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New Structure of Paracompact Spaces

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Abstract. In this paper author introduce and study the new class of paracompact spaces called g$^*\omega\alpha$-paracompact spaces as a generalization of paracompact spaces. Authors characterize g$^*\omega\alpha$-paracompact spaces and study their some of their basic properties.

Keywords: g$^*\omega\alpha$-closed sets, g$^*\omega\alpha$-locally finite collection, g$^*\omega\alpha$-paracompact, g$^*\omega\alpha$-expandable spaces.

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Introduction

A space P is said to be Paracompact if every open cover of a P has a locally finite open refinement. Al-Zoubi [1] defined S-Paracompact spaces using semi open sets which are generalization of paracompact spaces and obtained many interesting properties of S-paracompact spaces in 2006. P_1-paracompact and P_2-paracompact spaces were defined by Mashhour et al. [6].

In this paper, we introduce a new class of g$^*\omega\alpha$-paracompact spaces, characterized by the condition that every open cover of a space P has a $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-refinement. Also, we define and investigate the properties of $g^*\omega\alpha$-locally finite collections. Further, studied, $g^*\omega\alpha$-paracompact spaces and investigated their properties. Finally $g^*\omega\alpha$-expandable spaces are defined by using $g^*\omega\alpha$-open sets and $g^*\omega\alpha$-locally finite collections.

1. Preliminary

Definition 1.1 ([7]). *Let $A_1 \subset P$. Then A_1 is called*
(i) $g^*\omega\alpha$-closed if $cl(A_1) \subseteq U_1$ whenever $A_1 \subseteq U_1$ and U_1 is $\omega\alpha$-open in P.

Definition 1.2 ([3]). *A space P is said to be submaximal if each dense subset of P is open in P.*

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Definition 1.3 ([4]). *A collection $F_1 = \{F_\alpha : \alpha \in I\}$ of subsets of a space P is locally finite, if for each $p \in P$, there exists $U_{1_p} \in O(P,p)$ such that U_{1_p} intersects at most finitely many members of F_1.*

Theorem 1.4 ([3]). *If $\{ U_{1_\alpha} : \alpha \in I \}$ is a locally finite family of P and $V_{1_\alpha} \subseteq U_{1_\alpha}$ for each $\alpha \in I$ then the family $\{ V_{1_\alpha} : \alpha \in I \}$ is locally finite in P.*

Lemma 1.5 ([2]). *The union of locally finite family of locally finite collection of sets in a space is locally finite family of a sets.*

Lemma 1.6 ([11]). *If $\{ U_{1_\alpha} : \alpha \in I \}$ is a locally finite family of a space P then $\{cl(U_{1_\alpha}) : \alpha \in I\}$ is also a locally finite family of subsets of a space P.*

Definition 1.7 ([5]). *A space P is expandable if for every locally finite collection $F_1 = \{F_{1_\alpha} : \alpha \in I \}$ of subsets of P, there exists locally finite collection $G_1 = \{ G_{1_\alpha} : \alpha \in I \}$ of open subsets of P such that $F_{1_\alpha} \subseteq G_{1_\alpha}$ for each $\alpha \in I$.*

Lemma 1.8. *If $V_1 \in O(P)$ and $A_1 \in g^*\omega\alpha$-$O(P)$ then $V_1 \cap A_1 \in g^*\omega\alpha$-$O(P)$.*

Theorem 1.9 ([7]). *The following conditions are equivalent for a space P:*
(i) P is submaximal.
(ii) $g^*\omega\alpha$-$cl(A_1) = A_1$, where $A_1 \subset P$.

Theorem 1.10 ([8]). *A function $s : P \to Q$ is called*
(i) $g^*\omega\alpha$-irresolute if for each $V_1 \in G^*\omega\alpha C(P)$, $s^{-1}(V_1) \in G^*\omega\alpha C(Q)$.
(ii) pre $g^*\omega\alpha$-closed if for each $V_1 \in G^*\omega\alpha C(P)$, $s(V_1) \in G^*\omega\alpha C(Q)$.

Definition 1.11 ([10]). *A space P is said to be $g^*\omega\alpha$-compact if for every cover $\{V_{1_\alpha} : \alpha \in \lambda\}$ of P by $g^*\omega\alpha$-open sets, there exists a finite subset λ_0 of λ such that $P = \cup\{V_{1_\alpha} : \alpha \in \lambda_0\}$.*

Example 1.12. Let $P = \{p_1, p_2, p_3\}$ and $\tau = \{P, \phi, \{p_1\}, \{p_2\}, \{p_1, p_2\}, \{p_1, p_3\}\}$. Here every cover of $g^*\omega\alpha$-open sets can be expressed as a finite subcover and so (P, τ) is $g^*\omega\alpha$-compact.

2. g$^*\omega\alpha$-locally finite collection

Definition 2.1. *A collection $\xi = \{ F_{1_\alpha} : \alpha \in I \}$ of subsets of P is said to be $g^*\omega\alpha$-locally finite if for each $p \in P$, there exists $U_1 \in g^*\omega\alpha$-$O(P,p)$ and U_1 intersects F_{1_α} at most finitely many values of α.*

Remark 2.2. *Every locally finite collection is $g^*\omega\alpha$-locally finite.*

Remark 2.3. *In a submaximal space, every $g^*\omega\alpha$-locally finite collection of P is locally finite. However the converse need not be true follows from the example.*

Example 2.4. Let $P = \{p_1, p_2, p_3\}$ and $\tau = \{P, \phi, \{p_1\}, \{p_1, p_2\}\}$. Then P is not submaximal, since the set $A_1 = \{p_1, p_3\}$ is dense in P but not open. But, P is $g^*\omega\alpha$-locally finite collection of P.

Lemma 2.5. *The following properties holds for a collection $\xi = \{F_{1_\alpha} : \alpha \in I\}$:*
(a) if ξ is $g^*\omega\alpha$-locally finite collection and $G_{1_\alpha} \subset F_{1_\alpha}$ for each $\alpha \in I$, then $G_1 = \{G_{1_\alpha} : \alpha \in I\}$ is also $g^*\omega\alpha$-locally finite.
(b) ξ is $g^*\omega\alpha$-locally finite if and only if $\{g^*\omega\alpha\text{-}cl(F_{1_\alpha}) : \alpha \in I\}$ is also $g^*\omega\alpha$-locally finite.
(c) if ξ is $g^*\omega\alpha$-locally finite, then $\cup\, g^*\omega\alpha\text{-}cl(F_{1_\alpha}) = g^*\omega\alpha\text{-}cl(\cup F_{1_\alpha})$.

Proof. (a) Follows from Definition 2.1.
(b) Suppose ξ is g$^*\omega\alpha$-locally finite. Then for each $p \in P$, there exists $U_{1_p} \in g^*\omega\alpha\text{-}O(P,p)$ which meets finitely many of the sets F_{1_α}, say $F_{1_{\alpha 1}}$, $F_{1_{\alpha 2}}$, …, $F_{1_{\alpha n}}$. Since $F_{1_{\alpha K}} \subseteq g^*\omega\alpha\text{-cl}(F_{1_{\alpha K}})$ for each k = 1, 2, …, n. Thus, U_{1_p} meets g$^*\omega\alpha$-cl$(F_{1_{\alpha 1}})$, g$^*\omega\alpha$-cl$(F_{1_{\alpha 2}})$, …, g$^*\omega\alpha$-cl$(F_{1_{\alpha k}})$, that is U_{1_p} meets finitely many values of g$^*\omega\alpha$-cl(F_{1_α}). Therefore, { g$^*\omega\alpha$-cl(F_{1_α}) : $\alpha \in$ I } is g$^*\omega\alpha$-locally finite.

Conversely, let $p \in P$. Then, there exists $U_{1_p} \in g^*\omega\alpha\text{-}O(P,p)$ which meets finitely many of the sets $g^*\omega\alpha\text{-}cl(F_{1_\alpha})$, say $g^*\omega\alpha\text{-}cl(F_{1_{\alpha 1}})$, $g^*\omega\alpha\text{-}cl(F_{1_{\alpha 2}})$, …, g$^*\omega\alpha$-$cl(F_{1_{\alpha n}})$, implies $U_{1_p} \cap$ g$^*\omega\alpha$-$cl(F_{1_{\alpha k}}) \neq \phi$ for each k = 1, 2, …, n.

Let $q \in U_{1_p}$ and $q \in g^*\omega\alpha$-cl$(F_{1_{\alpha k}})$,then for every $V_{1_q} \in g^*\omega\alpha\text{-}O(P,q)$ such that $V_{1_q} \cap F_{1_{\lambda k}} \neq \phi$. But $U_{1_p} \in g^*\omega\alpha\text{-}O(P,q)$, so $U_{1_p} \cap F_{1_{\lambda k}} \neq \phi$ for each k = 1, 2, …, n. Hence ξ is g$^*\omega\alpha$-locally finite.
(c) Suppose ξ is $g^*\omega\alpha$-locally finite, then $\cup g^*\omega\alpha\text{-}cl(F_{1_\alpha}) \subseteq g^*\omega\alpha\text{-}cl(\cup F_{1_\alpha})$. On the other hand, let $p \in g^*\omega\alpha\text{-}cl(\cup F_{1_\alpha})$. Then every $V_{1_p} \in g^*\omega\alpha\text{-}O(P,p)$ such that $V_{1_p} \cap (\cup F_{1_\alpha}) \neq \phi$. Then there exists $U_{1_p} \in g^*\omega\alpha\text{-}O(P,p)$ which meets finitely many values of the sets F_{1_α}, that is $F_{1_{\alpha 1}}$, $F_{1_{\alpha 2}}$, …, $F_{1_{\alpha n}}$. Thus, for every $V_{1_p} \in g^*\omega\alpha\text{-}O(P,p)$, such that $V_{1_p} \cap (\cup F_{1_{\alpha k}}) \neq \phi$ for each k = 1, 2, …, n, that is $p \in g^*\omega\alpha\text{-}cl(\cup F_{1_{\alpha k}}) = \cup g^*\omega\alpha\text{-}cl(F_{1_{\alpha k}})$. Thus, there exists h_1 such that $p \in g^*\omega\alpha$-cl$(F_{1_{\alpha h_1}})$. Thus, $p \in \cup g^*\omega\alpha\text{-}cl(F_{1_\alpha})$ and hence $\cup g^*\omega\alpha\text{-}cl(F_{1_\alpha}) \subseteq g^*\omega\alpha$-cl$(\cup F_{1_\alpha})$. □

3. g$^*\omega\alpha$-paracompact spaces

We recall that a space P is said to be paracompact if every open cover of P has a locally finite open refinement.

Definition 3.1. *A space P is said to be $g^*\omega\alpha$-paracompact if every open cover of P has a $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-refinement.*

Example 3.2. Let $P = \{p_1, p_2, p_3\}$ and $\tau = \{P, \phi, \{p_1\}, \{p_2, p_3\}\}$. Here, every open cover of P has $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-refinement. Hence P is $g^*\omega\alpha$-paracompact.

Theorem 3.3. *Every paracompact space is $g^*\omega\alpha$-paracompact.*

Proof. It follows from Remark 2.2. □

Example 3.4. Let us consider a space P with $P = N^+ \cup N^-$ where N^+ is the set of all positive integers and N^- is the set of all negative integers.
Consider, the topology $\tau = \{U_1 \subseteq P : N \subseteq U_1\} \cup \{\phi\}$. Here, $G^*\omega\alpha O(P) = \{A_1 \subseteq P : A_1 \cap N \neq \phi\}$.
Consider, $\{\{p\} : p \in N\} \cup \{\{p, -p\} : p \in N\}$ is a $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-open covers of P. Hence, every open cover of P has a $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-open refinement. Hence (P, τ) is $g^*\omega\alpha$-paracompact.
However (P, τ) is not paracompact. Since the collection $\{N \cup \{p\} : p \in N\}$ which is an open cover of P, but this collection admits no locally finite open refinement.
Hence (P, τ) is not paracompact.

Remark 3.5. *Every compact space is $g^*\omega\alpha$-paracompact.*

Lemma 3.6. *Let $s : P \to Q$ be surjective. Then s is pre $g^*\omega\alpha$-closed if and only if for every $q \in Q$ and every $U_1 \in G^*\omega\alpha\text{-}O(P, s^{-1}(q))$, there exists $V_1 \in G^*\omega\alpha\text{-}O(Q,q)$ such that $s^{-1}(V_1) \subset U_1$.*

Proof. Necessity: Let s be pre g$^*\omega\alpha$-closed. Let $q \in Q$ and $U_1 \in G^*\omega\alpha\text{-}O(P)$ with $s^{-1}(q) \subset U_1$. Since $U_1 \in g^*\omega\alpha\text{-}O(P)$, then $P \setminus U_1 \in g^*\omega\alpha\text{-}C(P)$. As s is pre $g^*\omega\alpha$-closed, then $s(P \setminus U_1) \in g^*\omega\alpha\text{-}C(Q)$. Take $V_1 = Q \setminus s(P \setminus U_1)$. Then $V_1 \in g^*\omega\alpha\text{-}O(Q)$ with $q \in V_1$ and $s^{-1}(V_1) \subset U_1$.

Sufficient: Let $K_1 \in G^*\omega\alpha\text{-}C(P)$ and $q \in Q \setminus s(K_1)$. Then, $s^{-1}(q) \subset P \setminus K_1$. From hypothesis, there exists $V_{1_q} \in g^*\omega\alpha\text{-}O(Q,q)$ such that $s^{-1}(V_{1_1}) \subset Q \setminus s(K_1)$. Therefore, $q \in V_{1_q} \subset Q \setminus s(K_1)$. Thus, $Q \setminus s(K_1) = \cup\{\ V_{1_q}\ :\ \text{q} \in Q \setminus s(K_1)\ \}$. Thus $Q \setminus s(K_1) \in g^*\omega\alpha\text{-O(Q)}$ and so $s(K_1) \in g^*\omega\alpha\text{-}C(Q)$. □

Theorem 3.7. *Let $s : P \to Q$ be continuous open and pre $g^*\omega\alpha$-closed surjection with $s^{-1}(q)$ is $g^*\omega\alpha$-compact for each $q \in Q$. If P is $g^*\omega\alpha$-paracompact, then Q is also $g^*\omega\alpha$-paracompact.*

Proof. Let $U_1 = \{U_{1_\alpha} : \alpha \in I\}$ be an open cover of Q. As s is continuous, $s^{-1}(U_1) = \{s^{-1}(U_{1_\alpha}) : \alpha \in I\}$ is an open cover of the $g^*\omega\alpha$-paracompact space P, so it has a $g^*\omega\alpha$-open refinement say $V_1 = \{V_{1_\alpha} : \alpha \in I\}$. As s is pre $g^*\omega\alpha$-closed, then the collection $s(V_1) = \{s(V_{1_\alpha}) : \alpha \in I\}$ is a $g^*\omega\alpha$-open refinement of U_1. Now, we have to prove that $s(V_1)$ is $g^*\omega\alpha$-locally finite in Q.

Let $q \in Q$, then for each $q \in s^{-1}(q)$ there exists $U_{1_p} \in g^*\omega\alpha\text{-}O(P,p)$ such that U_{1_p} intersects at most finitely many members of V_1. The collection $\{U_{1_p} : p \in s^{-1}(q)\}$ is $g^*\omega\alpha$-open cover of $s^{-1}(q)$. Therefore, there exists a finite subset K_1 of $s^{-1}(q)$ with $s^{-1}(q) \subset U_{1_p},\ p \in K_1$. As s is pre $g^*\omega\alpha$-closed and by Lemma 3.6, there exists $P_{1_q} \in g^*\omega\alpha\text{-}O(Q,q)$ such that $s^{-1}(P_{1_q}) \subset \cup U_{1_p}$, $p \in K_1$. Then $s^{-1}(P_{1_q})$ intersects at most finitely many members of V_1. Thus, P_{1_q} intersects at most finitely many members of $s(V_1)$ and so $s(V_1)$ is $g^*\omega\alpha$-locally finite in Q. Hence Q is $g^*\omega\alpha$-paracompact. □

Theorem 3.8. *Let $s : P \to Q$ be $g^*\omega\alpha$-irresolute closed surjective function with $s^{-1}(q)$ is compact for each $q \in Q$. If Q is $g^*\omega\alpha$-paracompact then P is $g^*\omega\alpha$-paracompact.*

Proof. Let $U_1 = \{U_{1_\alpha} : \alpha \in I\}$ be an open cover of P. As $s^{-1}(q)$ is compact, there exists a finite subset I_0 of I such that $s^{-1}(q) \subseteq \cup U_{1_\alpha},\ \alpha \in I_0$. As s is a closed, there exists $V_{1_q} \in O(Q,q)$ with $s^{-1}(V_{1_q}) \subseteq \cup U_{1_\alpha},\ \alpha \in I_0$. Therefore $\{V_{1_q} : q \in Q\}$ is an open cover of the $g^*\omega\alpha$-paracompact space Q. Then V_{1_q} has $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-refinement say $W_1 = \{W_{1_\beta} : \beta \in B\}$. As s is $g^*\omega\alpha$-irresolute, $\{s^{-1}(W_{1_\beta}) : \beta \in B\}$ is $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-open cover of P. Then, for each $\beta \in B$, there exists $q(\beta) \in Q$ such that $W_{1_\beta} \subseteq V_{1_{q(\beta)}}$. Thus $s^{-1}(W_{1_\beta}) \subseteq s^{-1}(V_{1_{q(\beta)}}) \subseteq \cup U_{1_\alpha} : \alpha \in I_{0(q(\beta))} = F_{1_{q(\beta)}}$. Let $F_1 = \{s^{-1}(W_{1_\beta}) \cap U_{1_\alpha} : \beta \in B \text{ and } \alpha \in I(q(\beta))\}$. From Lemma 1.8, F_1 is $g^*\omega\alpha$-open subset of P. Then the family F_1 is $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-refinement of U_1. Thus P is $g^*\omega\alpha$-paracompact. □

Definition 3.9 ([9]). *A space P is $g^*\omega\alpha$-regular if for each $F_1 \in g^*\omega\alpha\text{-}C(P)$ and each point $q \notin F_1$, there exist disjoint $U_1, V_1 \in O(P)$ with $p \in U_1$ and $F_1 \subseteq V_1$.*

Theorem 3.10. *Every $g^*\omega\alpha$-regular submaximal space is regular.*

Proof. Let P be $g^*\omega\alpha$-regular submaximal and $U_1 \in O(P,p)$. Since P is $g^*\omega\alpha$-regular, then for each $p \in U_1$, there exists $V \in g^*\omega\alpha\text{-}O(P)$ such that $p \in V_1 \subseteq g^*\omega\alpha\text{-}cl(V_1) \subseteq U_1$. As P is submaximal, every $g^*\omega\alpha$-closed set is closed [7], that is $cl(V_1) \subseteq g^*\omega\alpha\text{-}cl(V_1)$. Thus $p \in V_1 \subseteq cl(V_1) \subseteq U_1$ and so P is regular space. □

Theorem 3.11. *Every $g^*\omega\alpha$-paracompact T_2-space is $g^*\omega\alpha$-regular.*

Proof. Let $A_1 \in C(P)$ and $p \notin A_1$. Then, for each $q \in A_1$, choose $U_q \in O(Q,q)$ and $x \notin cl(U_q)$. Thus the family $V_1 = \{U_q : q \in A_1\} \cup \{P \setminus A_1\}$ is an open cover of P. Since, P is $g^*\omega\alpha$-paracompact, V_1 has $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-refinement say H_1. Put $V_2 = \cup\{h \in H_1$ and $h \cap A_1 \neq \phi\}$. Then V_2 is $g^*\omega\alpha$-open set containing A_1 and $g^*\omega\alpha\text{-}cl(V_2) = \cup\{g^*\omega\alpha - cl(h) : h \in H_1$ and $h \cap A_1 \neq \phi\}$ follows from Lemma 2.5(c). Therefore, $U_1 = P \setminus g^*\omega\alpha\text{-}cl(V_2)$ is g$^*\omega\alpha$-open set containing p such that $U_1 \cap V_2 = \phi$. Thus P is $g^*\omega\alpha$-regular. □

Theorem 3.12. *Let P be a regular space. Then M is $g^*\omega\alpha$-paracompact if and only if every open cover V_1 of P has a $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-closed refinement U_1.*

Proof. Necessity: Let V_1 be an open cover of M. Then, for each $p \in P$, choose $U_{1_p} \in V_1$. As P is regular, there exists $V_{1_p} \in O(P)$ such that $p \in V_{1_p} \subseteq cl(V_{1_p}) \subseteq U_{1_p}$. Thus $V_1 = \{V_{1_p} : p \in P\}$ is an open cover of P. Then P has a $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-closed refinement say $\hbar = \{h_\alpha : \alpha \in \mathrm{B}\}$. Let $g^*\omega\alpha\text{-}cl(\hbar) = \{g^*\omega\alpha\text{-}cl(h_\alpha) : \alpha \in B\}$, then $g^*\omega\alpha\text{-}cl(\hbar)$ is $g^*\omega\alpha$-locally finite collection follows from Lemma 2.5(c). Thus for each $\alpha \in B$, $g^*\omega\alpha\text{-}cl(h_\alpha) \subseteq g^*\omega\alpha\text{-}cl(V_{1_p}) \subseteq cl(V_{1_p}) \subseteq U_{1_p}$, that is $g^*\omega\alpha\text{-}cl(\hbar)$ is a $g^*\omega\alpha$-refinement of V_1.
Sufficiency: Let V_1 be an open cover of P and U_1 be $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-closed refinement of V_1. For each $p \in P$, choose $W_{1_p} \in G^*\omega\alpha\text{-}O(P)$ such that $p \in W_{1_p}$ and W_{1_p} intersects at most finitely many members U_1. Let H_1 be a $g^*\omega\alpha$-closed $g^*\omega\alpha$-locally finite refinement of $W_1 = \{W_{1_p} : p \in P\}$. Then, for each $v \in U_1$, $V_1^1 = P - \{h \in H_1 : h \cap V_1 = \phi\}$ and so V_1^1 is $g^*\omega\alpha$-open, that is $\{V_1^1 : V_1 \in U_1\}$ is $g^*\omega\alpha$-open cover of P. Finally, for each $V_1 \in U_1$, choose $U_{1_v} \in U_1$ such that $V_1 \subseteq U_{1_v}$. Then, the collection $\{U_{1_v} \cap V_1^1 : v \in U_1\}$ is $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-open refinement of V_1 follows from Lemma 1.8. Thus P is $g^*\omega\alpha$-paracompact. □

4. g$^*\omega\alpha$-expandable spaces in topological spaces

Definition 4.1. *A space P is $g^*\omega\alpha$-expandable if for every locally finite collection $F_1 = \{F_{1_\alpha} : \alpha \in I\} \subset P$, then there exists $g^*\omega\alpha$-locally finite collection $G_1 = \{G_{1_\alpha} : \alpha \in I\}$ of $g^*\omega\alpha$-open subsets of P with $F_{1_\alpha} \subseteq G_{1_\alpha}$ for each $\alpha \in I$.*

Example 4.2. From the Example 3.4, we can observe that the space (P, τ) is $g^*\omega\alpha$-expandable space.

Theorem 4.3. *The following conditions are equivalent for a space P:*
(i) P is $g^*\omega\alpha$-expandable.
(ii) Every locally finite collection $F_1 = \{ F_{1_\alpha} : \alpha \in I \}$ of closed subsets of P, there exists a $g^*\omega\alpha$-locally finite collection $G_1 = \{ G_{1_\alpha} : \alpha \in I \}$ of $g^*\omega\alpha$-open subsets such that $F_{1_\alpha} \subseteq G_{1_\alpha}$ for each $\alpha \in I$.

Proof. (i) → (ii) Follows from the Definition 4.1.
(ii) → (i) Let $F_1 = \{F_{1_\alpha} : \alpha \in I\}$ be a locally finite collection of a space P. From Lemma 1.6, $\{cl(F_{1_\alpha}) : \alpha \in I\}$ is also locally finite collection. From hypothesis, there exists $g^*\omega\alpha$-locally finite collection $G_1 = \{G_{1_\alpha} : \alpha\ inI\}$ of $g^*\omega\alpha$-open subsets of P with $cl(F_{1_\alpha}) \subseteq G_{1_\alpha}$. But, $F_{1_\alpha} \subseteq cl(F_{1_\alpha}) \subseteq G_{1_\alpha}$, that is $F_{1_\alpha} \subseteq G_{1_\alpha}$. Thus P is $g^*\omega\alpha$-expandable. □

Theorem 4.4. *Every $g^*\omega\alpha$-paracompact space is $g^*\omega\alpha$-expandable.*

Proof. Let P be a $g^*\omega\alpha$-paracompact space and $F_1 = \{F_{1_\alpha} : \alpha \in I\}$ be a locally finite collection of closed subsets P. Let $\top$ be a collection of all finite subsets of I. Then, for each $\beta \in \top$, let $V_{1_\beta} = P \setminus \cup\{F_{1_\alpha} : \alpha \neq \beta\}$. As F_1 is locally finite, V_{1_β} is open and V_{1_β} meets only finitely many members of F_1. Let $\vartheta = \{V_{1_\beta} : \beta \in \top\}$, then ϑ is an open cover of P. As P is $g^*\omega\alpha$-paracompact, ϑ has a $g^*\omega\alpha$-locally finite $g^*\omega\alpha$-refinement, say $\omega = \{W_{1_\delta} : \delta \in \Delta\}$. Let $U_{1_\alpha} = \cup\{W_{1_\delta} \in \omega : W_{1_\delta} \cap F_{1_\alpha} \neq \phi\}$. Hence U_{1_α} is $g^*\omega\alpha$-open and so $F_{1_\alpha} \subseteq U_{1_\alpha}$.

Now, to show that $\{U_{1_\alpha} : \alpha \in I\}$ is $g^*\omega\alpha$-locally finite. Since ω is locally finite, then for each $p \in P$, there exists $U_{1_p} \in g^*\omega\alpha\text{-}O(P, p)$ with U_{1_p} intersects at most finitely many members of ω. Also $U_{1_{px}} \cap U_{1_\alpha} \neq \phi$ if and only if $U_{1_p} \cap W_{1_\delta} \neq \phi$ and $W_{1_\delta} \cap F_{1_\alpha} \neq \phi$ for some $\delta \in \Delta$. Again, ω is a refinement of ϑ, then there exists a member V_{1_β} of ϑ containing W_{1_δ} of ω. Then W_{1_δ} meets only finitely many members of F_{1_α} for each $\alpha \in I$. Thus $\{U_{1_\alpha} : \alpha \in I\}$ is $g^*\omega\alpha$-locally finite and so P is $g^*\omega\alpha$-expandable. □

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Новая структура паракомпактных пространств

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Аннотация. В данной статье вводится и изучается новый класс паракомпактных пространств, называемых g$^*\omega\alpha$-паракомпактными пространствами, как обобщение паракомпактных пространств. Авторы характеризуют g$^*\omega\alpha$-паракомпактные пространства и изучают некоторые их основные свойства.

Ключевые слова: g$^*\omega\alpha$-замкнутые множества, g$^*\omega\alpha$-локально конечный набор, g$^*\omega\alpha$-паракомпакт, g$^*\omega\alpha$-расширяемые пространства.