Journal of Siberian Federal University. Mathematics & Physics 2022, 15(3), 319–328

DOI: 10.17516/1997-1397-2022-15-3-319-328
УДК 517.9

A Further Generalization of the Reverse Minkowski Type Inequality via Hölder and Jensen Inequalities

Bouharket Benaissa*
Laboratory of Informatic anf Mathematics
Faculty of Material Science
University of Tiaret
Algeria, Algeria

Received 10.11.2021, received in revised form 25.01.2022, accepted 09.03.2022

Abstract. The main objective of this article is to establish new generalizations of the reverse Minkowski's integral inequalities by introducing weighted functions and two integrability parameters. Two new theorems will be proved using Jensen's integral inequality and Hölder's two-parameter inequality, some reverse Minkowski type Integral inequalities are also obtained.

Keywords: convex function, Hölder inequality, Minkowski inequality, Jensen inequality.

Citation: B.Benaissa, A further generalization of the reverse Minkowski Type Inequality via Hölder and Jensen Inequalities, J. Sib. Fed. Univ. Math. Phys., 2022, 15(3), 319–328.
DOI: 10.17516/1997-1397-2022-15-3-319-328.

Introduction

In recent years, many researchers have paid great attention to generalizations, extensions, and variations of Minkowski's inverse inequalities (see [1–7]). On the other hand, the convex functions have a very useful structure in terms of properties and play an important role in inequality theory, this class of functions has many applications in different branches of mathematics (functional analysis, numerical computation, probability theory, etc.). Many inequalities and results are obtained by the Jensen inequality, and many articles relating to different versions of this inequality have been found in the literature.

In this work, we will establish two results on the reverse Minkowski type integral inequalities, the first one involving Hölder inequality with two parameters, Also, we will investigate a second result via the Jensen integral inequality (convex function). Special cases will be given as generalizations to some known results.

1. Model inequalities

The following inequality is well known in the literature as Minkowski's inequality, it states that, for$p \geqslant 1$, if

$$0 < \int_a^b f^p(x)dx < \infty \quad \text{and} \quad 0 < \int_a^b g^p(x)dx < \infty,$$

*bouharket.benaissa@univ-tiaret.dz https://orcid.org/0000-0002-1195-6169

then

$$\left(\int_a^b (f(x)+g(x))^p dx\right)^{\frac{1}{p}} \leqslant \left(\int_a^b f^p(x)dx\right)^{\frac{1}{p}} + \left(\int_a^b g^p(x)dx\right)^{\frac{1}{p}}.$$

In this section, we give some recent results about the reverse Minkowski's inequality. Sulaiman [2] presented the following result related to the reverse Minkowski's inequality: for any $f, g > 0$, if $p \geqslant 1$ and

$$1 < m \leqslant \frac{f(x)}{g(x)} \leqslant M$$

for all $x \in [a, b]$, then

$$\begin{aligned} \frac{M+1}{M-1}\left(\int_a^b (f(x)-g(x))^p dx\right)^{\frac{1}{p}} &\leqslant \left(\int_a^b f^p(x)dx\right)^{\frac{1}{p}} + \left(\int_a^b g^p(x)dx\right)^{\frac{1}{p}} \leqslant \\ &\leqslant \frac{m+1}{m-1}\left(\int_a^b (f(x)-g(x))^p dx\right)^{\frac{1}{p}}. \end{aligned} \tag{1}$$

Banyat Sroysang in [3] proved a significant extension of the above inequality as follows: for any $f, g > 0$, if $p \geqslant 1$ and

$$0 < c < m \leqslant \frac{f(x)}{g(x)} \leqslant M$$

for all $x \in [a, b]$, then

$$\begin{aligned} \frac{M+1}{M-c}\left(\int_a^b (f(x)-cg(x))^p dx\right)^{\frac{1}{p}} &\leqslant \left(\int_a^b f^p(x)dx\right)^{\frac{1}{p}} + \left(\int_a^b g^p(x)dx\right)^{\frac{1}{p}} \leqslant \\ &\leqslant \frac{m+1}{m-c}\left(\int_a^b (f(x)-cg(x))^p dx\right)^{\frac{1}{p}}. \end{aligned} \tag{2}$$

Benaissa in [1] gave a new result to the inverse Minkowski inequality according to the following formula: For any $f, g > 0$, $\alpha > 0$, if $p \geqslant 1$ and

$$0 < c < m \leqslant \frac{\alpha f(x)}{g(x)} \leqslant M$$

for all $x \in [a, b]$, then

$$\begin{aligned} \frac{M+\alpha}{\alpha(M-c)}\left(\int_a^b (\alpha f(x)-cg(x))^p dx\right)^{\frac{1}{p}} &\leqslant \left(\int_a^b f^p(x)dx\right)^{\frac{1}{p}} + \left(\int_a^b g^p(x)dx\right)^{\frac{1}{p}} \leqslant \\ &\leqslant \frac{m+\alpha}{\alpha(m-c)}\left(\int_a^b (\alpha f(x)-cg(x))^p dx\right)^{\frac{1}{p}}. \end{aligned} \tag{3}$$

2. Main results

Motivated by the above Theorems, we give a further improvement of the reverse Minkowski Type inequality by introducing weight function and two parameters $p, q > 0$. Throughout this section, the functions f, g are measurable and non-negative on interval $(a,\, b)$, and w is weight function (measurable and positive) on $(a,\, b)$. In order to demonstrate our main results, we need the following Lemma:

Lemma 1. *Let* $0 < p \leqslant q < \infty$ *and* f, w *be non-negative measurable functions on* (a, b) *and suppose that* $0 < \int\limits_a^b f^q(t)w(t)dt < \infty$, *then*

$$\int_a^b f^p(t)w(t)dt \leqslant \left(\int_a^b w(t)dt\right)^{\frac{q-p}{q}} \left(\int_a^b f^q(t)w(t)dt\right)^{\frac{p}{q}}. \tag{4}$$

The inequality (4) hold for $-\infty < q \leqslant p < 0$ *and inverted for* $0 < q \leqslant p < \infty$.

Proof. Using Hölder inequality for using the parameter $\frac{q}{p} \geqslant 1$, we have

$$\begin{aligned} \int_a^b f^p(t)w(t)dt &= \int_a^b \left(w^{\frac{q-p}{q}}(t)\right)\left(f^p(t)w^{\frac{p}{q}}(t)\right)dt \leqslant \\ &\leqslant \left(\int_a^b w(t)dt\right)^{\frac{q-p}{q}} \left(\int_a^b f^q(t)w(t)dt\right)^{\frac{p}{q}}. \end{aligned}$$

Jensen's integral inequality
Let f be an integrable function defined on (a, b) and let $\phi : (a, b) \longrightarrow \mathbb{R}$ be a convex function. If $\phi \circ f \in L(a, b)$, then

$$\phi\left(\frac{1}{b-a}\int_a^b f(t)dt\right) \leqslant \frac{1}{b-a}\left(\int_a^b \phi(f(t))dt\right), \tag{5}$$

the above inequality (5) is inverted if ϕ is a concave function.
Taking $\phi(t) = t^\lambda$, thus the formula (5) can be rewritten in the following forms.

- If $1 \leqslant \lambda$, then
$$\int_a^b f^\lambda(t)dt \geqslant (b-a)^{1-\lambda}\left(\int_a^b f(t)dt\right)^\lambda, \tag{6}$$

- if $0 < \lambda < 1$, then
$$\int_a^b f^\lambda(t)dt \leqslant (b-a)^{1-\lambda}\left(\int_a^b f(t)dt\right)^\lambda. \tag{7}$$

Let $-\infty \leqslant a < b \leqslant +\infty$, for $p > 0$ we suppose that

$$0 < \int_a^b f^p(x)w(x)dx < \infty \quad and \quad 0 < \int_a^b g^p(x)w(x)dx < \infty,$$

and we denote by $L_p^w(a, b)$ the space of all Lebesgue measurable functions f on (a, b) for which

$$\|f\|_{L_p^w(a,b)} = \left(\int_a^b f^p(x)w(x)dx\right)^{\frac{1}{p}}.$$

Using the above lemmas, we give and prove the following theorems.

Theorem 1. *Let f, $g > 0$, $0 < p \leqslant q$, $\alpha > 0$, w be a weight function and*

$$0 < c < m \leqslant \frac{\alpha f(x)}{g(x)} \leqslant M \quad for\ all\ x \in [a,b], \tag{8}$$

then

$$\begin{aligned} \frac{M+\alpha}{\alpha(M-c)} (w(x))^{\frac{p-q}{pq}} \left(\int_a^b (\alpha f(x) - cg(x))^p w(x) dx\right)^{\frac{1}{p}} \leqslant \\ \leqslant \left(\int_a^b f^q(x) w(x) dx\right)^{\frac{1}{q}} + \left(\int_a^b g^q(x) w(x) dx\right)^{\frac{1}{q}} \leqslant \\ \leqslant \frac{m+\alpha}{\alpha(m-c)} \left(\int_a^b (\alpha f(x) - cg(x))^q w(x) dx\right)^{\frac{1}{q}}. \end{aligned} \tag{9}$$

Proof. From the hypothesi (8) we get

$$0 < \frac{1}{c} - \frac{1}{m} \leqslant \frac{1}{c} - \frac{g(x)}{\alpha f(x)} \leqslant \frac{1}{c} - \frac{1}{M}$$

then

$$\frac{M}{M-c} \leqslant \frac{\alpha f(x)}{\alpha f(x) - c\, g(x)} \leqslant \frac{m}{m-c}, \tag{10}$$

let $0 < p \leqslant q$, from the inequality (10) we have

$$\left[\frac{M}{\alpha(M-c)} (\alpha f(x) - c\, g(x))\right]^p w(x) \leqslant f^p(x) w(x),$$

and

$$f^q(x) w(x) \leqslant \left[\frac{m}{\alpha(m-c)} (\alpha f(x) - c\, g(x))\right]^q w(x).$$

Integrating the above inequalities on $[a,b]$, we get

$$\frac{M}{\alpha(M-c)} \left(\int_a^b (\alpha f(x) - c\, g(x))^p w(x) dx\right)^{\frac{1}{p}} \leqslant \left(\int_a^b f^p(x) w(x) dx\right)^{\frac{1}{p}}, \tag{11}$$

and

$$\left(\int_a^b f^q(x) w(x) dx\right)^{\frac{1}{q}} \leqslant \frac{m}{\alpha(m-c)} \left(\int_a^b (\alpha f(x) - c\, g(x))^q w(x) dx\right)^{\frac{1}{q}}, \tag{12}$$

from the inequalities (11) and (4), we get

$$\begin{aligned} \frac{M}{\alpha(M-c)} \left(\int_a^b (\alpha f(x) - c\, g(x))^p w(x) dx\right)^{\frac{1}{p}} \leqslant \\ \leqslant \left(\int_a^b w(t) dt\right)^{\frac{q-p}{pq}} \left(\int_a^b f^q(t) w(t) dt\right)^{\frac{1}{q}}, \end{aligned}$$

this is same us

$$\frac{M}{\alpha(M-c)} \left(\int_a^b w(t) dt\right)^{\frac{p-q}{pq}} \left(\int_a^b (\alpha f(x) - c\, g(x))^p w(x) dx\right)^{\frac{1}{p}} \leqslant \left(\int_a^b f^q(x) w(x) dx\right)^{\frac{1}{q}}. \tag{13}$$

From the hypothesi (8), we deduce that

$$0 < m - c \leqslant \frac{\alpha f(x) - c\, g(x)}{g(x)} \leqslant M - c,$$

thus

$$\frac{\alpha f(x) - c\, g(x)}{M - c} \leqslant g(x) \leqslant \frac{\alpha f(x) - c\, g(x)}{m - c}, \tag{14}$$

let $0 < p \leqslant q$, from the inequality (14), we obtain

$$\left[\frac{1}{M - c}(\alpha f(x) - c\, g(x))\right]^p w(x) \leqslant g^p(x) w(x),$$

and

$$g^q(x) w(x) \leqslant \left[\frac{1}{m - c}(\alpha f(x) - c\, g(x))\right]^q w(x),$$

integrating on $[a, b]$, we get

$$\left(\int_a^b g^q(x) w(x) dx\right)^{\frac{1}{q}} \leqslant \frac{1}{m - c}\left(\int_a^b (\alpha f(x) - c\, g(x))^q w(x) dx\right)^{\frac{1}{q}}, \tag{15}$$

and

$$\frac{1}{M - c}\left(\int_a^b (\alpha f(x) - c\, g(x))^p w(x) dx\right)^{\frac{1}{p}} \leqslant \left(\int_a^b g^p(x) w(x) dx\right)^{\frac{1}{p}}, \tag{16}$$

using the inequality (16) and (4), we get

$$\frac{1}{M - c}\left(\int_a^b w(t) dt\right)^{\frac{p-q}{pq}} \left(\int_a^b (\alpha f(x) - c\, g(x))^p w(x) dx\right)^{\frac{1}{p}} \leqslant \left(\int_a^b g^q(x) w(x) dx\right)^{\frac{1}{q}}. \tag{17}$$

By the inequalities (12), (15) and (13), (17) we result the inequality (9). □

Now we present a new result involving Jensen integral inequality.

Theorem 2. *Let $f, g > 0$, $\alpha > 0$, w be a weight function and*

$$0 < c < m \leqslant \frac{\alpha f(x)}{g(x)} \leqslant M \quad \textit{for all } x \in [a, b], \tag{18}$$

then, for $1 < p \leqslant q$

$$\begin{aligned} &\frac{M + \alpha}{\alpha(M - c)}(b - a)^{\frac{1-p}{q}} \left(\int_a^b (\alpha f(x) - c\, g(x))^{\frac{q}{p}} w(x) dx\right)^{\frac{p}{q}} \leqslant \\ &\leqslant \left(\int_a^b f^q(x) w(x) dx\right)^{\frac{1}{q}} + \left(\int_a^b g^q(x) w(x) dx\right)^{\frac{1}{q}} \leqslant \\ &\leqslant \frac{m + \alpha}{\alpha(m - c)}\left(\int_a^b (\alpha f(x) - c g(x))^q w(x) dx\right)^{\frac{1}{q}}, \end{aligned} \tag{19}$$

for $0 < q \leqslant p \leqslant 1$

$$\begin{aligned} &\frac{M+\alpha}{\alpha(M-c)}\left(\int_a^b (\alpha f(x) - c\,g(x))^{\frac{q}{p}} w(x)dx\right)^{\frac{p}{q}} \leqslant \\ &\leqslant \left(\int_a^b f^q(x)w(x)dx\right)^{\frac{1}{q}} + \left(\int_a^b g^q(x)w(x)dx\right)^{\frac{1}{q}} \leqslant \\ &\leqslant \frac{m+\alpha}{\alpha(m-c)}(b-a)^{\frac{1-p}{q}}\left(\int_a^b (\alpha f(x) - cg(x))^q w(x)dx\right)^{\frac{1}{q}}. \end{aligned} \tag{20}$$

Proof. Firstly let $1 \leqslant p \leqslant q$, from the inequality (10), we have

$$\left[\frac{M}{\alpha(M-c)}(\alpha f(x) - c\,g(x))\right]^{\frac{q}{p}} w(x) \leqslant f^{\frac{q}{p}}(x)w(x),$$

and

$$f^q(x)w(x) \leqslant \left[\frac{m}{\alpha(m-c)}(\alpha f(x) - c\,g(x))\right]^q w(x),$$

Integrating the above inequalities on $[a, b]$, we get

$$\left(\frac{M}{\alpha(M-c)}\right)^{\frac{q}{p}} \int_a^b (\alpha f(x) - c\,g(x))^{\frac{q}{p}} w(x)dx \leqslant \int_a^b f^{\frac{q}{p}}(x)w(x)dx, \tag{21}$$

and

$$\left(\int_a^b f^q(x)w(x)dx\right)^{\frac{1}{q}} \leqslant \frac{m}{\alpha(m-c)}\left(\int_a^b (\alpha f(x) - c\,g(x))^q w(x)dx\right)^{\frac{1}{q}}, \tag{22}$$

apply the Jensen inequality (7) for $\lambda = \dfrac{1}{p}$, hence from the inequality (21), we get

$$\begin{aligned} \left(\frac{M}{\alpha(M-c)}\right)^{\frac{q}{p}} \int_a^b (\alpha f(x) - c\,g(x))^{\frac{q}{p}} w(x)dx &\leqslant \int_a^b f^{\frac{q}{p}}(x)w(x)dx \leqslant \\ &\leqslant (b-a)^{1-\frac{1}{p}}\left(\int_a^b f^q(x)w(x)dx\right)^{\frac{1}{p}}, \end{aligned}$$

this give us

$$\frac{M}{\alpha(M-c)}(b-a)^{\frac{1-p}{q}}\left(\int_a^b (\alpha f(x) - c\,g(x))^{\frac{q}{p}} w(x)dx\right)^{\frac{p}{q}} \leqslant \left(\int_a^b f^q(x)w(x)dx\right)^{\frac{1}{q}}. \tag{23}$$

In another case, from the inequality (14) we result

$$g^q(x)w(x) \leqslant \left[\frac{1}{m-c}(\alpha f(x) - c\,g(x))\right]^q w(x),$$

and

$$\left[\frac{1}{M-c}(\alpha f(x) - c\,g(x))\right]^{\frac{q}{p}} w(x) \leqslant g^{\frac{q}{p}}(x)w(x),$$

integrating on $[a, b]$, we get

$$\left(\int_a^b g^q(x)w(x)dx\right)^{\frac{1}{q}} \leqslant \frac{1}{m-c}\left(\int_a^b (\alpha f(x) - c\,g(x))^q w(x)dx\right)^{\frac{1}{q}}, \tag{24}$$

and

$$\left(\frac{1}{M-c}\right)^{\frac{q}{p}} \int_a^b (\alpha f(x) - c\,g(x))^{\frac{q}{p}} w(x)dx \leqslant \int_a^b g^{\frac{q}{p}}(x)w(x)dx, \tag{25}$$

use Jensen integral inequality (7) and (25), we obtain

$$\frac{1}{M-c}(b-a)^{\frac{1-p}{q}} \left(\int_a^b (\alpha f(x) - c\,g(x))^{\frac{q}{p}} w(x)dx\right)^{\frac{p}{q}} \leqslant \left(\int_a^b g^q(x)w(x)dx\right)^{\frac{1}{q}}. \tag{26}$$

By the inequalities (22), (24) and (23), (26) we result the inequality (19).

Secondly let $1 \leqslant p \leqslant q$, from the inequality (10) we deduce that

$$\left[\frac{M}{\alpha(M-c)}(\alpha f(x) - c\,g(x))\right]^q w(x) \leqslant f^q(x)w(x),$$

and

$$f^{\frac{q}{p}}(x)w(x) \leqslant \left[\frac{m}{\alpha(m-c)}(\alpha f(x) - c\,g(x))\right]^{\frac{q}{p}} w(x),$$

Integrating the above inequalities on $[a, b]$, we get

$$\frac{M}{\alpha(M-c)}\left(\int_a^b (\alpha f(x) - c\,g(x))^q w(x)dx\right)^{\frac{1}{q}} \leqslant \int_a^b f^q(x)w(x)dx, \tag{27}$$

and

$$\int_a^b f^{\frac{q}{p}}(x)w(x)dx \leqslant \left(\frac{m}{\alpha(m-c)}\right)^{\frac{q}{p}} \int_a^b (\alpha f(x) - c\,g(x))^{\frac{q}{p}} w(x)dx, \tag{28}$$

apply the Jensen inequality (6) for $\lambda = \dfrac{1}{p}$, hence from the inequality (27), we get

$$\begin{aligned}(b-a)^{1-\frac{1}{p}}\left(\int_a^b f^q(x)w(x)dx\right)^{\frac{1}{p}} &\leqslant \int_a^b f^{\frac{q}{p}}(x)w(x)dx \leqslant \\ &\leqslant \left(\frac{m}{\alpha(m-c)}\right)^{\frac{q}{p}} \int_a^b (\alpha f(x) - c\,g(x))^{\frac{q}{p}} w(x)dx,\end{aligned}$$

this give us

$$\left(\int_a^b f^q(x)w(x)dx\right)^{\frac{1}{q}} \leqslant \frac{m}{\alpha(m-c)}(b-a)^{\frac{1-p}{q}} \left(\int_a^b (\alpha f(x) - c\,g(x))^{\frac{q}{p}} w(x)dx\right)^{\frac{p}{q}}. \tag{29}$$

In another case, from the inequality (14), we deduce that

$$\left[\frac{1}{M-c}(\alpha f(x) - c\,g(x))\right]^q w(x) \leqslant g^q(x)w(x),$$

and

$$g^{\frac{q}{p}}(x)w(x) \leqslant \left[\frac{1}{M-c}(\alpha f(x) - c\,g(x))\right]^{\frac{q}{p}} w(x),$$

integrating on $[a,b]$, we get

$$\frac{1}{M-c}\left(\int_a^b (\alpha f(x) - c\,g(x))^q w(x)dx\right)^{\frac{1}{q}} \leqslant \left(\int_a^b g^q(x)w(x)dx\right)^{\frac{1}{q}}, \tag{30}$$

and

$$\int_a^b g^{\frac{q}{p}}(x)w(x)dx \leqslant \left(\frac{1}{m-c}\right)^{\frac{q}{p}} \int_a^b (\alpha f(x) - c\,g(x))^{\frac{q}{p}} w(x)dx, \tag{31}$$

use the Jensen inequality (6) and (31), we obtain

$$\left(\int_a^b g^q(x)w(x)dx\right)^{\frac{1}{q}} \leqslant \frac{1}{m-c}(b-a)^{\frac{1-p}{q}} \left(\int_a^b (\alpha f(x) - c\,g(x))^{\frac{q}{p}} w(x)dx\right)^{\frac{p}{q}}. \tag{32}$$

By the inequalities (28), (30) and (29), (32) we result the inequality (20). □

3. Application

We now give some new results of the above Theorems.

3.1. Reverse Minkowski weight type inequality

Put $p = q$ in the Theorem 1 and $p = 1$ in the Theorem 2, we get the following corollary.

Corollary 1. *Let f, $g > 0$, $q > 0$, $\alpha > 0$, w be a weight function and*

$$0 < c < m \leqslant \frac{\alpha f(x)}{g(x)} \leqslant M \quad for\ all\ x \in [a,b],$$

then

$$\begin{aligned} &\frac{M+\alpha}{\alpha(M-c)}\left(\int_a^b (\alpha f(x) - cg(x))^q w(x)dx\right)^{\frac{1}{q}} \leqslant \\ &\leqslant \left(\int_a^b f^q(x)w(x)dx\right)^{\frac{1}{q}} + \left(\int_a^b g^q(x)w(x)dx\right)^{\frac{1}{q}} \leqslant \\ &\leqslant \frac{m+\alpha}{\alpha(m-c)}\left(\int_a^b (\alpha f(x) - cg(x))^q w(x)dx\right)^{\frac{1}{q}}. \end{aligned} \tag{33}$$

3.2. Reverse Minkowski type inequality

Using $w \equiv 1$ in Theorem 1 and Theorem 2, we get the following corollaries.

Corollary 2. *Let f, $g > 0$, $0 < p \leqslant q$, $\alpha > 0$ and*

$$0 < c < m \leqslant \frac{\alpha f(x)}{g(x)} \leqslant M \quad for\ all\ x \in [a,b],$$

then

$$\begin{aligned}
&\frac{M+\alpha}{\alpha(M-c)}(b-a)^{\frac{p-q}{pq}}\left(\int_a^b(\alpha f(x)-cg(x))^p dx\right)^{\frac{1}{p}} \leqslant \\
&\leqslant\left(\int_a^b f^q(x)dx\right)^{\frac{1}{q}}+\left(\int_a^b g^q(x)dx\right)^{\frac{1}{q}} \leqslant \\
&\leqslant \frac{m+\alpha}{\alpha(m-c)}\left(\int_a^b(\alpha f(x)-cg(x))^q dx\right)^{\frac{1}{q}}.
\end{aligned} \tag{34}$$

Corollary 3. *Let* $f, g>0,\ \alpha>0$ *and*

$$0<c<m \leqslant \frac{\alpha f(x)}{g(x)} \leqslant M \quad for\ all\ x \in[a, b],$$

then, for $1<p \leqslant q$

$$\begin{aligned}
&\frac{M+\alpha}{\alpha(M-c)}(b-a)^{\frac{1-p}{q}}\left(\int_a^b(\alpha f(x)-c\, g(x))^{\frac{q}{p}} dx\right)^{\frac{p}{q}} \leqslant \\
&\leqslant\left(\int_a^b f^q(x)dx\right)^{\frac{1}{q}}+\left(\int_a^b g^q(x)dx\right)^{\frac{1}{q}} \leqslant \\
&\leqslant \frac{m+\alpha}{\alpha(m-c)}\left(\int_a^b(\alpha f(x)-cg(x))^q dx\right)^{\frac{1}{q}}.
\end{aligned} \tag{35}$$

for $0<q \leqslant p \leqslant 1$

$$\begin{aligned}
&\frac{M+\alpha}{\alpha(M-c)}\left(\int_a^b(\alpha f(x)-c\, g(x))^{\frac{q}{p}} dx\right)^{\frac{p}{q}} \leqslant \\
&\leqslant\left(\int_a^b f^q(x)dx\right)^{\frac{1}{q}}+\left(\int_a^b g^q(x)dx\right)^{\frac{1}{q}} \leqslant \\
&\leqslant \frac{m+\alpha}{\alpha(m-c)}(b-a)^{\frac{1-p}{q}}\left(\int_a^b(\alpha f(x)-cg(x))^q dx\right)^{\frac{1}{q}}.
\end{aligned} \tag{36}$$

The inequalities (34), (35) and (36) are new generalizations of the revers Minkowski inequality with two parameters.

Conclusion

By using Hölder's inequality, Jensen's integral inequality and by introducing two parameters of integrability, new generalizations of the inverse of Minkowski's integral inequality have been established and demonstrated. Two results are given in the application section, the reverse Minkowski weight type inequality and we deduce a particular case the reverse Minkowski type inequality, this is a new generalization of the classic reverse Minkowski inequality known in the literature.

References

[1] B.Benaissa, More on reverses of Minkowski's inequalities and Hardy's integral inequalities, *Asian-Eur. J. Math.*, **13**(2020), no. 3, 1–7. DOI: 10.1142/S1793557120500643

[2] W.T.Sulaiman, Reverses of Minkowski's, Hölder's, and Hardy's integral inequalities, *Int. J. Mod. Math. Sci.*, **1**(2012), no. 1, 14–24.

[3] B.Sroysang, More on Reverses of Minkowski's Integral Inequality, *Mathematica Aeterna*, **3**(2013), no. 7, 597–600.

[4] S.Bobkov, M.Madiman, Reverse Brunn–Minkowski and reverse entropy power inequalities for convex measures, *J. Func. Anal.*, **262**(2012), 3309–3339.

[5] G.Rahman et al, The Minkowski inequalities via generalized proportional fractional integral operators, *Adv. Difference Equ.*, **2019**(2019), no. 287. DOI: 10.1186/s13662-019-2229-7

[6] S.Rashid, F.Jarad, Yu-M.Chu, A Note on Reverse Minkowski Inequality via Generalized Proportional Fractional Integral Operator with respect to Another Function, *Math. Probl. Eng.*, **2020**(2020), 1–12. DOI: 10.1155/2020/7630260

[7] S.Rashid et al, New generalized reverse Minkowski and related integral inequalities involving generalized fractional conformable integrals, *J. Inequal. Appl.*, **2020**(2020), no. 177. DOI: 10.1186/s13660-020-02445-2

Дальнейшее обобщение обратного неравенства типа Минковского с помощью неравенств Гельдера и Йенсена

Буаркет Бенаисса
Лаборатория информатики и математики
Факультет материаловедения
Тиаретский университет
Алжир, Алжир

Аннотация. Основная цель этой статьи — установить новые обобщения обратных интегральных неравенств Минковского путем введения весовых функций и двух параметров интегрируемости. Будут доказаны две новые теоремы с использованием интегрального неравенства Йенсена и двухпараметрического неравенства Гельдера, а также получены некоторые обратные интегральные неравенства типа Минковского.

Ключевые слова: выпуклая функция, неравенство Гельдера, неравенство Минковского, неравенство Йенсена.