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Integration of Mathematical and Computer Simulation Modeling in Engineering Education

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Abstract. The development of modern engineering, technology, science and education is impossible without the use of computer mathematics systems. Nowadays, an engineer should be equally proficient in both methods of mathematical modeling of technical objects and processes using theoretical models and methods of simulation modeling in computer mathematics systems. This article focuses on a laboratory class, in which students are suggested to perform mathematical modeling of a production process considering it as a Markov process, based on Kolmogorov equations. Students are asked to implement the theoretical solution to the model for specific numerical data and to carry out the computer modeling of the process of searching for the limiting probabilities of system states using the authors' virtual laboratory complex. This complex was developed on the platform for simulation of business systems AnyLogic, offering the three approaches to modelling: discrete event, agent based and system dynamics. The authors' lab-based system allows real-time modeling of such production process parameters as workload, equipment downtime, location of agents on attractors, etc. The article suggests the implementation of 2D and 3D production process visualization, which greatly simplifies the management and decision-making process.

Keywords: mathematical modeling, computer simulation modeling, engineering education, Markov process.

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Introduction

The most important trend in the development of modern society is the digitalization of economic and production processes. In the context of active innovative changes taking place in science and technology, an engineer is required to have integrative creative skills, readiness to carry out multifunctional research activities based on mathematical and computer modeling with the use of digital tools.

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The process of science and technology development based on modeling requires the improvement of mathematical foundations that make it possible to: model, develop algorithms, use the computer technology apparatus, evaluate models reliability in quantitative estimation, analyze and optimize. All this means that teaching mathematical modeling, based on the integration of mathematical and applied science, in combination with information technology, is a current trend in the development of modern engineering education. This corresponds to the fact that a significant part of the mathematical activity of a person in general and an engineer in particular, is associated with work in the field of information technology [1, 2].

Mathematical modeling, along with natural experiments, is the main way of research and obtaining new knowledge in various fields of natural science. The active use of mathematical modeling in various fields of human activity is due to many factors, the main of which are the complication of the class of problems under study, the study of which requires the creation of new expensive experimental facilities or model objects; the need to address environmental, social and other problems; the impossibility of carrying out full-scale (physical, chemical, economic, etc.) modeling (in this case, mathematical modeling is the only possible one) [3].

Analyzing the practice of teaching engineering students, it should be noted that mainly the process of studying such disciplines as Mathematics, Probability Theory, Applied Mathematics, Mathematical Programming, Operations Research, etc., is based on the use of the mathematical packages (Mathematica, Maxima, Maple, Derive, MathCad, Matlab), which are classified as engineering programs for computer-based design [4–6]. They make it possible to perform such routine operations as converting expressions, finding roots of equations, derivatives and indefinite integrals, etc., with little or no user intervention. However, this is not enough to study the modeling process, since many mathematical models cannot be solved due to their complexity and unsolvability in an analytic form. Therefore, simulation modeling becomes a universal tool in engineering education. By simulation modeling we usually mean the values calculation of some characteristics of a process, developing over time, by reproducing the course of this process on a computer using its mathematical model, and it is either impossible or extremely difficult to obtain the required results in other ways [7, 8]. Reproduction of the process on the computer with the help of a mathematical model is commonly called a simulation experiment.

The modeling process with the use of simulation models includes such stages as making a model, programming, conducting simulation experiments, processing and interpreting simulation results [9]. With the advent of simulation models, the concept of modeling has changed, which is now considered as the only process of building and researching models, that has a software support.

In the 90-s of the last century, simulation environments appeared (Arena, Extend, MicroSaint, Enterprise Dynamics, etc.) containing a non-programming user interface, input and output analyzers, and the ability to animate simulation. Such environments do not require programming in the form of a sequence of commands. Instead of compiling a program, the user builds the model by transferring ready-made blocks from the library to the working field and establishing links between them. Visual modeling packages enable the user to enter the modeled system description in a natural for the application area and mainly graphical form, as well as visually present the simulation results, for example, in the form of diagrams or animation [10].

One of the main advantages of simulation modeling systems is that they enable the user not to worry about the model software implementation as a sequence of executable statements. Thus they create some extremely convenient environment on the computer in which you can create virtual parallel systems and conduct experiments with them. The graphical environment becomes similar to a physical test bench, but instead of heavy metal boxes, cables and real measuring equipment, oscillographs and recorders, the user deals with their images on the display screen [11]. Besides, the user can see and evaluate the simulation results during the experiment and, if neces-sary, actively intervene in it.

An interesting direction in computer modeling is virtual reality. The term "virtual reality" appeared in the late 70-s of the twentieth century (the so-called three-dimensional macro models of reality, which were created on a computer and gave the effect of a person's presence in the virtual world) [12]. Virtual reality is a highly developed form of computer simulation that allows users to immerse themselves in a virtual world and immediately act in it using special touch-sensitive devices. Such devices (virtual reality helmet, glasses, gloves, capsules, etc.) associate the user's movements with audiovisual effects. The user's visual, auditory, tactile and motor sensations are replaced by their computer-generated imitation. As studies by domestic and foreign authors have shown, the alternative world is attractive to many precisely because of its "virtuality" [13].

Thus, modern mathematical education of engineering students should be based on the integration of mathematical and computer simulation modeling. At the same time, it is necessary to pay more attention to the design of methodological strategies for teaching mathematical modeling [14–17].

The introduction of innovative teaching methods based on the integration of mathematical and computer modeling into the process of mathematical training of future engineers is one of the ways to increase the efficiency of the mathematical component of engineering education, which enables to form in future engineers both mathematical and digital competencies that they need in their future professional activities.

The purpose of the article is to show the implementation of a mathematical model in the form of the Markov process based on the Kolmogorov equations in the production process as an integration of mathematical and computer modeling, using the example of laboratory work for engineering students.

1. Materials and methods

The range of problems to be solved determined the methods of the present study.

The first group of methods is connected with solving problems of teaching higher and applied mathematics to engineering students. In the theory and methodology of teaching mathematics, the most important teaching methods are search methods. One of these methods is mathematical modeling, which contributes to the development of all mathematical activity components of a future engineer in students:

1. actual knowledge, skills determined by the educational program;
2. mental operations and methods inherent in mathematical activity;
3. mathematical style of thinking;
4. mastering the methods of modeling real processes, including computer simulation [7, 18–20].

This method is implemented by solving professionally-oriented problems, the condition and requirement of which deal with engineering activity objects. Such problems are aimed at mastering the techniques of mathematical modeling through reaching the following stages of solution:

- examining the object of modeling, formalizing the problem data;
- conceptual setting of the modeling problem, determining the functioning patterns of the modeling object;
- mathematical setting of the original problem, reducing it to the solution of the mathematical problem;
- choice and substantiation of the method for solving the obtained mathematical problem;
- solution of the mathematical model of the modeling object;
- analysis of the obtained solution, validation of the model [20].

The second group of methods is related to the tasks of developing digital competencies of future engineers [21, 22]. Computer technology is constantly changing and fundamentally altering mathematics education [23]. Therefore, in teaching mathematical disciplines, it is necessary to use simulation modeling methods, which presuppose, on the basis of a mathematical model, creating a simulation model of a process or a simulation object. These methods make it possible to analyze the mathematical model in action, to study processes and make changes to the simulation model in the course of work, to better analyze the operation of the system and quickly solve the problem [11]. Such models, developed with the help of special software, reproduce the events occurring in a real system step by step [24]. The simulation models advantage is the ability to replace the process of changing events in the system under study in real time with an accelerated process of changing events at the program pace.

The simulation models development is possible on the AnyLogic platform, a tool that offers the possibility of multi-method simulation using all three modern approaches: discrete event, agent based and system dynamics. These three methods can be used in any combination on the basis of one software to model a system of any complexity. AnyLogic has different visual modeling languages: process diagrams, state diagrams, block diagrams, stock and flow diagrams.

We used the AnyLogic platform to develop a virtual laboratory complex that contains laboratory works on the topics listed in Tab. 1 and is focused on training road transport engineers majoring in fields: "Traffic Organization", "Transport technologies", "Organization of transportation and management in road transport". For example, in road transport engineers' professional activity there arise tasks related to optimal management of traffic flows, organization of traffic in road transport and ensuring its safety. Such problems can be solved using linear, dynamic and stochastic programming methods.

The virtual laboratory complex suggested is used in teaching Applied Mathematics to engineering students of any specialization.

As an example of a virtual laboratory work from the laboratory complex developed by us, let us consider the one on the topic "Markov processes", in which systems designed for multiple use in industry are modeled using the queuing theory.

Such systems play an important role in many areas of economy, finance, production and everyday life. Systems such as computer networks, systems for collecting, storing and processing information, transport systems can be considered as a kind of QS. Each QS includes in its structure a certain number of service devices, which are called service channels (devices, lines).

The mathematical analysis of the queuing systems operation is considerably simplified if they are considered as a Markov process or a stochastic process with no consequence. An example of a Markov process is any technological process related to transportations and traffic management, logistics, etc.

Table 1. Topics and content of the virtual laboratory complex

Number	Topic of laboratory work	Modeled processes and methods
1.	Graphical methods of solving linear programming problems	Integer linear programming model. Cutting plane method
2.	Simplex method for solving linear programming problems	Optimization of logistics processes in transport. Gomory method
3.	Methods for solving the transport problem	Method of potentials as applied to transport infrastructure models
4.	Dynamic programming	Bellman's principle of optimality. Finding the most reliable route in relation to transport infrastructure models
5.	Equipment replacement tasks	Finding optimal terms for equipment replacement in transport and technological models
6.	Bellman equations	Allocation of funds between branches for n years in transport infrastructure models
7.	Assignment model	The Hungarian method for solving assignment problems
8.	Network models	Problem of the maximum flow in relation to the urban traffic optimization model
9.	Floyd model	Model of optimizing the route network of goods and passengers transportation
10.	Combinatorial models	Branch-and-Bound method of implementing the "travelling salesman" problem
11.	Probability models	Deterministic and stochastic models of inventory management
12.	Elements of game theory	Game models 2×2. Solving games in mixed strategies
13.	Game models $2 \times n$	Graphical solution of games with a given payoff matrix $2 \times n$
14.	Solving matrix games	Typical model for solving matrix games using linear programming methods
15.	Decision making under uncertainty	Decision-making models in transport systems based on Laplace, minimax, Savage, Hurwicz criteria
16.	Markov stochastic processes	Practical implementation of Markov processes study in transport systems modeling
17.	Queuing systems (QS)	The process of death and reproduction. Multichannel system with failures (Erlang problem)

The article deals with a virtual laboratory work on the topic "Markov processes" and studies the practical implementation of the investigation of queuing systems in industrial systems modeling. In the paper, students are asked to build a mathematical description of the system based on the Kolmogorov equations on the basis of a given state graph of a certain technological process associated with road transport. Then they model the same process using a simulation model, in which such parameters of the production process as workload of workplaces, equipment downtime, location of agents on attractors, etc., vary in the model time mode. Moreover, it becomes possible to use 2D and 3D visualization of production process, which significantly simplifies the management and decision-making. This approach to the educational process organization is a mixed form of education that is being actively implemented in many educational organizations [25, 26].

2. Results

In the laboratory work under consideration, students are asked to determine the optimal operating mode of a motor industrial enterprise, for example transport, that has two conditional workplaces (for example, a warehouse for finished products, a garage, a repair shop, etc.), which ensures minimal loss of time while meeting the needs of a certain production process. The simulated production process is considered as a queuing system, described using Markov process [27], the labeled state graph of which is shown in Fig. 1.

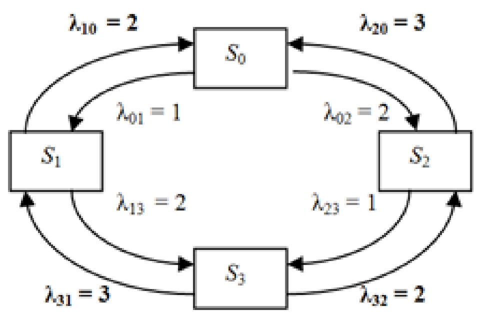

Fig. 1. Labeled system state graph, where S_i — system state; λ_{ij} — flow of events intensity

It is assumed that the system has four states, which can be conditionally described as: S_0 (both workplaces are occupied or not available), S_1 (workplace 1 is available when workplace 2 is not available), S_2 (workplace 1 is not available when workplace 2 is available), S_3 (both workplaces are not occupied, i.e. available). Flow of events intensities λ_{ij} represents the frequency of transitions from state S_i to state S_j.

The modeling is carried out step by step. Let us describe these steps.

Step 1. Drawing up a mathematical model of the queuing system under consideration. At this stage, group educational work is carried out with students, which can be described with the help of the following reasoning.

Ideally, we assume that the process considered in the system is Markov process, and also that all system transitions from state S_i to state S_j are influenced by the simplest flows of events with intensities λ_{ij}.

If we consider, for example, warehouse terminals as workplaces, then the transition from one state to another is influenced by the flow of events — the arrival of machines for unloading, which causes the occupancy of workplaces, and therefore their unavailability to other machines. The reverse transition takes place when the terminal is free from the unloaded machine, which makes the workplace available for other machines (Tab. 2).

The $p_i(t)$ is the probability that at time t the system will be in state S_i, $i = 0, 1, 2, 3$. The mathematical description of the simulated Markov random process with discrete states and continuous time is the system of Kolmogorov differential equations for probabilities of states.

Table 2. Interpretation of system states

Workplaces state	System state	Production situation	Symbol
Both workplaces are functioning	S_0	Both terminals are occupied for unloading	$(+,+)$
Workplace 1 is not functioning, workplace 2 is functioning	S_1	Terminal 1 is available, terminal 2 is occupied for unloading the machine	$(-,+)$
Workplace 1 is functioning, workplace 2 is not functioning	S_2	Terminal 1 is occupied for unloading the machine, terminal 2 is available	$(+,-)$
Both workplaces are not functioning	S_3	Both terminals are not occupied and available for unloading	$(-,-)$

Writing out the Kolmogorov equations for the states given in the example under consideration, we obtain:

$$\begin{cases} \dfrac{dp_0}{dt} = \lambda_{10}p_1 + \lambda_{20}p_2 - (\lambda_{01} + \lambda_{02})\, p_0, \\ \dfrac{dp_1}{dt} = \lambda_{01}p_0 + \lambda_{31}p_3 - (\lambda_{10} + \lambda_{13})\, p_1, \\ \dfrac{dp_2}{dt} = \lambda_{02}p_0 + \lambda_{32}p_3 - (\lambda_{20} + \lambda_{23})\, p_2, \\ \dfrac{dp_3}{dt} = \lambda_{13}p_1 + \lambda_{23}p_2 - (\lambda_{31} + \lambda_{32})\, p_3. \end{cases} \tag{1}$$

The Kolmogorov equations make it possible to find all probabilities of states as a function of time, but the probabilities of system $p_i(t)$ in the limiting stationary mode, i.e., at $t \to \infty$, are of particular interest. They are called limiting probabilities of states, which are to be found.

The limiting probability has a clear meaning: it shows the average relative time the system spends in this state.

Since the limiting probabilities are constant, replacing their derivatives $\dfrac{dp_i}{dt}$ in the Kolmogorov equations (1) with zero values, we obtain a system of linear algebraic equations describing the stationary mode:

$$\begin{cases} (\lambda_{01} + \lambda_{02})\, p_0 = \lambda_{10}p_1 + \lambda_{20}p_2, \\ (\lambda_{10} + \lambda_{13})\, p_1 = \lambda_{01}p_0 + \lambda_{31}p_3, \\ (\lambda_{20} + \lambda_{23})\, p_2 = \lambda_{02}p_0 + \lambda_{32}p_3, \\ (\lambda_{31} + \lambda_{32})\, p_3 = \lambda_{13}p_1 + \lambda_{23}p_2. \end{cases} \tag{2}$$

Thus, a mathematical model in the form of a system of linear equations is obtained.

Step 2. Solution of the mathematical model obtained at step 1. Students are given individual initial data in the form of numerical values of intensities λ_{ij} and are asked to simulate the system. For example, such tasks can include the following: do the following tasks for system S, the state graph of which is shown in Fig. 2:

- find the limiting probabilities if
  $\lambda_{01} = 1,\ \lambda_{02} = 2,\ \lambda_{10} = 2,\ \lambda_{13} = 2,\ \lambda_{20} = 3,\ \lambda_{23} = 1,\ \lambda_{31} = 3,\ \lambda_{32} = 2;$

- find the total profit from system S operating in the stationary mode, if it is known that the operation of the first and the second workplaces per time unit brings the income respectively $I_1 = 10$ monetary units and $I_2 = 6$ monetary units, and their availability entails costs respectively $E_1 = 4$ monetary units and $E_2 = 2$ monetary units;
- substantiate whether the modernization of the system will be effective if it entails a halving of the average time of unavailability of each workplace, provided that this entails a doubling of the expenses of their availability.

Students need to understand that the predictive properties of a model strongly depends on the values of the model parameters. Identifying adequate model parameters λ_{ij} $(i,\ j = 0,\ 1,\ 2,\ 3)$ is often considered as a delicate task in mathematical modeling. This process is achieved based either on expert knowledge or inference procedures. Students can participate in this process, for example, during practical training by conducting a natural experiment.

A student, substituting the given numerical data into the system (2), receives a linear equations system with respect to the variables p_i, $i = 0,\ 1,\ 2,\ 3$.

$$\begin{cases} 3p_0 = 2p_1 + 3p_2, \\ 4p_1 = p_0 + 3p_3, \\ 4p_2 = 2p_0 + 2p_3, \\ p_0 + p_1 + p_2 + p_3 = 1. \end{cases} \tag{3}$$

Having solved the system (3), students obtain the values of limiting probabilities $p_0 = 0.4$, $p_1 = 0.2$, $p_2 = 0.27$, $p_3 = 0.13$. The interpretation of the found values of limiting probabilities is given in Tab. 3.

Table 3. Interpretation of the limiting probabilities of system states

System state	Symbol	Limiting probabilities	Production interpretation
S_0	(+,+)	$p_0 = 0.4$	40% of time both terminals are occupied for unloading
S_1	(–,+)	$p_1 = 0.2$	20% of time Terminal 1 is available, terminal 2 is occupied for unloading the machine
S_2	(+,–)	$p_2 = 0.27$	27% of time terminal 1 is occupied for unloading the machine, terminal 2 is available
S_3	(–,–)	$p_3 = 0.13$	13% of time both terminals are not occupied

Thus, at the second step of modeling, students find a connection between the states of workplaces and the limiting probabilities of the system. Students should pay special attention to the fact that the sum of the limiting probabilities of states should be equal to 1: $\sum_{i=0}^{3} p_i = 1$, which they can easily verify [28].

Step 3. Calculation of profits and expenses from the functioning of workplaces 1 and 2. The student is asked to consider elementary events: A_i — i-workplace is functioning, as well as opposite events: A_i' — i-workplace is not functioning, where $i = 1,\ 2$.

The incomes and expenses set in step 2 are associated with the occurrence of these events (Tab. 4). Then, students are asked to express S_i through complementary events A_i and A_i', considering S_i as a random events, and write the results in Tab. 5.

Since the condition $\sum_{i=0}^{3} p_i = 1$ is satisfied, then the events S_i constitute a complete group of events. The calculation of possible incomes can be made taking into account the reasoning that students can do on their own or with the help of a teacher.

Table 4. Correspondence of incomes and expenses to the state of workplaces

Event	Symbol for event	Income, monetary	Expenses, monetary units
Workplace 1 is functioning	A_1	$I_1 = 10$	–
Workplace 2 is functioning	A_2	$I_2 = 6$	–
Workplace 3 is not functioning	A_1'	–	$E_1 = 4$
Workplace 4 is not functioning	A_2'	–	$E_2 = 2$

Table 5. Correspondence of incomes and expenses to the state of workplaces

System state	Expression through complementary events	Probability
S_0	$S_0 = A_1 \cdot A_2$	$P(S_0) = p_0$
S_1	$S_1 = A_1' \cdot A_2$	$P(S_1) = p_1$
S_2	$S_2 = A_1 \cdot A_2'$	$P(S_2) = p_2$
S_3	$S_3 = A_1' \cdot A_2'$	$P(S_3) = p_3$

1) Profit from the functioning of workplace 1 can be obtained if the system is in state S_0 or S_2. Event B_1, consisting in the fact that workplace 1 operates in the system under consideration, is the sum of events S_0 and S_2, that is $B_1 = S_0 + S_2$.

To find the probability of event B_1, we use Theorem 2.1 [28] (p. 56):

Theorem 2.1 *If events A and B are mutually exclusive, then* $P(A + B) = P(A) + P(B)$.

Since events S_0 and S_2 cannot occur at the same time, they are mutually exclusive. According to Theorem 1:

$$P(B_1) = P(S_0 + S_2) = P(S_0) + P(S_2). \quad (4)$$

In consideration that $P(S_0) = p_0$, $P(S_2) = p_2$ and taking into account (4), we can get the probability of event B_1 :

$$P(B_1) = p_0 + p_2 = 0.4 + 0.27 = 0.67.$$

2) The probability of event B_1', opposite to event B_1 — workplace 1 does not function in the system under consideration, can be found according to Theorem 2.2 [28] (p. 58):

Theorem 2.2 *If A and A' are complementary events, then* $P(A) + P(A') = 1$.

It follows from Theorem 2.2 that $P(B_1) + P(B_1') = 1$, from which we find

$$P(B_1') = 1 - P(B_1) = 1 - 0.67 = 0.33.$$

The same probability can be found using Theorem 2.1:

$$P(B_1') = P(S_1 + S_3) = p_1 + p_3 = 0.2 + 0.13 = 0.33.$$

3) Considering similarly event B_2, consisting in the fact that workplace 2 functions in the system under consideration, as well as opposite event B_2' — workplace 2 functions in the system under consideration, we have:

according to Theorem 2.1: $P(B_2) = P(S_0 + S_1) = p_0 + p_1 = 0.4 + 0.2 = 0.6$;

according to Theorem 2.2: $P(B_2') = 1 - p(B_2) = 1 - 0.6 = 0.4$;

or according to Theorem 2.1: $P(B_2') = P(S_2 + S_3) = p_2 + p_3 = 0.27 + 0.13 = 0.4$.

4) Students are asked to calculate economic indicators by filling in Tab. 6.

Table 6. Calculation of economic system indicators before modernization

Economic indicator	Formula	Value, monetary units
Workplace 1 functioning income	$P(B_1) \cdot I_1$	$0.67 \cdot 10 = 6.7$
Workplace 2 functioning income	$P(B_2) \cdot I_2$	$0.6 \cdot 6 = 3.6$
Total system income	$I = P(B_1) \cdot I_1 + P(B_2) \cdot I_2$	$I = 6.7 + 3.6 = 10.3$
Workplace 1 non-functioning expense	$P(B_1') \cdot E_1$	$0.33 \cdot 4 = 1.32$
Workplace 2 non-functioning expense	$P(B_2') \cdot E_2$	$0.4 \cdot 2 = 0.8$
Total system expenses	$E = P(B_1') \cdot E_1 + P(B_2') \cdot E_2$	$E = 1.32 + 0.8 = 2.12$
Total profit from the system operation	$P = I - E$	$P = 10.3 - 2.12 = 8.18$

Step 4. Substantiation of possible efficiency of production modernization. To do this, changes are made to the initial data of the problem. Thus, the necessity to halve the average time of working places availability will lead to the increase in the frequency of returning to work.

With new data, students deal with the following task:

For the system S, the state graph of which is shown in Fig. 1, find:

- limiting probabilities if the intensities of the inner circle do not change: $\lambda_{01} = 1$, $\lambda_{02} = 2$, $\lambda_{10} = 2$, $\lambda_{13} = 2$, $\lambda_{23} = 1$, and the intensities of the outer circle of the labeled state graph shown in Fig. 1 are doubled: $\lambda_{10} = 4$, $\lambda_{31} = 6$, $\lambda_{32} = 4$, $\lambda_{20} = 6$;
- changes in the total profit from operation of system S in the stationary mode, if it is known that the operation of workplace 1 and workplace 2 per unit of time brings income, respectively, in $I_1 = 10$ monetary units and $I_2 = 6$ monetary units, and the costs associated with their availability will double and amount to $E_1' = 2E_1 = 8$ monetary units and $E_2' = 2E_2 = 4$ monetary units for workplace 1 and 2, respectively;
- evaluate the effectiveness of halving the average time of unavailability of each workplace, while doubling the cost of their availability.

The student, substituting new numerical values of intensities into system (2), obtains a system of equations:

$$\begin{cases} (1+2) \cdot p_0 = 4 \cdot p_1 + 6 \cdot p_2, \\ (4+2) \cdot p_1 = 1 \cdot p_0 + 6 \cdot p_3, \\ (6+1) \cdot p_2 = 2 \cdot p_0 + 4 \cdot p_3, \\ p_0 + p_1 + p_2 + p_3 = 1. \end{cases} \Leftrightarrow \begin{cases} 3 \cdot p_0 = 4 \cdot p_1 + 6 \cdot p_2, \\ 6 \cdot p_1 = 1 \cdot p_0 + 6 \cdot p_3, \\ 7 \cdot p_2 = 2 \cdot p_0 + 4 \cdot p_3, \\ p_0 + p_1 + p_2 + p_3 = 1. \end{cases} \Leftrightarrow \begin{cases} p_0 = 0.6, \\ p_1 = 0.15, \\ p_2 = 0.2, \\ p_3 = 0.05. \end{cases} \quad (5)$$

Step 5. Calculation of the total profit of the system, taking into account the modernization, connected with the production need. To do this, students can use the reasoning done in Step 3.

As a result, taking into account (5) and formulas from Tab. 2, the following should be calculated:

- income from the operation of workplace 1 and workplace 2:

$$P(B_1) \cdot I_1 = (p_0 + p_2) \cdot 10 = (0.6 + 0.2) \cdot 10 = 0.8 \cdot 10 = 8,$$

$$P(B_2) \cdot I_2 = (p_0 + p_1) \cdot 6 = (0.6 + 0.15) \cdot 6 = 0.75 \cdot 6 = 4.5.$$

- income from availability of workplace 1 and workplace 2:

$$P(B_1') \cdot E_1 = (p_1 + p_3) \cdot 8 = (0.15 + 0.05) \cdot 8 = 0.2 \cdot 8 = 1.6,$$

$$P(B_2') \cdot E_2 = (p_2 + p_3) \cdot 4 = (0.2 + 0.05) \cdot 4 = 0.25 \cdot 4 = 1.$$

- total income and expenditure of the system:

$$I' = P(B_1) \cdot I_1 + P(B_2) \cdot I_2 = 8 + 4.5 = 12.5,$$

$$E' = P(B_1') \cdot E_1 + P(B_2') \cdot E_2 = 1.6 + 1 = 2.6.$$

- total system profit: $P' = I' - E' = 12.5 - 2.6 = 9.9$ (monetary units).

Analyzing the feasibility of upgrading production in the context of economic benefits of changes made to the technological process, students should compare the total profit of the system before and after the modernization. Since the profit received after the modernization $P' = 9.9$ monetary units is larger than before it $P = 8.18$ monetary units, we come to the conclusion that the modernization is economically feasible.

Step 6. Modeling the considered production system using an idealized virtual model of the Markov process developed on the AnyLogic platform using a discrete-event approach.
Fig. 2 shows the image of the computer screen in the virtual laboratory work developed by us and the tables with the calculation of expenses and profits for the workplaces' functioning.

Working with a virtual model students observe that:

1. With an increase in the model time interval, the empirical values of the virtual model tend to the theoretical values found using the mathematical apparatus;

2. All given numerical values dynamically change over the course of model time in accordance with built-in functions;

3. Text fields allow an interactive change of the problem condition in order to obtain answers in dynamics, which is very important in the process of understanding the further development of production models;

4. State graphs (initial and modernized) have an interactive behavioral coloring of the current states, dynamically changing when the model time changes.

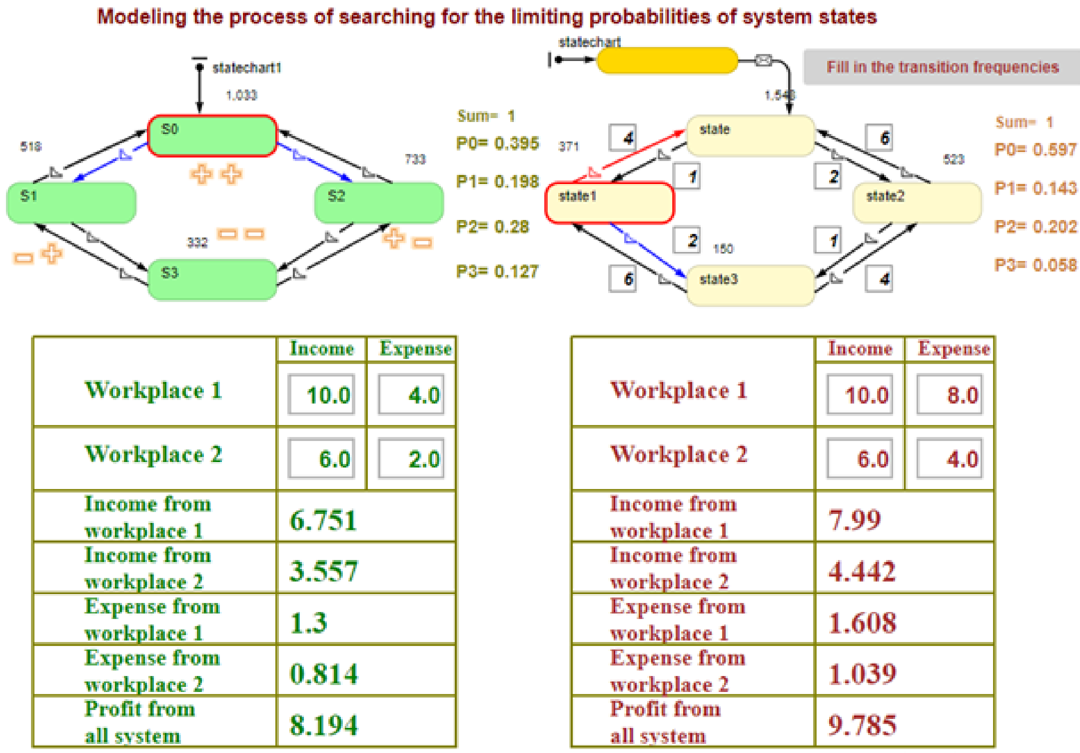

Fig. 2. The image of the computer screen in the virtual laboratory work with the calculation of ex-penses and profits for the workplaces' functioning

Step 7. Visualization of the considered production system using a simulation model developed on the AnyLogic platform using an agent-based approach (Fig. 3).

Fig. 4 shows the current 2D dynamic model, but no longer idealized, where transitions from state to state occur instantly (which is naturally impossible in a real life process). But it is based on the use of ***Queue*** blocks that model the queue of agents, ***Delay*** blocks that delay agents for a specified period of time, as well as ***Service*** blocks that capture a specified amount of resources for the agent, delay them, and then release the resources captured by it. The management of the working capacity of "workplace 1" and "workplace 2" that corresponds to states S_1 and S_2, is achieved with the use of ***Hold*** blocks which block (unblock) the flow of agents in a certain section of the flowchart.

Step 8. Virtual laboratory work also makes it possible to visualize the production site in 3D mode (Fig. 5).

Fig. 4 shows a working 3D model of the production process modeled in the laboratory work on the topic "Markov Processes". The student can observe the workload of workplaces S_1 and S_2, as well as downtime S_3, when S_1 and S_2 are not working at the same time. It is possible to do this in the model time mode (the model time can be calculated in seconds, minutes, hours, years, etc. and can be changed in accordance with the controls). As agents located on attractors (allows you to set the exact location of agents) — S_1, S_2 and S_3, we chose the figure of a person (in relation to the production setting), which enables to visually identify the workload of workplaces and downtime places when the latter do not function.

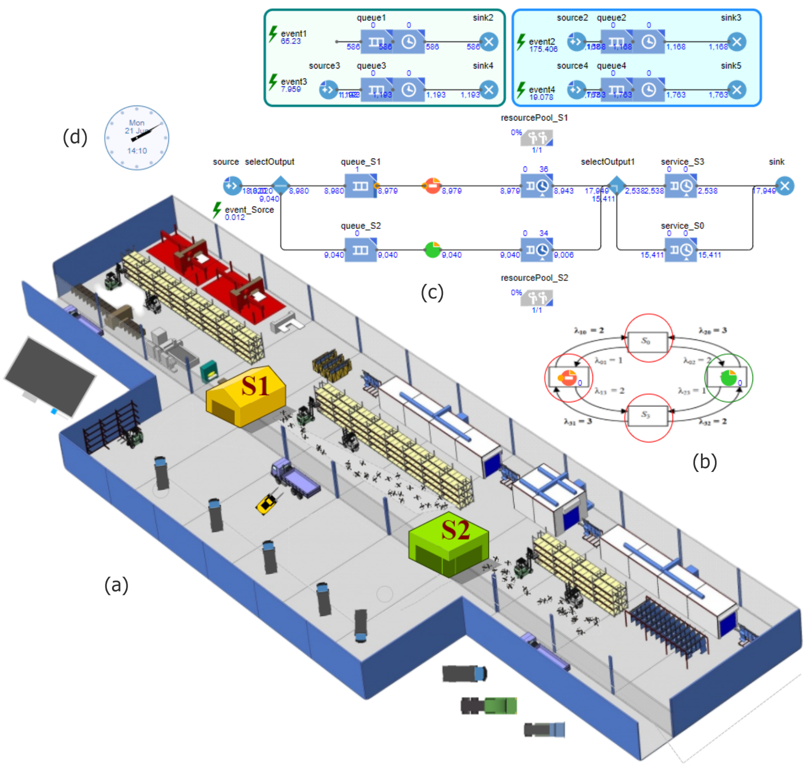

Fig. 3. Production 2D model implementation: a) production site; b) system state graph; c) flowchart of the system management process; d) system model time

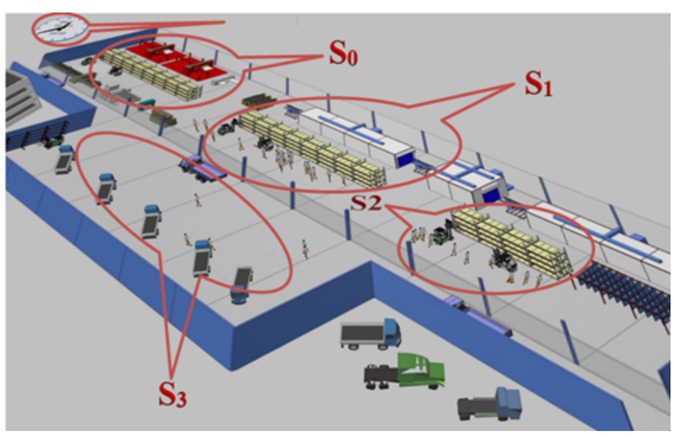

Fig. 4. Production 3D model implementation

Thus, when doing each laboratory work included in the laboratory complex developed by us, students go through the following stages:

1. Drawing up a mathematical model.
2. Solution of the mathematical model for initial data.
3. Variation of initial data and calculations for new values of variables.

4. Evaluation of the economic efficiency of the initial and modified model.
5. Performing calculations on an idealized model.
6. Working with 2D model of the production process.
7. Process control using a virtual 3D model.

The laboratory complex suggested can be used to organize the work of students in practical classes, which will contribute to effective mastering of the material in Applied Mathematics.

3. Discussion

The developed virtual laboratory complex is a digital tool for teaching mathematics to future engineers. Unlike the educational digital simulation tools offered by other researchers [29, 30], it does not require from engineering students to have programming skills, which enables them to focus on the formation of mathematical activity techniques and visualization of the studied pro-cesses using the offered simulation models.

The importance of teaching mathematical modeling to future engineers is noted by many scientists, for example [7, 11, 17]. We agree with the opinion that mathematical modeling is both an important tool for professional engineers and a teaching method in engineering education [8]. Be-sides, mathematical modeling is a means of forming the professional competence of an engineer [17]. Many scientists think it is necessary to integrate mathematical and simulation modeling. They note that a dominant trend today is the interpenetration of all types of modeling, the symbiosis of various information technologies in the field of modeling, especially for complex applications and complex modeling projects [10].

Our investigations testify to the fact, that only a reasonable combination of traditional teaching methods and educational technologies of knowledge engineering, machine learning methods will ensure the quality of mathematical education in the future [26].

Mathematical modeling is considered by us as a strategy for preparing students for solving poorly structured mathematical problems, necessary for the development of critical skills of the 21st century and a productive attitude towards problem setting and problem solving [18].

At present, two types of mathematical modeling are widely used in practice: analytical and simulation ones. In analytical modeling, mathematical (abstract) models of a real object are studied in the form of algebraic, differential and other equations, as well as those presupposing the use of an unambiguous computational procedure leading to their exact solution [16]. The traditional approach to teaching mathematics to engineering students involves the use of analytical mathematical modeling as one of teaching methods. So, when teaching mathematics using the activity-based approach, the goals of teaching higher mathematics include mastering mathematical modeling methods. To achieve these goals, special teaching aids (systems of tasks, teaching aids) are being developed. They contain systems of professionally oriented tasks, the solution of which requires the use of the mathematical modeling method [20, 31].

It is also proposed to apply the method of mathematical modeling in combination with mathematical software for calculations, such as Maple, Derive, MathCad, Mathematica, MatLAB, which make it possible not only to carry out calculations, but to solve the most complex engineering problems. However, these software tools cannot be used to create simulation models for teaching mathematics [4–6]. When doing the described laboratory work, students can use computer mathematics systems to solve a system of linear algebraic equations in case of its high dimension. At the same time, this does not exclude the use of simulation models in virtual laboratory work.

Unlike other researchers [8, 32], who suggest using such simulation environments, as Arena, Extend, MicroSaint, Enterprise Dynamics, etc., we believe that the AnyLogic environment is the most convenient for teaching mathematics to future engineers, the advantage of which is the ability to present simulation models in 2D and 3D, which makes any ideas and concepts more visual. In contrast to table-based analytics or linear optimization, modeling provides the ability to observe the behavior of a real system over time with the required level of detail. For example, it is possible to check the occupancy level of a warehouse on a given date.

The effect of using simulation modeling in teaching mathematics may be enhanced with the help of heuristic methods of teaching. These methods are used to teach students how to build search strategies when solving engineering problems. For example, in the described laboratory work, using the heuristic dialogue method, the teacher can lead students to building a solution algorithm, designing their own tables for presenting and structuring the values obtained during the solution, finding alternative methods for calculating the probabilities of the considered events. To do this, the teacher can use hints of various levels (hard, algorithmic, soft guidance), heuristic constructions [24], and organize design heuristic activities [33].

Further research should be aimed at measuring digital and mathematical competencies [19, 22] in engineering students, which they master by performing virtual laboratory works.

Conclusions

Thus, the introduction of analytical and simulation mathematical modeling methods in teaching engineering students makes it possible to achieve the following didactic goals:

- acquisition of fundamental mathematical knowledge necessary in engineering for mathematical description of technical objects and processes;
- forming the competence of using mathematical apparatus for professional problems solution;
- mastering digital competences of using software packages to carry out calculations when solving problems;
- forming the simulation modeling techniques to study complex processes and phenom-ena in real time;
- developing engineering students' creative thinking, mathematical style of thinking, as well as fostering their interest to mathematical models studying.

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Интеграция математического и компьютерного имитационного моделирования в инженерном образовании

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Аннотация. Развитие современной техники, технологий, науки и образования невозможно без использования систем компьютерной математики. Современный инженер должен владеть в равной мере как методами математического моделирования технических объектов и процессов с использованием теоретических моделей, так и методами имитационного моделирования в системах компьютерной математики. На примере лабораторной работы реализации в производственном процессе математической модели в виде марковского процесса, построенного на основе уравнений Колмогорова, в статье раскрывается интеграция математического и компьютерного моделирования. Студентам предлагается реализовать теоретическое решение модели для конкретных числовых данных и осуществить компьютерное моделирование процесса поиска предельных вероятностей состояний системы с помощью авторского виртуального лабораторного комплекса, созданного на базе платформы для имитационного моделирования бизнес-систем AnyLogic. Такая платформа предлагает три подхода к моделированию: дискретно-событийный; агентный; системной динамики. Авторский лабораторный комплекс позволяет в режиме реального времени моделировать такие параметры производственного процесса, как загруженность рабочих мест, время простоя оборудования, расположение агентов на аттракторах и др. В статье демонстрируется реализация возможности визуализации производственного процесса в 2D- и 3D-формате, что значительно упрощает управление и процесс принятия решения.

Ключевые слова: математическое моделирование, компьютерное имитационное моделирование, инженерное образование, марковский процесс.