}^{(1,2)}=-\frac{q_{nm}^2(P+1)}{2P}\pm\frac{1}{2}\sqrt{\frac{q_{nm}^4(P-1)^2}{P^2}+4G},\tag{15}$$

and constants

$$D_{nm}^{(1)}=\frac{F_{nm}^0(q_{nm}^2+\lambda_{nm}^{(2)})-S_{nm}^0}{\lambda_{nm}^{(2)}-\lambda_{nm}^{(1)}},\quad D_{nm}^{(2)}=\frac{S_{nm}^0-F_{nm}^0(q_{nm}^2+\lambda_{nm}^{(1)})}{\lambda_{nm}^{(2)}-\lambda_{nm}^{(1)}},$$

are found from initial conditions (14). The formal solution of the problem will be functions

$$\begin{aligned}w(x,y,t)=&\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\lambda_{nm}^{(2)}-\lambda_{nm}^{(1)}}\Bigg[F_{nm}^0\left(q_{nm}^2+\lambda_{nm}^{(1)}\right)\left(q_{nm}^2+\lambda_{nm}^{(2)}\right)\left(e^{\lambda_{nm}^{(1)}t}-e^{\lambda_{nm}^{(2)}t}\right)+\\ &+S_{nm}^0\left\{\left(q_{nm}^2+\lambda_{nm}^{(2)}\right)e^{\lambda_{nm}^{(2)}t}-\left(q_{nm}^2+\lambda_{nm}^{(1)}\right)e^{\lambda_{nm}^{(1)}t}\right\}\Bigg]\cdot\sin\frac{\pi nx}{l}\sin\pi my,\\ T(x,y,t)=&\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\lambda_{nm}^{(2)}-\lambda_{nm}^{(1)}}\Bigg[F_{nm}^0\left\{\left(q_{nm}^2+\lambda_{nm}^{(2)}\right)e^{\lambda_{nm}^{(1)}t}-\left(q_{nm}^2+\lambda_{nm}^{(1)}\right)e^{\lambda_{nm}^{(2)}t}\right\}+\\ &+S_{nm}^0\left(e^{\lambda_{nm}^{(2)}t}-e^{\lambda_{nm}^{(1)}t}\right)\Bigg]\cdot\sin\frac{\pi nx}{l}\sin\pi my.\end{aligned}\tag{16}$$

Suppose that $0<G\leqslant q_{nm}^4/P$ (it suffices to require fulfillment of inequality $0<G\leqslant q_{11}^4/P$), thereat $\lambda_{nm}^{(1,2)}\leqslant 0$ and

$$\lambda_{nm}^{(1,2)}=-q_{nm}^2\left(\frac{P+1}{2P}\mp\frac{1}{2}\sqrt{\frac{(P-1)^2}{P^2}+\frac{4G}{q_{nm}^4}}\right)\equiv -q_{nm}^2d_{nm}^{(1,2)},\tag{17}$$

where $d_{nm}^{(1,2)} \geqslant 0$. It's clear that $\lambda_{nm}^{(1,2)} \to -\infty$ when $n, m \to \infty$. From formula (17) we obtain

$$\lambda_{nm}^{(2)} - \lambda_{nm}^{(1)} = -q_{nm}^2 \sqrt{\frac{(P-1)^2}{P^2} + \frac{4G}{q_{nm}^4}} < 0. \tag{18}$$

So

$$\frac{1}{|\lambda_{nm}^{(2)} - \lambda_{nm}^{(1)}|} \leqslant \frac{P}{|P-1| q_{nm}^2} = \frac{Pl^2}{\pi^2 |P-1|(n^2 + m^2 l^2)}. \tag{19}$$

Using equalities (15) and (17) we find

$$\begin{gathered} q_{nm}^2 + \lambda_{nm}^{(1,2)} = q_{nm}^2 (1 - d_{nm}^{(1,2)}), \quad (q_{nm}^2 + \lambda_{nm}^{(1)})(q_{nm}^2 + \lambda_{nm}^{(2)}) = -G, \\ |1 - d_{nm}^{(1,2)}| \leqslant \frac{1}{2} \left(\frac{|P-1|}{P} + \sqrt{\frac{(P-1)^2}{P^2} + \frac{4G}{q_{11}^4}} \right) \equiv B. \end{gathered} \tag{20}$$

Let us prove that for $0 < G \leqslant q_{11}^4/P$ series (16) are a classical solution of problem (7) for all $t \geqslant 0$ if the series of initial data $w_0(x, y), T_0(x, y)$ absolutely converge

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} |S_{nm}^0| < \infty, \quad \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} |F_{nm}^0| < \infty. \tag{21}$$

Employing (19), (20), from representations of the solution in form of series (16) we find

$$\begin{aligned} |w(x,y,t)| &\leqslant \frac{Pl^2}{\pi^2 |P-1|} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left[\frac{G|F_{nm}^0|}{n^2 + m^2 l^2} + \frac{B\pi^2 |S_{nm}^0|}{l^2} \right] \left(e^{\lambda_{nm}^{(1)} t} + e^{\lambda_{nm}^{(2)} t} \right) \leqslant \\ &\leqslant \frac{Pl^2}{\pi^2 |P-1|} \max\left(\frac{G}{1+l^2}, \frac{B\pi^2}{l^2} \right) \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left(|F_{nm}^0| + |S_{nm}^0| \right) \left(e^{\lambda_{nm}^{(1)} t} + e^{\lambda_{nm}^{(2)} t} \right), \end{aligned} \tag{22}$$

$$\begin{aligned} |T(x,y,t)| &\leqslant \frac{Pl^2}{\pi^2 |P-1|} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{n^2 + m^2 l^2} \left(Bq_{nm}^2 |F_{nm}^0| + |S_{nm}^0| \right) \left(e^{\lambda_{nm}^{(1)} t} + e^{\lambda_{nm}^{(2)} t} \right) \leqslant \\ &\leqslant \frac{Pl^2}{\pi^2 |P-1|} \max\left(\frac{B\pi^2}{l^2}, \frac{1}{1+l^2} \right) \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left(|F_{nm}^0| + |S_{nm}^0| \right) \left(e^{\lambda_{nm}^{(1)} t} + e^{\lambda_{nm}^{(2)} t} \right). \end{aligned} \tag{23}$$

Series (22), (23) converge since $\exp\left(\lambda_{nm}^{(1,2)} t\right) \leqslant 1$. Moreover, the functions $w(x,y,t), T(x,y,t)$ tend exponentially to zero when $t \to \infty$. Indeed due to (18)

$$\begin{aligned} e^{\lambda_{nm}^{(1)} t} + e^{\lambda_{nm}^{(2)} t} = e^{\lambda_{11}^{(1)} t} \exp\left[\left(\lambda_{nm}^{(1)} - \lambda_{11}^{(1)} \right) t \right] \left\{ 1 + \exp\left[\left(\lambda_{nm}^{(2)} - \lambda_{nm}^{(1)} \right) t \right] \right\} \leqslant \\ \leqslant 2 e^{\lambda_{11}^{(1)} t} \exp\left[\left(\lambda_{nm}^{(1)} - \lambda_{11}^{(1)} \right) t \right]. \end{aligned} \tag{24}$$

It is easy to see that for $G > 0$ the quantity $\lambda_{nm}^{(1)} - \lambda_{11}^{(1)} \leqslant 0$, then from (22), (23) and (24) we derive estimates

$$|w(x,y,t)| \leqslant H_1 e^{\lambda_{11}^{(1)} t}, \quad |T(x,y,t)| \leqslant H_2 e^{\lambda_{11}^{(1)} t}, \tag{25}$$

with constants $H_1 > 0, H_2 > 0$. Recall that $\lambda_{11}^{(1)} = -q_{11}^1 d_{11}^{(1)} < 0$, and therefore $w \to 0, T \to 0$ when $t \to \infty$ uniformly in a rectangle Ω.

Remark 1. By what was proved above, series (16) converge absolutely and uniformly, their terms are continuous, which means that their sums (functions $w(x,y,t), T(x,y,t)$) are also continuous on $\Omega \cup \Gamma, t \geqslant 0$.

Let's now prove that the functions $w(x, y, t)$, $T(x, y, t)$ (sums of series (16)) have first derivatives with respect to t and second derivatives with respect to x and y for $t > 0$. To this end it suffices to prove that differentiation of series (16) with respect to x, y, and t, corresponding number of times results in series that converge uniformly in $\Omega \cup \Gamma$ and $t \geqslant \varepsilon$, where ε is arbitrary positive number. Truly, when differentiating series (16) with respect to t, the expressions $\lambda_{nm}^{(1,2)} \exp(\lambda_{nm}^{(1,2)} t)$ arise. Since $\lambda_{nm}^{(1,2)} < 0$, then $\left|\lambda_{nm}^{(1,2)} \exp(\lambda_{nm}^{(1,2)} t)\right| < \left|\lambda_{nm}^{(1,2)}\right| \exp(-\left|\lambda_{nm}^{(1,2)}\right| \varepsilon) < < L_{1,2}/\varepsilon$ with positive constants $L_{1,2}$. This is a consequence of fact that function $g(x) = x^\alpha e^{-x}$, $\alpha > 0$ is bounded $\forall x > 0$,namely $x^\alpha e^{-x} \leqslant \alpha^\alpha e^{-\alpha} \equiv L$. In our case $\alpha = 1$. Thus series for w_t, T_t converge absolutely and uniformly in Ω for $t \geqslant \varepsilon$.

When differentiating series (16) twice with respect to x (with respect to y), the expressions $n^2 \exp(\lambda_{nm}^{(1,2)} t)$, $m^2 \exp(\lambda_{nm}^{(1,2)} t)$ arise. Insofar as

$$n^2 < \frac{l^2 \left|\lambda_{nm}^{(1,2)}\right|}{\pi^2 d_{nm}^{(1,2)}} < \left\{ \begin{array}{c} \dfrac{l^2 P d_{11}^{(2)} \left|\lambda_{nm}^{(1)}\right|}{\pi^2 \left(1 - \frac{GP}{q_{11}^4}\right)} \\ \dfrac{2 l^2 P \left|\lambda_{nm}^{(2)}\right|}{\pi^2 (P+1)} \end{array} \right. , \quad m^2 < \left\{ \begin{array}{c} \dfrac{P d_{11}^{(2)} \left|\lambda_{nm}^{(1)}\right|}{\pi^2 \left(1 - \frac{GP}{q_{11}^4}\right)} \\ \dfrac{2 P \left|\lambda_{nm}^{(2)}\right|}{\pi^2 (P+1)} \end{array} \right. .$$

then, by the same considerations as above, series for w_{xx}, w_{yy}, T_{xx}, T_{yy} converge absolutely and uniformly in Ω for all $t \geqslant \varepsilon$ with arbitrary $\varepsilon > 0$. This proves

Theorem 1. *Let us $0 < G \leqslant q_{11}^4/P$ and series (21) converge absolutely in the rectangle $\Omega \cup \Gamma$. Then solution of problem (7) is classical and estimates (25) are satisfied.*

Remark 2. In fact, solution of the problem for $t > 0$ has derivatives of all orders in x, y and t, that is, it is infinitely differentiable (one should use the inequality $x^\alpha e^{-x} \leqslant M = const$ for natural α).

Remark 3. The fluid flow rate Q in case of a rectangular section is equal to

$$Q(t) = \iint\limits_{\Omega} w(x, y, t) d\Omega = \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \frac{4l}{\pi^2 (2k-1)(2j-1)\left(\lambda_{2k-1,2j-1}^{(2)} - \lambda_{2k-1,2j-1}^{(1)}\right)} \times$$
$$\times \left[F_{2k-1,2j-1}^0 \left(q_{2k-1,2j-1}^2 + \lambda_{2k-1,2j-1}^{(1)}\right)\left(q_{2k-1,2j-1}^2 + \lambda_{2k-1,2j-1}^{(2)}\right)\left(e^{\lambda_{2k-1,2j-1}^{(1)} t} - e^{\lambda_{2k-1,2j-1}^{(2)} t}\right) + \right.$$
$$\left. + S_{2k-1,2j-1}^0 \left(\left(q_{2k-1,2j-1}^2 + \lambda_{2k-1,2j-1}^{(2)}\right) e^{\lambda_{2k-1,2j-1}^{(2)} t} - \left(q_{2k-1,2j-1}^2 + \lambda_{2k-1,2j-1}^{(1)}\right) e^{\lambda_{2k-1,2j-1}^{(1)} t} \right) \right].$$

Note that if $F_{nm}^0 = 0$, $S_{nm}^0 = 0$ for odd n and m, then the fluid flow rate is zero.

Remark 4. Solution of the problem in case of MS-20 oil flow at a temperature of $0°C$ [6] in a vertical heat exchanger of rectangular cross section with initial data $w_0(x, y) = \sin(\pi x/l) \sin \pi y$, $T_0(x, y) = \sin(\pi x/l) \sin \pi y$ and constants $l_1 = 0.05$ m, $l_2 = 0.1$ m, $A = 1$ K/m, $\rho = 903.6$ kg/m^3, $\beta = 6.27 \cdot 10^{-4}$ $1/K$, $\chi = 62.06 \cdot 10^{-3}m^2/c$, $\nu = 7.59 \cdot 10^{-3}$m$^2/c$ with $G = 0.144 < q_{11}^4/P = 6.036$ has the form

$$w(x, y, t) = (1.00325 e^{\lambda^1 t} - 0.00325 e^{\lambda^2 t}) \sin 2\pi x \sin \pi y,$$
$$T(x, y, t) = (0.02316 e^{\lambda^1 t} + 0.97684 e^{\lambda^2 t}) \sin 2\pi x \sin \pi y,$$

where $\lambda^1 = -6.032925$, $\lambda^2 = -49.35134$. Fig. 1 shows the vertical velocity profile $w(x, y, t)$ at $t = 0$ and at $t = 0.15$ in dimensionless coordinates; as t increases $w(x, y, t)$ tends to zero.

If in previous example we take initial data $w_0(x, y) = \sin(2\pi x/l) \sin 2\pi y$, $T_0(x, y) = = \sin(2\pi x/l) \sin 2\pi y$ then $G = 0.144 < q_{22}^4/P = 24.145$ and solution of the problem has representation

$$w(x,y,t) = (1.0607e^{\lambda^1 t} - 0.0607e^{\lambda^2 t})\sin 4\pi x \sin 2\pi y,$$
$$T(x,y,t) = (0.02316e^{\lambda^1 t} + 0.97684e^{\lambda^2 t})\sin 4\pi x \sin 2\pi y,$$

with $\lambda^1 = -24.0838$, $\lambda^2 = -197.4532$. Fig. 2 shows the vertical velocity profile $w(x,y,t)$ at $t = 0$ and at $t = 0.05$ in dimensionless coordinates; as t grows the quantity $w(x,y,t)$ also tends to zero. Here the fluid flow rate is $Q(t) = 0$ and there are zones of reverse motion near the corners of rectangle.

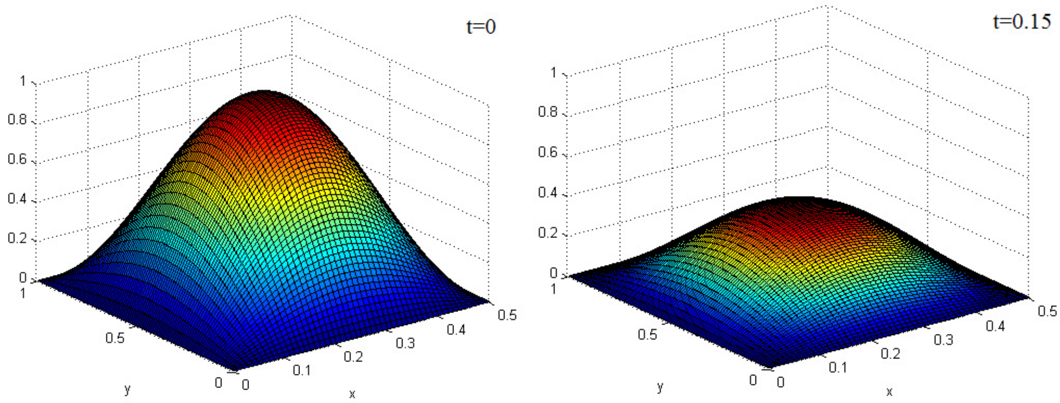

Fig. 1. Velocity profile $w(x,y,t)$ at $t = 0$, $t = 0.15$

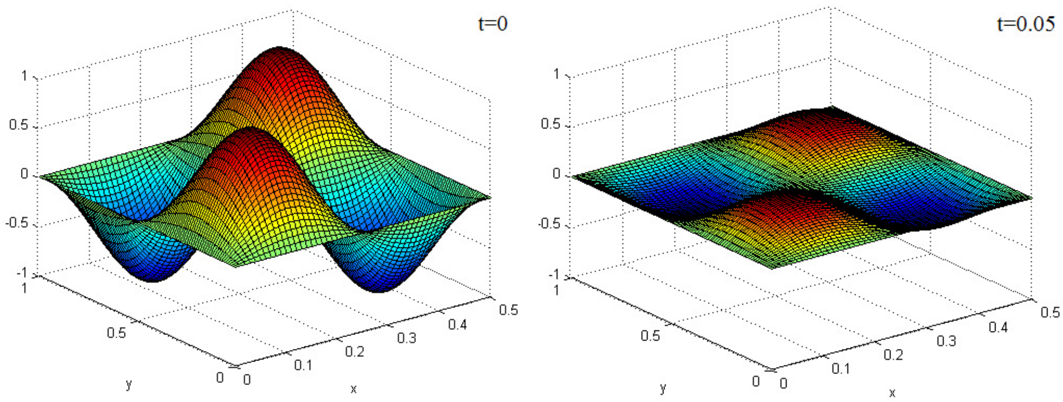

Fig. 2. Velocity profile $w(x,y,t)$ at $t = 0$, $t = 0.05$

3. Solution of the problem in case of a circular cross section

Consider region Ω in the form of a circle with boundary Γ

$$\Omega = \{r,\phi \,|r < 1, \phi \in [0,2\pi]\}, \quad \Gamma = \{r,\phi \,|r = 1, \phi \in [0,2\pi]\}.$$

In general, the radius of a circle is a, so $d = a$. Problem (7) can be written as

$$\begin{aligned} & w_t = \frac{1}{P}\Big(w_{rr} + \frac{1}{r}w_r + \frac{1}{r^2}w_{\phi\phi}\Big) + GT, \\ & T_t = T_{rr} + \frac{1}{r}T_r + \frac{1}{r^2}T_{\phi\phi} + w, \quad (r,\phi) \in \Omega \\ & w|_{t=0} = w_0(r,\phi), \quad T|_{t=0} = T_0(r,\phi), \\ & w(1,\phi,t) = 0, \quad T(1,\phi,t) = 0. \end{aligned} \qquad (26)$$

For the a priori estimate w, T by inequality (10) let's find constant C. In case of a circular section $\mu = (\xi_0^1)^2$, where $\xi_0^1 = 2.40482$ is first zero of the zero-order Bessel function [4, 7]. Thus, the solution to problem (26) is bounded in L_2 from 0 to 1 with weight r by inequality (10), where $C = 0.172915$. Moreover, if

$$G \in (-11.56632 \min(1, P^{-1}) - 1, 11.56632 \min(1, P^{-1}) - 1),$$

then solution tends to zero as $t \to +\infty$ in the norm $L_2(\Omega)$ with weight r.

To solve problem, we also use the Fourier method. Solution is sought in form

$$\begin{aligned} w(r,\phi,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} S_{nm}(t) J_n\left(\xi_n^{(m)} r\right) \left\{ \begin{array}{c} \cos n\phi \\ \sin n\phi \end{array} \right\}, \\ T(r,\phi,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} F_{nm}(t) J_n\left(\xi_n^{(m)} r\right) \left\{ \begin{array}{c} \cos n\phi \\ \sin n\phi \end{array} \right\}, \end{aligned} \tag{27}$$

where $\xi_n^{(m)}$ is mth zero of nth order Bessel function. Substitution (27) into (26) gives the equalities

$$\begin{aligned} &\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \left(S'_{nm}(t) + \frac{\left(\xi_n^{(m)}\right)^2}{P} S_{nm}(t) - G F_{nm}(t) \right) J_n\left(\xi_n^{(m)} r\right) \left\{ \begin{array}{c} \cos n\phi \\ \sin n\phi \end{array} \right\} = 0, \\ &\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \left(F'_{nm}(t) + \left(\xi_n^{(m)}\right)^2 F_{nm}(t) - S_{nm}(t) \right) J_n\left(\xi_n^{(m)} r\right) \left\{ \begin{array}{c} \cos n\phi \\ \sin n\phi \end{array} \right\} = 0. \end{aligned} \tag{28}$$

If we denote $q_{nm}^2 = \left(\xi_n^{(m)}\right)^2$, then system (28) reduces to a system similar to (13)

$$S'_{nm} + \frac{q_{nm}^2}{P} S_{nm} = G F_{nm}, \quad F'_{nm} + q_{nm}^2 F_{nm} = S_{nm}, \tag{29}$$

with initial data [5]

$$S_{nm}(0) = \int_0^1 \int_0^{2\pi} w_0(r,\phi) J_n\left(\xi_n^{(m)} r\right) \left\{ \begin{array}{c} \cos n\phi \\ \sin n\phi \end{array} \right\} dr d\phi \equiv S_{nm}^0,$$

$$F_{nm}(0) = \int_0^1 \int_0^{2\pi} T_0(r,\phi) J_n\left(\xi_n^{(m)} r\right) \left\{ \begin{array}{c} \cos n\phi \\ \sin n\phi \end{array} \right\} dr d\phi \equiv F_{nm}^0.$$

The general solution of system (29) has representation

$$S_{nm}(t) = K_{nm}^{(1)} \left(q_{nm}^2 + \lambda_{nm}^{(1)}\right) e^{\lambda_{nm}^{(1)} t} + K_{nm}^{(2)} \left(q_{nm}^2 + \lambda_{nm}^{(2)}\right) e^{\lambda_{nm}^{(2)} t},$$

$$F_{nm}(t) = K_{nm}^{(1)} e^{\lambda_{nm}^{(1)} t} + K_{nm}^{(2)} e^{\lambda_{nm}^{(2)} t},$$

where

$$\lambda_{nm}^{(1,2)} = -\frac{q_{nm}^2}{2P}(P+1) \pm \frac{1}{2}\sqrt{\frac{q_{nm}^4}{P^2}(P-1)^2 + 4G}, \tag{30}$$

and constants

$$K_{nm}^{(1)} = \frac{F_{nm}^0 \left(q_{nm}^2 + \lambda_{nm}^{(2)}\right) - S_{nm}^0}{\lambda_{nm}^{(2)} - \lambda_{nm}^{(1)}}, \quad K_{nm}^{(2)} = \frac{S_{nm}^0 - F_{nm}^0 \left(q_{nm}^2 + \lambda_{nm}^{(1)}\right)}{\lambda_{nm}^{(2)} - \lambda_{nm}^{(1)}},$$

are found from initial conditions. The formal solution of problem will be functions

$$
\begin{aligned}
w(r,\phi,t) =& \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\lambda_{nm}^{(2)}-\lambda_{nm}^{(1)}}\Big[F_{nm}^0\left(q_{nm}^2+\lambda_{nm}^{(1)}\right)\left(q_{nm}^2+\lambda_{nm}^{(2)}\right)\left(e^{\lambda_{nm}^{(1)}t}-e^{\lambda_{nm}^{(2)}t}\right)+\\
&+S_{nm}^0\left\{\left(q_{nm}^2+\lambda_{nm}^{(2)}\right)e^{\lambda_{nm}^{(2)}t}-\left(q_{nm}^2+\lambda_{nm}^{(1)}\right)e^{\lambda_{nm}^{(1)}t}\right\}\Big]J_n\left(\xi_n^{(m)}r\right)\left\{\begin{array}{c}\cos n\phi\\ \sin n\phi\end{array}\right\},\\
T(r,\phi,t) =& \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\lambda_{nm}^{(2)}-\lambda_{nm}^{(1)}}\Big[F_{nm}^0\left\{\left(q_{nm}^2+\lambda_{nm}^{(2)}\right)e^{\lambda_{nm}^{(1)}t}-\left(q_{nm}^2+\lambda_{nm}^{(1)}\right)e^{\lambda_{nm}^{(2)}t}\right\}+\\
&+S_{nm}^0\left(e^{\lambda_{nm}^{(2)}t}-e^{\lambda_{nm}^{(1)}t}\right)\Big]J_n\left(\xi_n^{(m)}r\right)\left\{\begin{array}{c}\cos n\phi\\ \sin n\phi\end{array}\right\}.
\end{aligned}
\tag{31}
$$

Assume that $0 < G \leqslant q_{nm}^4/P$ (here it is also sufficient to require inequality $0 < G \leqslant q_{11}^4/P$), then $\lambda_{nm}^{(1,2)} \leqslant 0$ and

$$
\lambda_{nm}^{(1,2)} = -q_{nm}^2\left(\frac{P+1}{2P}\mp\frac{1}{2}\sqrt{\frac{(P-1)^2}{P^2}+\frac{4G}{q_{nm}^4}}\right)\equiv -q_{nm}^2 z_{nm}^{(1,2)}, \tag{32}
$$

with $z_{nm}^{(1,2)} \geqslant 0$. Here it is seen that $\lambda_{nm}^{(1,2)} \to -\infty$ when $n,m\to\infty$. From formula (32) we obtain

$$
\lambda_{nm}^{(2)}-\lambda_{nm}^{(1)} = -q_{nm}^2\sqrt{\frac{(P-1)^2}{P^2}+\frac{4G}{q_{nm}^4}} < 0. \tag{33}
$$

So,

$$
\frac{1}{|\lambda_{nm}^{(2)}-\lambda_{nm}^{(1)}|}\leqslant\frac{P}{|P-1|q_{nm}^2}=\frac{P}{|P-1|\left(\xi_n^{(m)}\right)^2}. \tag{34}
$$

Using equalities (30) and (32), we find

$$
\begin{aligned}
q_{nm}^2+\lambda_{nm}^{(1,2)} = q_{nm}^2(1-z_{nm}^{(1,2)}),\quad (q_{nm}^2+\lambda_{nm}^{(1)})(q_{nm}^2+\lambda_{nm}^{(2)}) = -G,\\
|1-z_{nm}^{(1,2)}|\leqslant\frac{1}{2}\left(\frac{|P-1|}{P}+\sqrt{\frac{(P-1)^2}{P^2}+\frac{4G}{q_{11}^4}}\right)\equiv\psi.
\end{aligned}
\tag{35}
$$

Let us prove that for $0 < G \leqslant q_{11}^4/P$ series (31) are a classical solution of problem (7) for all $t \geqslant 0$ if the series of initial data $w_0(r,\phi), T_0(r,\phi)$ converge

$$
\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}|S_{nm}^0| < \infty,\quad \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}|F_{nm}^0| < \infty. \tag{36}
$$

Utilazing (34), (35) and the fact that $\xi_n^{(1)} < \xi_{n+1}^{(1)} < \xi_n^{(2)} < \xi_{n+1}^{(2)} < \ldots$ [8] from the representations of solution in form of series (31) we find

$$
\begin{aligned}
|w(r,\phi,t)| \leqslant& \frac{P}{|P-1|}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{G|F_{nm}^0|}{\left(\xi_n^{(m)}\right)^2}+\psi|S_{nm}^0|\right]\left(e^{\lambda_{nm}^{(1)}t}+e^{\lambda_{nm}^{(2)}t}\right)\leqslant\\
\leqslant& \frac{P}{|P-1|}\max\left(\frac{G}{\left(\xi_1^{(1)}\right)^2},\psi\right)\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(|F_{nm}^0|+|S_{nm}^0|\right)\left(e^{\lambda_{nm}^{(1)}t}+e^{\lambda_{nm}^{(2)}t}\right),\\
|T(r,\phi,t)| \leqslant& \frac{P}{|P-1|}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\psi|F_{nm}^0|+\frac{|S_{nm}^0|}{\left(\xi_n^{(m)}\right)^2}\right]\left(e^{\lambda_{nm}^{(1)}t}+e^{\lambda_{nm}^{(2)}t}\right)\leqslant\\
\leqslant& \frac{P}{|P-1|}\max\left(\psi,\frac{1}{\left(\xi_1^{(1)}\right)^2}\right)\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(|F_{nm}^0|+|S_{nm}^0|\right)\left(e^{\lambda_{nm}^{(1)}t}+e^{\lambda_{nm}^{(2)}t}\right).
\end{aligned}
\tag{37}
$$

Series (37) converge because $\exp\left(\lambda_{nm}^{(1,2)} t\right) \leqslant 1$. Moreover, the functions $w(r, \phi, t), T(r, \phi, t)$ tend exponentially to zero as $t \to \infty$. Really, due to (33)

$$e^{\lambda_{nm}^{(1)} t} + e^{\lambda_{nm}^{(2)} t} = e^{\lambda_{11}^{(1)} t} \exp\left[\left(\lambda_{nm}^{(1)} - \lambda_{11}^{(1)}\right) t\right] \left\{1 + \exp\left[\left(\lambda_{nm}^{(2)} - \lambda_{nm}^{(1)}\right) t\right]\right\} \leqslant$$
$$\leqslant 2e^{\lambda_{11}^{(1)} t} \exp\left[\left(\lambda_{nm}^{(1)} - \lambda_{11}^{(1)}\right) t\right]. \quad (38)$$

It is clear that for $G > 0$ the quantity $\lambda_{nm}^{(1)} - \lambda_{11}^{(1)} \leqslant 0$, then from (37), (38) the estimates follow

$$|w(r, \phi, t)| \leqslant R_1 e^{\lambda_{11}^{(1)} t}, \quad |T(r, \phi, t)| \leqslant R_2 e^{\lambda_{11}^{(1)} t}, \quad (39)$$

with constants $R_1 > 0$, $R_2 > 0$. Recall that $\lambda_{11}^{(1)} = -q_{11}^1 z_{11}^{(1)} < 0$, and therefore $w \to 0$, $T \to 0$ uniformly in a circle Ω when $t \to \infty$.

Remark 5. By what was proved above, series (31) converge absolutely and uniformly, their terms are continuous so their sums $\big($functions $w(r, \phi, t), T(r, \phi, t)\big)$ also are continuous on $\Omega \cup \Gamma, t \geqslant 0$.

To prove that the functions $w(r, \phi, t)$, $T(r, \phi, t)$ are a classical solution of problem (7), we need to show that the series w_t, T_t, w_{rr}, T_{rr}, w_r/r, T_r/r, $w_{\phi\phi}/r^2$, $T_{\phi\phi}/r^2$ converge uniformly in $\Omega \cup \Gamma$ and $t \geqslant \varepsilon$, where ε is an arbitrary positive number.

When once differentiating series (31) with respect to t expressions $\lambda_{nm}^{(1,2)} \exp(\lambda_{nm}^{(1,2)} t)$ arise. Since $\lambda_{nm}^{(1,2)} < 0$ then $\left|\lambda_{nm}^{(1,2)} \exp(\lambda_{nm}^{(1,2)} t)\right| < \left|\lambda_{nm}^{(1,2)}\right| \exp(-\left|\lambda_{nm}^{(1,2)}\right| \varepsilon) < L_{1,2}/\varepsilon$ with positive constants $L_{1,2}$. Therefore the series for w_t, T_t converge absolutely and uniformly in Ω for $t \geqslant \varepsilon$.

If series (31) are differentiated twice with respect to r, then the expression $\left(\xi_n^{(m)}\right)^2 J_n''\left(\xi_n^{(m)} r\right)$ is formed. It is known from [9] that for uniform convergence on the segment $[0, 1]$ of a series

$$\sum_{m=1}^{\infty} Z_{nm} \left(\xi_n^{(m)}\right)^2 J_n''\left(\xi_n^{(m)} r\right),$$

where $n = 1, n \geqslant 2$, it is sufficient that all coefficients Z_{nm} satisfy the inequality $|Z_{nm}| \leqslant$ $\leqslant Z/\big(\xi_n^{(m)}\big)^{3+\delta}$, $\delta > 0$, $Z = const$. Actually, the double series also converges

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Z_{nm} \left(\xi_n^{(m)}\right)^2 J_n''\left(\xi_n^{(m)} r\right),$$

so far as $\xi_n^{(m)}$ is equivalent to n for $n \gg 1$ (just like $\xi_n^{(m)}$ is equivalent to m for $m \gg 1$)[10]. Hence and from estimates (37) we obtain that if $|F_{nm}^0| + |S_{nm}^0| \leqslant Z/\big(\xi_n^{(m)}\big)^{3+\delta}$, then the series w_{rr}, T_{rr} converge absolutely and uniformly in Ω for all $t \geqslant \varepsilon$, $\varepsilon > 0$. Moreover, the series w_r/r, T_r/r also converge under this condition.

Consider now the series $w_{\phi\phi}/r^2, T_{\phi\phi}/r^2$. Differentiating functions $w(r, \phi, t)$, $T(r, \phi, t)$ twice with respect to ϕ gives the expression $n^2 J_n\left(\xi_n^{(m)} r\right)$. Since $J_n\left(\xi_n^{(m)} r\right)$ is a solution to the Bessel equation we have

$$\frac{n^2}{r^2} J_n\left(\xi_n^{(m)} r\right) = \left(\xi_n^{(m)}\right)^2 J_n''\left(\xi_n^{(m)} r\right) + \frac{\xi_n^{(m)}}{r} J_n'\left(\xi_n^{(m)} r\right) + \left(\xi_n^{(m)}\right)^2 J_n\left(\xi_n^{(m)} r\right). \quad (40)$$

Replacing in series $w_{\phi\phi}/r^2$, $T_{\phi\phi}/r^2$ expression $n^2 J_n\left(\xi_n^{(m)} r\right)/r^2$ to the right side of identity (40) we get that, by what was proved earlier, for $|F_{nm}^0| + |S_{nm}^0| \leqslant Z/\left(\xi_n^{(m)}\right)^{3+\delta}$, $\delta > 0$, $Z = const$ these series converge absolutely and uniformly in Ω for all $t \geqslant \varepsilon$, $\varepsilon > 0$.

Thereby theorem is proved

Theorem 2. *Let be* $0 < G \leqslant q_{11}^4/P$*, series (36) absolutely converge in circle* $\Omega \cup \Gamma$ *and* $|F_{nm}^0| + |S_{nm}^0| \leqslant Z/\left(\xi_n^{(m)}\right)^{3+\delta}$, $\delta > 0$, $Z = const$*, where* $\xi_n^{(m)}$ *is mth zero of the nth order Bessel function. Then the solution of problem (7) is classical and estimates (39) are satisfied.*

Remark 6. The fluid flow rate Q in case of a circular cross section will be equal to zero:

$$Q(t) = \int_0^1 rdr \int_0^{2\pi} \left[\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} S_{nm}(t) J_n\left(\xi_n^{(m)} r\right) \left\{ \begin{array}{c} \cos n\phi \\ \sin n\phi \end{array} \right\} \right] d\phi = 0.$$

Remark 7. Solution of the problem in case of MS-20 oil flow at a temperature of $0°C$ [6] in a vertical heat exchanger of circular cross section with initial data $w_0(r,\phi) = J_1(\xi_1^{(1)} r) \sin\phi$, $T_0(r,\phi) = J_1(\xi_1^{(1)} r)\sin\phi$ and constants $d = 0.1$ m, $A = 1$ K/m, $\rho = 903.6$ kg/m^3, $\beta = 6.27 \cdot 10^{-4}$ $1/K$, $\chi = 62.06 \cdot 10^{-3}m^2/c$, $\nu = 7.59 \cdot 10^{-3}$m$^2/c$ with $G = 0.144 < q_{11}^4/P = 26.36745$ has the form

$$w(r,\phi,t) = \left(1.0103 e^{\lambda^1 t} - 0.0103 e^{\lambda^2 t}\right) J_1(3.83171 r) \left\{ \begin{array}{c} \cos\phi \\ \sin\phi \end{array} \right\},$$

$$T(r,\phi,t) = \left(-0.07833 e^{\lambda^1 t} + 0.92166 e^{\lambda^2 t}\right) J_1(3.83171 r) \left\{ \begin{array}{c} \cos\phi \\ \sin\phi \end{array} \right\},$$

where $\lambda^1 = -1.784727$, $\lambda^2 = -14.693177$. Fig. 3 shows the vertical velocity profile $w(r,\phi,t)$ at $t = 0$ and at $t = 0.6$ in dimensionless coordinates; as t increases, $w(r,\phi,t)$ tends to zero. Here the fluid flow rate is $Q(t) = 0$ and a reverse flow occurs.

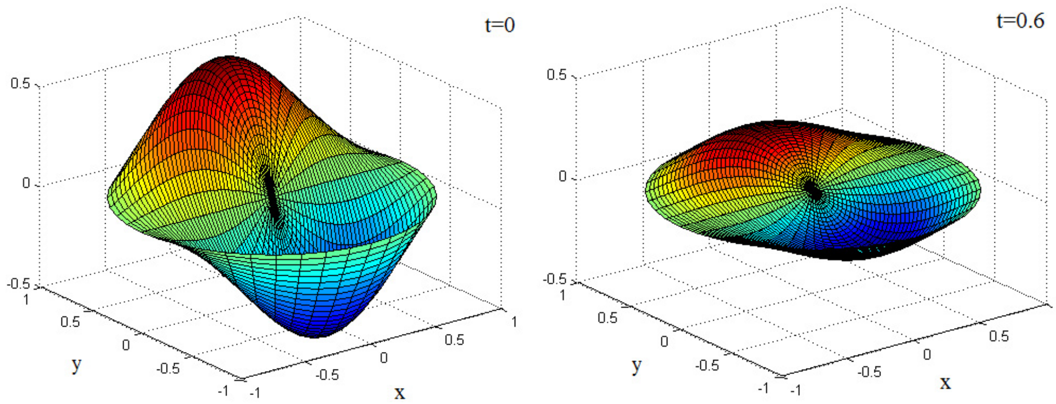

Fig. 3. Velocity profile $w(r,\phi,t)$ at $t = 0$, $t = 0.6$

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Начально-краевая задача о движении вязкой теплопроводной жидкости в вертикальной трубе

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Аннотация. Исследуется начально-краевая задача, возникающая при моделировании нестационарного однонаправленного конвективного течения в вертикальных теплообменниках с поперечным сечением произвольной формы. Получена априорная оценка в L_2 и доказана единственность решения задачи. В случае прямоугольного и круглого сечения решение найдено в виде двойных рядов Фурье. Даны достаточные условия стабилизации с ростом времени решения к покою.

Ключевые слова: начально-краевая задача, априорная оценка, ряды Фурье, конвекция.